Christian Ullerich

Advanced Disassembly Planning

Flexible, Price-Quantity Dependent, and Multi-Period Planning Approaches

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Christian Ullerich
TU Dresden, Germany

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To my parents

Gerhard and Karin

Foreword

Scarce natural resources, legal requirements, and cost savings are some of the reasons why product recovery is given increased attention. Disassembly processes are obviously an essential aspect of product recovery and, therefore, the corresponding planning is crucial. It turns out that this planning can be applied not only to product recovery, but also to repair and maintenance. In this remarkable thesis, Christian Ullerich chooses a managerial economics perspective and concentrates on disassembly-to-order planning. He develops comprehensive mathematical models reflecting the disassembly process, in order to determine the optimal number of cores to be procured and to specify the optimal types and quantities of modules, items, and material to be sold.

The thesis is divided into two main parts: complete disassembly planning and flexible disassembly planning. In the first main part he develops a new consistent model which integrates e.g. conditions of cores, purity requirements for material recycling, and demand restrictions. In contrast to existing models, this linear model is extended by considering linear price-quantity dependencies, with the result, that a non-linear problem arises. To solve the problem Christian Ullerich combines modified non-linear methods and commercial solvers in an innovative and impressive manner. Moreover, the initial static model is extended to a dynamic approach by taking multiple periods into account. A newly developed rolling horizon planning is applied and the excellent solution quality is proven by a comparison with the solution gained from a total planning approach.

In the second main part a completely new disassembly model is presented. In contrast to the complete disassembly planning, several modules can be maintained. The question arises to what extent the cores have to be disassembled (disassembly depth) and which disassembly sequence has to be taken. The already existing simultaneous approaches are significantly extended by Christian Ullerich. The new idea is that for each type of a core more than one disassembly state is allowed. That means that one type of a core could be disassembled in different ways which offers a completely new degree of freedom. Since this new approach includes different disassembly models as special cases, Christian Ullerich denotes this model as flexible planning. Further, he provides different recommendations to solve the problem heuristically in order to cope with the high model complexity.

Summarising the above, it should be noted that the disassembly research is considerably enriched by a lot of new and future-oriented ideas and methods. Therefore, I hope that Christian Ullerichs excellent dissertation finds large distribution.

Prof. Dr. Udo Buscher

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Acronyms

BILP	Binary Integer Linear Program
DfA	Design for Assembly
DfD	Design for Disassembly
DfE	Design for Environment
DfR	Design for Recycling
ILP	Integer Linear Program
LP	Linear Program
MIP	Mixed Integer Program
MILP	Mixed Integer Linear Program
MIQLP	Mixed Integer Quadratic Program with Linear Constraints
$\mathcal{NP}$	Non-deterministic Polynomial-time
QLP	Quadratic Program with Linear Constraints
QLLP	Quadratic and Linear Program with Linear Constraints
QPCO	Quadratic Program with Constrained Optimum
QPCOS	Quadratic Program with Constrained Optimum and Sections
QPUO	Quadratic Program with Unconstrained Optimum

List of Symbols

Chapter 2 and Appendix A

D	item demand [unit
D_j	demand of item [unit]
DR	recycling demand [kg]
g_3	goal
i	core index
j	item index
L	disposal quantity [unit]
NRC	number of items recycled
NRU	number of items reused
PRC	recyclable percentage
R	recycling quantity [unit]
RQ	recycling material quantity [kg]
V	storage quantity [unit]
W	weight [kg/unit]
X	reuse quantity [unit]
X_{ij}	reuse quantity of item of core [unit]
Y	quantity of a core [unit]
α_j	defective rate of item
β_j	damaging rate of item
γ	fraction

γ_j	replacement rate of item
δ	fraction
λ	objective value
$\mu_3(g_3)$	achievement level function

Chapter 3 and Appendix B

a	arbitrary variable
$\mathbf{B}$	arbitrary matrix
b	arbitrary variable
$\mathbf{b}$	arbitrary vector
C	cost [€]
C_t	cost in period [€]
C_t^{S}	shortage cost in period [€]
C_t^{V}	inventory holding cost in period [€]
c	core index and arbitrary variable
$\mathbf{c}$	coefficient vector
c_c^{A}	acquisition unit cost [€/unit]
c_{tc}^{A}	acquisition unit cost in period [€/unit]
$\bar{c}_c^{\mathrm{A}}$	fix acquisition unit cost [€/unit]
$\hat{c}_c^{\mathrm{A}}$	marginal acquisition unit cost [€/unit^2]
$\hat{c}_{cs}^{\mathrm{A}}$	marginal acquisition unit cost of section [€/unit^2]
c_d^{D}	disposal unit cost [€/kg]
c_{td}^{D}	disposal unit cost in period [€/kg]
$\bar{c}_d^{\mathrm{D}}$	fix disposal unit cost [€/kg]
$\hat{c}_d^{\mathrm{D}}$	marginal disposal unit cost [€/kg^2]
$\hat{c}_{ds}^{\mathrm{D}}$	marginal disposal unit cost of section [€/kg^2]
c_c^{J}	disassembly unit cost [€/unit]
c_{ci}^{JF}	destructive disassembly cost factor [€/unit]
c_{ci}^{JN}	non-destructive disassembly cost factor [€/unit]
$\mathbf{D}$	arbitrary matrix
D^{I}	demand of items to reuse [unit]

D^{I}_{e}	demand of items to reuse [unit]
D^{I}_{te}	contracted quantity in period of items to reuse [unit]
D^{R}	demand of material to recycle [kg]
D^{R}_{r}	demand of material to recycle [kg]
D^{R}_{tr}	contracted quantity in period of material to recycle [kg]
d	disposal bin index
e	demand position of items
$f(x)$	arbitrary function
$g(x)$	arbitrary function
$\mathcal{H}$	set of hazardous core item combination
$\mathbf{h}$	arbitrary vector
$\mathbf{h}_i$	arbitrary vector
h^{C}_{c}	inventory holding unit cost of core [€/unit/period]
h^{D}_{d}	inventory holding unit cost of disposal [€/kg/period]
h^{I}_{e}	inventory holding unit cost of item [€/unit/period]
h^{R}_{r}	inventory holding unit cost of recycling [€/kg/period]
$h(x)$	arbitrary function
$\bar{I}_c$	number of items of core
i	item index and arbitrary index
j	index
$\bar{L}$	labour time limitation [h]
m	arbitrary variable
n	number of items and arbitrary variable
$\mathbf{n}$	normal vector
$\mathbf{o}$	vector
P	profit [€]
$\mathcal{P}_e$	demand sets of core item combinations
P_t	profit in period [€]
$P(\mathbf{x})$	profit function of vector $\mathbf{x}$
p	number of periods in the past
Q	arbitrary quantity
Q_s	quantity of section
$\check{Q}_s$	quantity limit of section

Q^{C}	quantity of a core [unit]
Q^{C}_{c}	quantity of a core [unit]
Q^{C}_{cs}	quantity of a core of section [unit]
Q^{C}_{tc}	quantity of a core of period [unit]
$\overline{Q}^{\mathrm{C}}_{c}$	upper quantity limit of a core [unit]
$\overline{Q}^{\mathrm{C}}_{tc}$	contracted quantity in period of a core [unit]
$\check{Q}^{\mathrm{C}}_{cs}$	quantity limit of a core of section [unit]
$\widetilde{Q}^{\mathrm{C}}_{tc}$	quantity of a core of period [unit]
$\underline{Q}^{\mathrm{C}}_{c}$	lower quantity limit of a core [unit]
Q^{D}	quantity of material to dispose [kg]
Q^{D}_{d}	quantity of material to dispose [kg]
Q^{D}_{ds}	quantity of material to dispose of section [kg]
Q^{D}_{td}	quantity of material to dispose in period [kg]
$\overline{Q}^{\mathrm{D}}_{d}$	upper quantity limit of material to dispose [kg]
$\overline{Q}^{\mathrm{D}}_{td}$	contracted quantity in period of material to dispose [kg]
$\check{Q}^{\mathrm{D}}_{ds}$	quantity limit of material to dispose of section [kg]
$\widetilde{Q}^{\mathrm{D}}_{td}$	quantity of material to dispose in period [kg]
$\underline{Q}^{\mathrm{D}}_{d}$	lower quantity limit of material to dispose [kg]
Q^{I}	quantity of item to reuse [unit]
Q^{I}_{e}	quantity of item to reuse [unit]
Q^{I}_{es}	quantity of item to reuse of section [unit]
Q^{I}_{te}	quantity of item to reuse in period [unit]
$\check{Q}^{\mathrm{I}}_{es}$	quantity limit of item to reuse of section [unit]
$\widetilde{Q}^{\mathrm{I}}_{te}$	quantity of item to reuse in period [unit]
$\underline{Q}^{\mathrm{I}}_{e}$	lower quantity limit of item to reuse [unit]
$\widetilde{Q}^{\mathrm{I,expost}}_{te}$	quantity of item to reuse in period [unit]
Q^{opt}_{s}	optimal quantity of section
Q^{R}	quantity of material to recycle [kg]
Q^{R}_{r}	quantity of material to recycle [kg]
Q^{R}_{rs}	quantity of material to recycle of section [kg]
Q^{R}_{tr}	quantity of material to recycle in period [kg]
$\check{Q}^{\mathrm{R}}_{rs}$	quantity limit of material to recycle of section [kg]
$\widetilde{Q}^{\mathrm{R}}_{tr}$	quantity of material to recycle in period [kg]

$\underline{Q}_r^{\mathrm{R}}$	lower quantity limit of material to recycle [kg]
R	revenues [€]
R_t	revenues in period [€]
r	recycling box index and arbitrary price
$\bar{r}$	basic price
$\hat{r}_s$	marginal price
$r(Q)$	price function
r_e^{I}	price of item to reuse [€/unit]
r_{te}^{I}	price of item to reuse in period [€/unit]
$\bar{r}_e^{\mathrm{I}}$	fix price of item to reuse [€/unit]
$\hat{r}_e^{\mathrm{I}}$	marginal price of item to reuse [€/unit^2]
$\hat{r}_{es}^{\mathrm{I}}$	marginal price of item to reuse in section [€/unit^2]
$r_e^{\mathrm{I}}(Q_e^{\mathrm{I}})$	price function of item to reuse [€/unit]
r_r^{R}	price of recycling material [€/kg]
r_{tr}^{R}	price of recycling material in period [€/kg]
$\bar{r}_r^{\mathrm{R}}$	fix price of recycling material [€/kg]
$\hat{r}_r^{\mathrm{R}}$	marginal price of recycling material [€/kg^2]
$\hat{r}_{rs}^{\mathrm{R}}$	marginal price of recycling material of section [€/kg^2]
s	section index and study horizon length
$\mathbf{s}$	direction vector
$\tilde{s}$	section index
T	number of periods
$\widetilde{T}$	number of periods
$\widetilde{T}'$	number of periods
t	period index
t_c^{J}	disassembly unit time [h/unit]
t_{ci}^{JF}	destructive disassembly time [h/unit]
t_{ci}^{JN}	non-destructive disassembly time [h/unit]
$\bar{V}^1$	inventory limit [space units]
$\bar{V}^2$	inventory limit [space units]
$\bar{V}^3$	inventory limit [space units]
V_{tc}^{C}	inventory of core [unit]
V_{td}^{D}	inventory of disposal material [kg]

V^{I}_{te}	inventory of demand position [unit]
V^{R}_{tr}	inventory of recycling material [kg]
w	weight [kg/unit]
w_{ci}	weight [kg/unit]
X^{D}	number of items assigned to disposal [unit]
X^{D}_{cid}	number of items assigned to disposal [unit]
X^{D}_{tcid}	number of items assigned to disposal in period [unit]
X^{F}	number of non-destructively disassembled items [unit]
X^{F}_{ci}	number of non-destructively disassembled items [unit]
X^{I}	number of items assigned to reuse [unit]
X^{I}_{ci}	number of items assigned to reuse [unit]
X^{I}_{tci}	number of items assigned to reuse in period [unit]
X^{N}	number of destructively disassembled items [unit]
X^{N}_{ci}	number of destructively disassembled items [unit]
X^{R}	number of items assigned to recycling [unit]
X^{R}_{cir}	number of items assigned to recycling [unit]
X^{R}_{tcir}	number of items assigned to recycling in period [unit]
x	arbitrary variable
$\mathbf{x}$	arbitrary vector/point
x_i	arbitrary variable
$\mathbf{x}_i$	arbitrary vector/point
$\mathbf{x}_{\mathrm{opt}}$	optimal vector/point
$\hat{\mathbf{x}}$	vector/point
$\mathbf{x}^{\mathrm{B}}_j$	points on section border
$\mathbf{x}^{\mathrm{int}}_{\mathrm{opt}}$	optimal integer vector/point
y	arbitrary variable
$\mathbf{y}$	arbitrary vector
z	arbitrary variable and discount factor
$z(x)$	arbitrary objective function
$\mathbb{Z}$	domain of integer numbers
$\mathbb{Z}^*$	domain of non-negative integer numbers
α	factor
β	factor

β_c	guaranteed level of core acquisition
ϵ	small value, maximal underrun of purity level, and arbitrary variable
$\epsilon(s, p)$	arbitrary function
ζ_{ci}	non-genuine probability
η_{ci}	defective probability
θ_{ci}	damaging probability
ι_{ci}	wrong material probability
λ	arbitrary factor
ν_c^{C}	storage usage factor for core
ν_d^{D}	storage usage factor for disposal
ν_e^{I}	storage usage factor for item
ν_r^{R}	storage usage factor for recycling
π_{cir}	beneficial fraction
ρ	step size
ρ_j^{B}	step size to section border
σ^{C}	shortage cost factor for core
σ^{D}	shortage cost factor for disposal
σ^{I}	shortage cost factor for item
σ^{R}	shortage cost factor for recycling
τ	period
$\bar{\tau}$	period
$\mathcal{I}_r$	number of periods in the past
ω	purity level
ω_r	purity requirement
$\bar{\omega}_t$	average purity level
$\nabla z(x)$	gradient of $z(x)$
$\otimes$	selective combination operator

Chapter 4 and Appendix C

$\mathbf{A}$	coefficient matrix

A_{js}	coefficient matrix elements
a	arbitrary variable
b	arbitrary variable
C	cost [€]
c	core index and coefficient vector
$\tilde{c}$	core index
c_c^{A}	acquisition unit cost [€/unit]
c_d^{D}	disposal unit cost [€/kg]
c_c^{J}	disassembly unit cost [€/unit]
c_{cm}^{J}	disassembly unit cost of module [€/unit]
D^{I}	demand of items to reuse [unit]
D_e^{I}	demand of items to reuse [unit]
D^{M}	demand of modules to reuse [unit]
D_f^{M}	demand of modules to reuse [unit]
D^{R}	demand of material to recycle [kg]
D_r^{R}	demand of material to recycle [kg]
d	disposal bin index
$E_{v\tilde{v}}^{\mathrm{C}}$	definition of edge in core graph from node to node
$E_{cv\tilde{v}}^{\mathrm{C}}$	definition of edge in core graph from node to node of core
$E_{w\tilde{w}}^{\mathrm{D}}$	definition of edge in distribution graph from node to node
$E_{cw\tilde{w}}^{\mathrm{D}}$	definition of edge in distribution graph from node to node of core
$E_{w\tilde{w}}^{\mathrm{I}}$	definition of edge in distribution graph from node to node
$E_{w\tilde{w}}^{\mathrm{R}}$	definition of edge in recycling graph from node to node
e	demand position of items
f	demand position of modules
$\mathcal{H}$	set of hazardous core item combination
$\bar{I}$	number of items of core
$\bar{I}_c$	number of items of core
i	item index
$\tilde{J}$	set of rows
j	index
k	arbitrary variable

$\bar{L}$	labour time limitation [h]
L_w^{A}	set of module item indices for disposal, distribution, or recycling graph node
L_{cw}^{A}	set of module item indices for disposal, distribution, or recycling graph node of core
L_w^{D}	set of core graph nodes for disposal graph node
L_{cw}^{D}	set of core graph nodes for disposal graph node of core
L_w^{I}	set of core graph nodes for distribution graph node
L_{cw}^{I}	set of core graph nodes for distribution graph node of core
L_w^{R}	set of core graph nodes for recycling graph node
L_{cw}^{R}	set of core graph nodes for recycling graph node of core
M	arbitrary large value
$\overline{M}_c$	number of modules of core
m	module index
$\tilde{m}$	module index
n	arbitrary variable and unit index
P	profit [€]
$\mathcal{P}_e$	demand sets of core item combinations
$P(\cdot)$	probability function
Q^{C}	quantity of a core [unit]
Q_c^{C}	quantity of a core [unit]
$\overline{Q}_c^{\mathrm{C}}$	upper quantity limit of a core [unit]
$\underline{Q}_c^{\mathrm{C}}$	lower quantity limit of a core [unit]
Q^{D}	quantity of material to dispose [kg]
Q_d^{D}	quantity of material to dispose [kg]
$\overline{Q}_d^{\mathrm{D}}$	upper quantity limit of material to dispose [kg]
$\underline{Q}_d^{\mathrm{D}}$	lower quantity limit of material to dispose [kg]
Q^{I}	quantity of item to reuse [unit]
Q_e^{I}	quantity of item to reuse [unit]
$\underline{Q}_e^{\mathrm{I}}$	lower quantity limit of item to reuse [unit]
Q^{M}	quantity of module to reuse [unit]
Q_f^{M}	quantity of module to reuse [unit]
$\underline{Q}_f^{\mathrm{M}}$	lower quantity limit of module to reuse [unit]

Q^{R}	quantity of material to recycle [kg]
Q^{R}_r	quantity of material to recycle [kg]
$\underline{Q}^{\mathrm{R}}_r$	lower quantity limit of material to recycle [kg]
Q^{S}_s	state quantity
Q^{S}_{cs}	state quantity
q^{I}_{iu}	planned quantity of item and usage
q^{M}_{mu}	planned quantity of module and usage
$\mathbf{q}^{\mathrm{S}}$	state quantity vector
q^{S}_s	planned state quantity
R	revenues [€]
$\mathbb{R}$	domain of real numbers
$\mathcal{R}_f$	demand sets of core module combinations
R_j	elements of right hand side vector
r	recycling box index
$\mathbf{r}$	right hand side vector
r^{I}_e	price of item to reuse [€/unit]
r^{M}_f	price of module to reuse [€/unit]
r^{R}_r	price of recycling material [€/kg]
S	set of state indices
$\bar{S}_c$	number of states per core
$\tilde{S}$	set of state indices
$\tilde{S}_c$	allowed number of states per core
s	state index
$\tilde{s}$	state index
t_{ni}	tested item condition of unit
t^{J}_{cm}	disassembly unit time [h/unit]
U_{cs}	state selection variable
U_{ns}	unit state selection variable
u	number of usage categories and usage index
V^{C}_v	output of core graph at node
V^{C}_{cv}	output of core graph at node of core
v	node index
$\tilde{v}$	node index

w	node index
w_{ci}	weight [kg/unit]
$\hat{w}$	node index
$\tilde{w}$	node index
X_i	number of item [unit]
X_{niu}	item usage
X_i^{A}	number of damaged items [unit]
X_{ci}^{A}	number of damaged items [unit]
X^{D}	number of items assigned to disposal [unit]
X_i^{D}	number of items assigned to disposal [unit]
X_{id}^{D}	number of items assigned to disposal [unit]
X_{cid}^{D}	number of items assigned to disposal [unit]
$\widetilde{X}_i^{\mathrm{D}}$	number of items assigned to disposal [unit]
$\widetilde{X}_{ci}^{\mathrm{D}}$	number of items assigned to disposal [unit]
X^{I}	number of items assigned to reuse [unit]
X_i^{I}	number of items assigned to reuse [unit]
X_{ci}^{I}	number of items assigned to reuse [unit]
$\dot{X}_{\tilde{c}i}^{\mathrm{I}}$	number of items assigned to reuse [unit]
$\widetilde{X}_i^{\mathrm{I}}$	number of items with reuse condition [unit]
X^{R}	number of items assigned to recycling [unit]
X_i^{R}	number of items assigned to recycling [unit]
X_{ir}^{R}	number of items assigned to recycling [unit]
X_{cir}^{R}	number of items assigned to recycling [unit]
$\dot{X}_{\tilde{c}ir}^{\mathrm{R}}$	number of items assigned to recycling [unit]
$\widetilde{X}_i^{\mathrm{R}}$	number of items assigned to recycling [unit]
$\mathbf{x}$	vector of number of items
Y_m	number of module [unit]
Y_{nmu}	module usage
Y^{D}	number of modules assigned to disposal [unit]
Y_m^{D}	number of modules assigned to disposal [unit]
Y_{md}^{D}	number of modules assigned to disposal [unit]
Y_{cmd}^{D}	number of modules assigned to disposal [unit]
$\dot{Y}_{\tilde{c}md}^{\mathrm{D}}$	number of modules assigned to disposal [unit]

Y^{M}	number of modules assigned to reuse [unit]
Y^{M}_{m}	number of modules assigned to reuse [unit]
Y^{M}_{cm}	number of modules assigned to reuse [unit]
$\dot{Y}^{\mathrm{M}}_{\tilde{c}m}$	number of modules assigned to reuse [unit]
Y^{R}	number of modules assigned to recycling [unit]
Y^{R}_{m}	number of modules assigned to recycling [unit]
Y^{R}_{mr}	number of modules assigned to recycling [unit]
Y^{R}_{cmr}	number of modules assigned to recycling [unit]
$\dot{Y}^{\mathrm{R}}_{\tilde{c}mr}$	number of modules assigned to recycling [unit]
$\mathbf{y}$	vector of number of modules
$Z^{\mathrm{C}}_{v\tilde{v}}$	flow between nodes in core graph
$Z^{\mathrm{C}}_{cv\tilde{v}}$	flow between nodes in core graph of core
$Z^{\mathrm{D}}_{w\tilde{w}}$	flow between nodes in disposal graph
$Z^{\mathrm{D}}_{cw\tilde{w}}$	flow between nodes in disposal graph of core
$Z^{\mathrm{I}}_{w\tilde{w}}$	flow between nodes in distribution graph
$Z^{\mathrm{I}}_{cw\tilde{w}}$	flow between nodes in distribution graph of core
$Z^{\mathrm{R}}_{w\tilde{w}}$	flow between nodes in recycling graph
$Z^{\mathrm{R}}_{cw\tilde{w}}$	flow between nodes in recycling graph of core
$\mathbb{Z}$	domain of integer numbers
$\mathbb{Z}^*$	domain of non-negative integer numbers
α	arbitrary value
α_{cmi}	additional item
γ^{I}_{is}	state item definition
$\gamma^{\mathrm{I}}_{cis}$	state item definition
γ^{M}_{ms}	state module definition
$\gamma^{\mathrm{M}}_{cms}$	state module definition
δ_{mi}	module definition
δ_{cmi}	module definition
$\epsilon^{\mathrm{I}}_{iu}$	expected quantity of item and usage
$\epsilon^{\mathrm{M}}_{mu}$	expected quantity of module and usage
ζ	non-genuine probability
ζ_{i}	non-genuine probability
ζ_{ci}	non-genuine probability

η	defective probability
η_i	defective probability
η_{ci}	defective probability
θ	damaging probability
θ_i	damaging probability
θ_{ci}	damaging probability
ι	wrong material probability
ι_i	wrong material probability
ι_{ci}	wrong material probability
π_{cir}	beneficial fraction
$\pi^{\mathrm{I}}(i,u)$	priority value function of item and usage index
$\pi^{\mathrm{M}}(m,u)$	priority value function of module and usage index
$\pi^{\mathrm{S}}(s)$	priority value function of a state
ρ_v	percentage of incoming units at this node
ρ_{cv}	percentage of incoming units at this node of core
$\tilde{\rho}_v$	percentage of incoming units at this node
$\tilde{\rho}_{cv}$	percentage of incoming units at this node
$\sigma^{\mathrm{I}}(i,u)$	scarcity value function of item and usage index
$\sigma^{\mathrm{M}}(m,u)$	scarcity value function of module and usage index
ω_r	purity requirement
ω^{M}	module preference weight
ω^{U}_u	usage weight

Chapter 1
Introduction

1.1 Motivation

The product recovery and thus the disassembly process become more and more important these days. The motivation for product recovery is manifold. Typical motives are legal restrictions or environmental guidelines. Examples are the Waste Electrical and Electronic Equipment (WEEE) Directive for electrical and electronic products and the End-of-Life Vehicles Directive. Furthermore, there is also the fact of the environmentally friendly image. So it seems that product recovery becomes part of the competitive strategy of a company. Hence, companies are economically interested in product recovery, which increases the quantities of recovered products and makes a planning even more necessary. In addition, companies aim at maximising their profit or minimising their loss. So if it is possible to gain a profit out of the product recovery, companies should be more than willing to recover products. A further aspect is that manufacturing a new product utilising remanufactured parts causes only 50–60 % of the cost.[1] In addition, disassembly companies do not just exist since today. The existing specialised companies have already operated profitable. Hence, economic reasons exist, too.[2]

Next to the legal requirements and the cost saving, another reason causes the increase of recovery volumes. The natural resources for selected material become scarce, which leads to an increasing price for such material and increased cost for the exploitation. This, on the other hand, makes the product recovery more beneficial so that natural resources and cost are saved. Hence,

[1] Cf. Thierry et al. (1995): *Strategic issues in product recovery*, p. 119.

[2] Cf. Fleischmann et al. (1997): *Quantitative models for reverse logistics*, pp. 2 et seq.

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the quantities for product recovery increase. And this increase of quantities together with the goal to make the recovery process as profitable as possible requires an optimal planning. This applies to any associated research field, and in particular to the disassembly planning.

Disassembly planning is one of the key issues in product recovery. Moreover, disassembly planning is also applicable in repair and maintenance. Thereby, disassembly is the entity of all planned processes that systematically separate a core into modules, items, material and waste.[3] In this context a core is a returned or recovered product. A module or sub-assembly is a set of items, which are connected among each other. An item is the smallest single piece a core or module consists of. We subsume modules and items as parts. When the value added is of interest, i.e., the module and item is demanded for some kind of reuse, we still call them module and item, respectively. If, on the other hand, the value added is not of interest, i.e., only the material value is important, the parts are subsumed as material, even though the parts are entire. Lastly, if not even the material is of interest anymore, the parts are disposed of, which means they are characterised as waste. This way, the planned usage category of a part is already represented by the notation.

The disassembly planning is in the area of conflict of managerial economics, industrial management, engineering, and practice—maybe even more. Depending on the point of interest, different research fields have emerged. These are the disassembly sequencing, disassembly scheduling, disassembly-to-order planning, and disassembly line balancing—to name a few. And as if that were not enough, the influence of the disassembly reaches as far as to the research and development stage of a product. This aspect is considered in the research area of design for disassembly. Basically, aspects of disassembly are omnipresent during the complete life cycle of a product.

From a managerial economics point of view, the main interest is the cost minimisation or profit maximisation. This applies to conceptual models as well as quantitative models where, e.g., supply and distribution quantities are determined. In the context of disassembly planning, the line balancing, scheduling, and disassembly-to-order planning focus such problems. Thereby, the disassembly-to-order planning—and especially the generalised disassembly-to-order planning, which is focussed on in this work—can be seen as somewhat superior to the other planning problems, which makes it interesting as a kind of central planning problem. The other planning prob-

[3] Cf. Brennan / Gupta / Taleb (1994): *Operations planning in assembly/disassembly*, p. 59, Zussman / Zhou (1999): *Methodology for modeling disassembly processes*, p. 190, and Seliger (2011): *Montage und Demontage*, p. S97.

lems like line balancing, scheduling, or sequencing use the results of the disassembly-to-order planning, because disassembly-to-order planning aims at fixing the supply and distribution quantities the other planning problems take as given.

As indicated in the literature, the disassembly planning is not just the reverse of production planning. In the disassembly planning a diverging product structure is dealt with, whereas in production planning a converging product structure is prevalent, i.e., in production planning the construction guidance is given and the result is usually one final product. However, in disassembly planning not only many "products" result from one unit of a core, but, in addition, the decision is necessary how far a core is disassembled, i.e., which "products" are the result of the process. Furthermore, the assembly is not always simply reversible and the value added is lower in disassembly than in assembly.[4]

Moreover, disassembly processes are mainly conducted by humans, i.e., they are labour intensive.[5] This gives the disassembly processes a flexibility that is necessary for a wide range of companies and products. One reason is surely the uncertainty about the core specific condition. Another reason is the fact that companies take back cores of different make and model. This means that it might not be clear which core is recovered, how old the core is, and how many of them are recovered.[6] Nonetheless, this does not mean that automatic (flexible) disassembly should not be enforced. Thinking of the growing quantities the automatic disassembly becomes necessary—surely not for all products, but where it suits. When the disassembly can be conducted very flexible, the planning of the process should account for this flexibility. And especially in the disassembly-to-order planning this flexibility has to be integrated.

A consideration of the flexibility in the disassembly-to-order planning—be it the uncertainty or the individual disassembly depth in incomplete (or partial) disassembly—does not only account for the flexible process to realise the maximal profit. In addition, it might reduce the complexity of subsequent planning problems like scheduling, sequencing, and line balancing. If the disassembly-to-order planning fixes the optimal disassembly states (i.e., a specific set of parts is gained out of a unit of a core), the subsequent planning problems could possibly be transformed to a multi-core complete disassembly problem. This should reduce the size of the subsequent models

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[4] Cf. Lambert (2003): *Disassembly sequencing: A survey*, pp. 3721 et seq.

[5] Cf. Bley et al. (2004): *Human involvement in disassembly*, p. 487, and Kopacek / Kopacek (2006): *Intelligent, flexible disassembly*, p. 554.

[6] Cf. Lambert (2003): *Disassembly sequencing: A survey*, pp. 3721 et seq.

and will most likely speed up the solving, because the incomplete disassembly planning incorporating condition uncertainties as well as the usage options reuse, recycling, and disposal make the planning model complex, which is illustrated in this work.

1.2 Objectives

This work seeks to provide an insight into the disassembly planning. Thereby, facets are considered, that go beyond the considerations which can be found in today's literature. The focus is on a consistent integration of core condition, damaging during the disassembly, hazardous items, and recycling material purity requirements in the planning. Furthermore, there always exist the three usage options (i.e., distribution channels): reuse, material recycling, and disposal. In addition, the supply of cores as well as the distribution of disassembly output, e.g., parts for reuse, material for recycling, and waste, might be limited. Even though, damaging of items during the disassembly is considered, we assume non-destructive disassembly. This does not mean that nothing gets damaged—be it an item or a joint that connects items—, but we believe that because of human labour the differences between destructive and non-destructive disassembly with regard to cost and time are not too significant. In addition, the cores do not have to be disassembled down to every nut and bolt and the destructive disassembly can be expressed as a special case of the non-destructive disassembly,[7] which makes a differentiation possible, even if it is not explicitly considered.

Since most disassembly companies do not just obtain a single core, a multi-core planning is necessary. Otherwise, the influence of a resource allocation for one core on the other cores cannot be reflected satisfactorily. The condition and damaging contain uncertainty, because it is usually not clear which condition a specific unit of a core is in and whether an item gets damaged during the disassembly process or not. We assume that the companies are able to estimate fractions of certain conditions and the damaging. This estimating can be done by educated guessing or by some kind of monitoring. For example, the Stadtreinigung Dresden GmbH (a German sanitation company) monitors the yields of material they get out of clusters of cores. Anyhow, as long as this information is available, it should be

[7] The special case occurs when an item is either always disassembled non-destructively or always destructively. In the case of destructive disassembly the damaging is set to 100 % and the cost factor is adjusted accordingly. If both disassemble methods are possible, the planning for this core has to be transformed into a multi-core planning.

integrated in the planning. Of course, it is usually not detailed enough for a thorough stochastic modelling. Hence, the inclusion of this information, i.e., the fractions, in the deterministic planning makes the consideration of uncertainty possible and we call it quasi-stochastic planning.

With (quasi) stochastic planning the realisation of the stochastic values is not known in advance, so that—especially for small quantities—the realisation can be very different to the expected value. The same applies to the fractions. Nevertheless, they are used for planning in a deterministic model.[8] Once the optimal quantities are gained, there is a link missing to the control of the disassembly process, because for a particular unit of a core the disassembly state (for incomplete disassembly with more than one resulting state per core) and the assignment of the resulting items and modules of that particular unit to a usage option is necessary. All the aforementioned aspects should be considered in disassembly planning.

Speaking of more than one disassembly state per core as a possible result of the disassembly process, leads to the aspect of flexible disassembly planning. Flexible disassembly planning allows several different disassembly states for a core to be planned at the same time. And this should be possible not only for tree-like core structures. Thus, the complete disassembly and the disassembly of tree-like core structures are special cases of the flexible disassembly planning. This planning should still incorporate all above mentioned aspects. With this increased degree of freedom in the planning, a new decision dimension is introduced, i.e., the disassembly depth. Hence, it is expected, that the planning is more difficult compared to the complete disassembly planning. Whether or not this extra effort leads to a plus, has to be revealed.

Next to the flexibility, two further aspects are of interest. One is the planning of quantities over several periods. A predominant approach is multi-period planning with fixed initial and final inventory, e.g., zero inventory. It is believed, that this approach prevents good solutions for a business that continuous a long time, i.e., beyond the planning horizon. In addition, it might be of interest how the planning reacts on data, which changes during the planning of neighbouring periods. Furthermore, it is interesting to know, whether the multi-period planning is suitable for delivering decision support for the contracting in periods ahead.

The second aspect is the consideration of quantity dependent prices and unit cost. Usually fixed prices and unit cost are assumed in the disassembly planning. However, JORJANI / LEU / SCOTT consider a piecewise linear program to find the optimal disassembly policy. Thereby, the piecewise linear

[8] Cf. KONGAR / GUPTA (2006b): *Disassembly to order*.

function is used to model decreasing revenues as the volume available for a usage option (e.g., recycling) increases. Thereby, the prices change in discrete steps.[9] But not all usage options and no supply of cores is considered. Hence, an extension by these and a continuous price or unit cost change for changing quantities is of interest.

The above discussed points shall now be summarised to four research questions. Based on the complete disassembly considering core condition, recycling purity, item damaging, hazardous items, etc. advances towards price-quantity dependencies, dynamic multi-period planning, and flexible disassembly planning shall be made to answer the following questions:

Q1: Can price-quantity and cost-quantity dependencies with external sources (supplier & customer) be integrated in the planning and to what extend?
Q2: How can the contracting with suppliers and customers be supported by a multi-period dynamic planning?
Q3: Is the (optimal) flexible disassembly planning beneficial? And if so:
Q4: How can the resulting quantities be interpreted to find a concrete disassembly guideline?

Obviously, question four is only to be considered if question three is answered positively. In order to discuss the question this work is structured as illustrated in the following section.

1.3 Structure of this work

Following the introduction in this chapter, the topical placement, the review of relevant literature, as well as a first disassembly-to-order planning model considering complete disassembly is illustrated in Chap. 2. Thereby, this first disassembly model is modified to fit the needs. The result is the developed basic model in Chap. 3. This basic model is the underlying model for all other developments in this work—as depicted in Fig. 1.1. In this figure we find arrows starting from the basic model and pointing to the section with the linear price-quantity dependency, one to the rolling horizon disassembly planning, and a third to the flexible disassembly model. This indicates that these three sections are enhancements of the basic model.

[9] Cf. Jorjani / Leu / Scott (2004): *Allocation of electronics components to reuse options*, p. 1135.

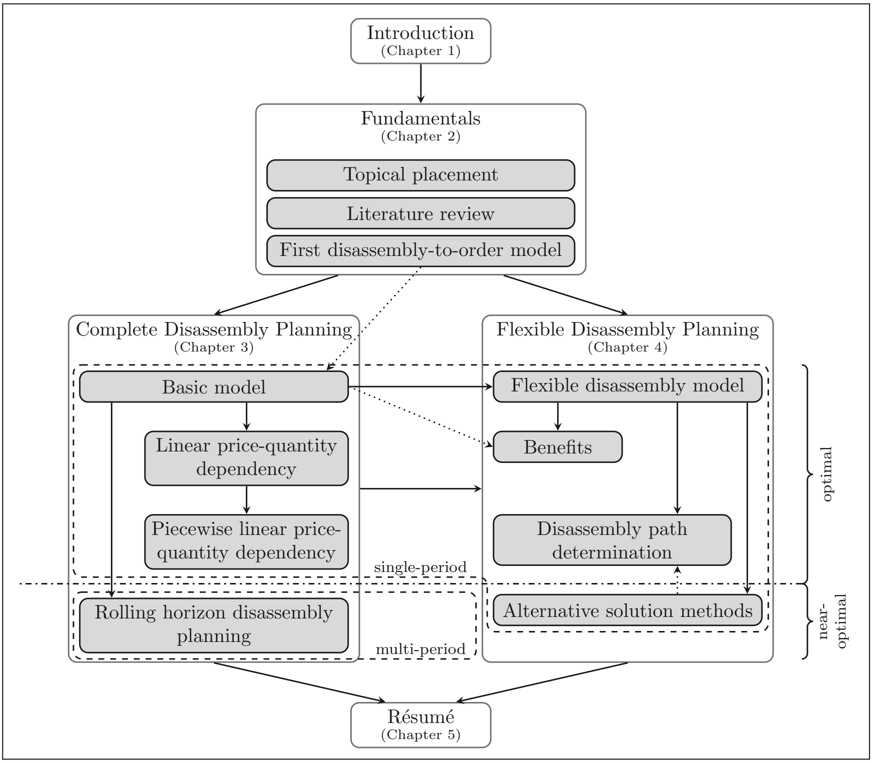

Fig. 1.1 Structure of this work

The first extension of the basic model, (i.e., the profit optimal single-period multi-core complete disassembly planning considering core condition, purity requirements, hazardous items, damaging, supply, labour time, distribution restrictions, demand, disposal, recycling, reuse, commonality, multiplicity, acquisition, disassembly, and disposal cost, as well as revenues from reuse and recycling) is the incorporation of linear price-quantity and cost-quantity dependencies for the core acquisition and item, material, and waste distribution. (In the sequel price and cost are subsumed as price.) This allows the selective removal of supply or distribution limits of the basic model, because with changing quantities, the prices change so that the market behaviour of particular scenarios is approximated to some extent. Here, an optimal solution with the mixed integer quadratic programming is gained.

To increase the freedom of approximation for the price-quantity dependency, the approach is extended to piecewise linear price-quantity dependencies. This allows a more detailed approximation, but leads to a model with other properties for solving. And these properties require a different solution method to find the optimal solution. In Sect. 3.3 an approach is developed, which aims at using standard solver software (for mixed integer quadratic and linear problems) so that the piecewise linear dependencies can be adopted in practice.

A further extension of the basic model is the modelling of a multi-period scenario with changing data and uncertain future data. In Sect. 3.4 a rolling horizon planning model is developed, which tries to maintain a steady policy and still allows for flexibility for changing data. The results of the planning are evaluated with an ex-post optimal solution. This indicates that the result of the rolling horizon planning is generally not an optimal but near-optimal solution. Note that the model is not a stochastic one, even though uncertainties are accounted for.

Another enhancement of the basic model is the consideration of incomplete disassembly. This introduces the disassembly depth (or level) as decision variable. A comprehensive model is developed in Sect. 4.2. This newly added aspect includes portions of the disassembly sequencing. These portions can be summarised as disassembly state, which is not the disassembly sequence as such, but the result of it, i.e., the set of items and modules gained after applying a disassembly sequence. The modelling of this problem is based on graphs and the optimal solution is gained by solving the mixed integer linear problem with a standard solver. The integration of the disassembly depth accounts for the flexibility in the disassembly processes. To evaluate the benefits of this flexible disassembly planning, the solution is compared to the basic model and the best two-stage approach.

With the gained solution of the planning, the quantities for the acquisition of cores as well as the distribution of items, modules, material, and waste are determined. Yet, these quantities are not detailed enough in order to derive a specific instruction of how to disassemble a particular unit of a core and to which usage options the items and modules should be assigned to. The necessary steps of determining the required disassembly states and the assignment of a particular unit of a core to a proper disassembly state and the best usage options are developed in Sect. 4.4.

As becomes evident in Sect. 4.2, the problem of the flexible disassembly planning, considering core conditions and the three usage options (reuse, recycling, and disposal), leads to a large sized model. This is usually linked with a relatively long solution time. In Sect. 4.5 four possibilities of speeding up the solving are presented. These possibilities are derived from the content of this work and not so much a dedicated development of a heuristic solution method. Nonetheless, the gained solutions are not necessarily optimal. But a near-optimal solution is found in less time compared to the optimal one.

The alternative solution methods only regard the flexible disassembly planning model. This means that the subsequent disassembly state and usage option assignments have to be applied to these solutions as well. This indicates the dotted arrow from “alternative solution methods” to “disassembly path determination” in Fig. 1.1. Finally, Chap. 5 summarises the work and gives implications for future research. Note that all figures in this work are created by the author himself.

Chapter 2
Fundamentals

2.1 Topical placement

The disassembly planning belongs to the field of environmentally conscious manufacturing and product recovery.[1] This already indicates that the manufacturing seems to play a role or the disassembly has influence on the manufacturing, or both. This is a little bit surprising, but the steps before the manufacturing, e.g., the design of a product, have a major influence on the complete life cycle, which includes manufacturing and disassembling. Because of this all-embracing influence of the design, several design directions with explicit environmental focus have been established. The main three are design for environment (DfE), design for disassembly (DfD), and design for recycling (DfR). The DfE focusses on the environmental impact on the whole life cycle of a product. This starts with the extraction of materials and ends with the final disposal. In between the start and end, the material or product could be reused and recycled many times in order to avoid emission of harmful substances and excessive use of energy.[2]

Thus, the DfE can be understood as umbrella for all more special design directions, like DfD, DfR, and even design for assembly (DfA). But since the focus is rather on the end of life of the products, DfD and DfR are more relevant than DfA. The DfR concentrates on design attributes for separating and recycling the comprised material in a cost-effective manner at the product's end of life.[3] But the DfR should not be in the main focus

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[1] Cf. Ilgin / Gupta (2010): *ECMPRO: A review*, pp. 563–565.

[2] Cf. Bevilacqua / Ciarapica / Giacchetta (2007): *Development of a sustainable product lifecycle*, pp. 4073 et seq.

[3] Cf. Masanet / Horvath (2007): *Assessing the benefits of design for recycling*, p. 1801.

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when it comes to disassembly planning, because the interest is primarily on the embodied material. When items or modules should also be reused, the DfD is the relevant design direction. According to the definition of disassembly—which includes all planned processes that separate products into modules, items, and/or material[4]—the DfR can be seen as one part of the DfD. The reason is that deriving design issues for a planned process with the result of material to recycle equals the goal of the DfR. Thereby, the DfD is the combination of all design considerations to facilitate the disassembly process (i.e., minimising the complexity of the structure of the product, increasing the use of common materials and items, and easily removable fasteners and joint types).[5] This especially includes an evaluation of a current product design with, e.g., index-based approaches,[6] and, moreover, design recommendations to facilitate the disassembly of a product.[7]

But even though an optimal design is found for all phases of the life cycle, the design does not have to be optimal at the end of life of the product. This problem might occur because of changing needs for material or parts for recycling or reuse, respectively, changes in legislation, too little estimated abrasion of items, etc. Hence, the longer the life cycle the greater is the danger of a suboptimal design. Note that long life cycles are positive in terms of an environmental conscious manufacturing and product recovery. Thereby, not necessarily the complete product has to have a long life cycle. At least the embodied materials and items should have one. On the other hand, it means that all aspects with regard to the product recovery need to be flexible in terms of quantities, conditions, and limitations. This affects not only the disassembly planning, but also the logistics.

The reverse logistics (as pendant to the forward logistics) is a branch of logistics that focusses on reverse flows, i.e., from the consumer back to the manufacturer or a company that is entrusted with the recovery of used

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[4] Cf. Seliger (2011): *Montage und Demontage*, p. S97.

[5] Cf. Güngör (2006): *Evaluation of connection types in DFD*, p. 36.

[6] Cf. Kroll / Hanft (1998): *Quantitative evaluation of product disassembly*, Kroll / Carver (1999): *Disassembly analysis*, Veerakamolmal / Gupta (1999): *Design efficiency for disassembly*, Zeid / Gupta (2002): *Disassembly cost index*, Villalba et al. (2004): *Recyclability index as a tool for DfD* (in combination with Villalba et al. (2002): *Quantifying the recyclability of materials*), Desai / Mital (2003): *Evaluation of disassemblability*, and Desai / Mital (2005): *Design for disassembly*. In a broader view the index can also be a maximal process value reduced by disassembly cost, cf. Kwak / Hong / Cho (2009): *Eco-architecture analysis for end-of-life*.

[7] Cf. Viswanathan / Allada (2001): *Configuration analysis to support product redesign*, Viswanathan / Allada (2006): *Product configuration optimization for disassembly planning*, and Giudice / Kassem (2009): *End-of-life impact reduction*.

products so that the product or material is again usable in a market.[8] This includes the planning, implementing, and controlling of the backward flows to the recovery or proper disposal site.[9] Thereby, the retrieval of the used products should be efficiently dealt with.[10] Moreover, the view of reverse logistics can also be extended to the inclusion of material selection aspects. Hence, it is not just collecting and transporting products and material, but, furthermore, the decision which recovery option (see below) is used.[11] This selection is already an aspect that is common with the disassembly planning, only that after disassembling we call them usage option to differentiate between general recovery option and the result of the disassembly.

The reverse flow is not independent from the forward flow. There exists a strong impact from both flow directions on the capacities for storage, transportation, etc. Thus, a simultaneous consideration is favourable, if not necessary for an optimal planning. This combined consideration is focus of the closed-loop supply chain.[12] Thereby, the system is only a closed loop when the product returns to the original producer. Otherwise, it is an open-loop system with forward and reverse flow, which starts at a producer, goes to a consumer, and ends at a different recovery company.[13] Note that not only end-of-life products cause a reverse flow. Also product recalls, service and warranty returns, even rework, etc. can be seen as reverse flow.[14]

A main logistical aspect of the closed-loop supply chain is the placement of facilities and the allocation of flows between them, i.e., the network design. For this aspect it is already of interest what recovery options exist at this stage.[15] The common options are:[16]

- (direct) reuse,
- repair,

[8] Cf. Fleischmann et al. (1997): *Quantitative models for reverse logistics*, p. 2.

[9] Cf. de Brito (2003): *Managing reverse logistics*, p. 20.

[10] Cf. Demirel / Gökçen (2009): *MIP model for remanufacturing in reverse logistics*, p. 1197.

[11] Cf. Jamshidi (2011): *Reverse Logistics*, p. 254.

[12] Cf. Guide Jr. / Jayaraman / Linton (2003): *Contingency planning for CLSC*, p. 278, Ilgin / Gupta (2010): *ECMPRO: A review*, p. 567, and Savaskan / Bhattacharya / Van Wassenhove (2004): *CLSC models with product remanufacturing*, p. 239.

[13] Cf. Fleischmann et al. (1997): *Quantitative models for reverse logistics*, p. 4.

[14] Cf. Zarandi / Sisakht / Davari (2011): *Design of a CLSC model*, p. 809, and Inderfurth / Teunter (2001): *Production planning and control of CLSC*, p. 1.

[15] Cf. Ilgin / Gupta (2010): *ECMPRO: A review*, pp. 571 et seq.

[16] Cf. here and in the sequel Thierry et al. (1995): *Strategic issues in product recovery*, pp. 118–120, de Brito (2003): *Managing reverse logistics*, pp. 61 et seq., and Goggin / Browne (2000): *Towards a taxonomy of recovery*, pp. 179 et seq.

- refurbishing,
- remanufacturing,
- cannibalisation,
- recycling,
- incineration, and
- disposal.

Thereby, the reuse option is the one with the least extra effort. This means that the product can either be directly reused or minor repairs are done.[17] This might apply to unused spare parts, resold products, or carriers and packaging. The repair option returns a used product back to working order. This comes along with a lower quality than the new product and could be executed at the consumer's location. It requires already some small amount of disassembly and reassembly. The refurbishing is analogue to the repair, but the used product is brought to a specific quality including a possible upgrade in functionality. Nevertheless, the overall quality is lower than that of the new product. The remanufacturing brings the product back to a quality of a new product.[18] Therefore, these products are "as good as new".[19] Cannibalisation or retrieval denotes the recovery of a limited number of parts from the used product to be used for, e.g., repair work. Thereby, the focus is not on the complete product anymore, but shifted to the constituent parts. These parts can be single items or modules. When it comes to recycling, incineration, and disposal, the original product (and its constituent parts) is not of interest anymore. With recycling and incinerating material and energy, respectively, are recovered. Lastly, the disposal represents the loss of any value for today. It is quite possible, that land-filled waste is going to be recovered in the future. This depends on technology and whether it becomes economic beneficially.

These recovery options can be ordered according to several criteria. In pursuance of Gerrard / Kandlikar, the preference ordering is reuse, remanufacturing, recycling, incineration, and lastly disposal.[20] Even legislative regulations provide a preference ordering for waste, nowadays. According-

[17] Cf. Fleischmann et al. (1997): *Quantitative models for reverse logistics*, p. 11.

[18] Cf. Guide Jr. / Jayaraman / Linton (2003): *Contingency planning for CLSC*, p. 278.

[19] Companies like Caterpillar have even set up an extra brand (e.g., Cat Reman) to distribute these remanufactured products. Cf. `http://catreman.cat.com/`. Other companies like Jungheinrich established a series for refurbished trucks (e.g., Jungheinrich JungSTARs). Cf. `http://www.jungheinrich.de/en/used-trucks/jungheinrich-jungstars/`.

[20] Cf. Gerrard / Kandlikar (2007): *Assessing the impact of the ELV Directive*, p. 23.

ing to German law, five waste handling options exist in the following ordering:[21]

1. avoiding,
2. preparing for reuse,
3. recycling,
4. other recovery, especially energy recycling and filling, and
5. disposal.[22]

Thereby, the reuse corresponds to avoiding, remanufacturing, refurbish, repair, and cannibalisation, the preparing for reuse, material recycling to recycling, energy recycling to other recovery, and disposal to disposal. Note that "filling" is seen as disposal in this work.

Taking a look at the recovery options, we find that in many of them disassembly is undertaken. It applies to repair, refurbishing, remanufacturing, and cannibalisation, because only with disassembly the parts to exchange or retrieve can be accessed. But most likely, disassembly is also necessary for recycling, because not the complete core consists of material that is going to be processed in the same way. Moreover, in an environmentally conscious system disposal should be reduced as much as possible. This means that not the complete core is going to be disposed of. At least hazardous material and possibly recyclable material or reusable parts should be extracted and processed separately. This again makes disassembly necessary. Furthermore, we see that not only one recovery option has to be applied to a core (except for product reuse, repair, refurbishing, and remanufacturing). All in all, disassembly is one of the key issues in the product recovery.

When it comes to the planning of the disassembly, one might think that it equals the assembly planning or that it is just the reverse of it. For some aspects this might be the case, but definitely not for all. One indicator for the necessity of a separate disassembly planning is the existence of quite a few research articles in the literature. Moreover, Lambert lists significant differences. These are:[23]

1. a not completely reversible assembly process,
2. less value added obtained in disassembly processes,

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[21] Cf. 6 KrWG (Gesetz zur Frderung der Kreislaufwirtschaft und Sicherung der umweltvertrglichen Bewirtschaftung von Abfllen – Kreislaufwirtschaftsgesetz).

[22] In the literature (land) filling and disposal are synonymously used terms. Why the filling is differentiated from the disposal in German law and equal to energy recycling is not clear. It might be that with filling the theoretical possibility exists to recover this waste later and process it with new techniques. In this context, the disposal might be an option, which expresses the final loss of the material (e.g., ocean dumping).

[23] Cf. Lambert (2003): *Disassembly sequencing: A survey*, pp. 3721 et seq.

3. uncertainty about the condition of the constituent parts,
4. uncertainty about the quantity of core supply,
5. a variety in supplied products,
6. mainly human labour instead of automated assembly lines and robots, and
7. usually not complete disassembly, which introduces the disassembly depth into the consideration.

In addition to these properties, there might also exist uncertainty about what parts the core consists of. Depending on the product the consumer might have replaced parts of the product by different ones. If this is the case, non-genuine parts are inserted into the product.

For the above reasons the separate research field of disassembly planning is established. As mentioned above, disassembly occurs in different recovery options, which results in planning problems with particular properties. In addition, keeping the developed models understandable and usable the models should include all necessary aspects. And in general, the necessary aspects are problem dependent. A first group of such problems is the repair and refurbishing. Here, the cores are partly disassembled in order to reach the damaged parts and afterwards reassembled. The main focus is clearly to gain a functioning product with as little disassembly and reassembly effort as possible.[24] This group of problems shows parallels to maintenance planning.

A second group is the disassembly with regard to remanufacturing. Thereby, the cores are disassembled "completely" and reassembled to gain an "as good as new" product. From a disassembly point of view, it is a special case of the repair or refurbishing—namely, the worst case—, because all other disassembly options result in an incomplete disassembly. On the other hand, from a planning point of view, this worst case of the disassembly—the complete disassembly—is easier than the planning of incomplete disassembly, which incorporates the disassembly depth as a further decision to make. The term completely with regard to the disassembly does not literally mean a complete disassembly. If it would be a complete disassembly in the literal meaning, all items, i.e., every semiconductor, screw, etc., would be separated from each other and no connection between items would remain. This is usually not the case. In general, groups of items stay together, e.g., a relay. Such a relay could be further disassembled but no one does it, because it is seen as an item.[25] This view is a level of abstraction necessary

[24] The interpretation of disassembly and reassembly effort depends on the decision maker.

[25] Note that there might exist situations where a relay must be disassembled, e.g., for valuable material.

for modelling disassembly processes. Otherwise, the model becomes too big to handle. Hence, depending on the level of abstraction, an item could be as big as a complete engine of a ship (or even bigger) or as small as a transistor only visible with an electron microscope.

The last group is the disassembly planning. It embodies the recovery options cannibalisation, recycling, incineration, and disposal. These options do not aim to have a complete product in the end. The individual items and modules after the disassembly might be used for reuse, recycling, incineration, and disposal.[26] Again, the disassembly can be complete or incomplete. Especially this group of the disassembly planning is characterised by the diverging structure, i.e., one core leads to several items and modules. Since the disassembly planning is the focus of this work, a more detailed look onto the existing planning problems can be found in the following.

2.2 Literature review on disassembly planning

The term disassembly planning comprises many aspects around the concrete disassembly process. It includes product representation, related product design/redesign issues, and disassembly sequencing with disassembly level and end-of-life options.[27] But when taking a look in the literature, one might find the term as a kind of generic term. Hence, in the sequel we shall use it as such. Under this generic disassembly planning five main fields have emerged that preferably consider quantitative problem statements. These five are disassembly sequencing, disassembly-to-order planning, disassembly scheduling, disassembly line balancing, and flexible disassembly system planning. Thereby, a

Disassembly sequence is a listing of subsequent disassembly actions, where an action is, e.g., dividing an assembly into two or more modules or separating one or more connections between parts.[28] Finding the preferably optimal sequence of all possible sequences is the goal of the disassembly sequencing. The

Disassembly-to-order planning aims at finding the optimal quantities of cores to be disassembled in order to meet the demand of parts and material from a mix of cores. Thereby, these cores can have parts in common. If the common parts occur across different cores, the term *commonality*

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[26] Note that incineration is seen as a material recycling option in the sequel.

[27] Cf. Lee / Kang / Xirouchakis (2001): *Disassembly planning and scheduling*, p. 697.

[28] Cf. Lambert (2003): *Disassembly sequencing: A survey*, p. 3721.

is used, and if they occur within a core, we find the term *multiplicity* in the literature.[29] In general, the optimisation criterion is either a cost minimisation or a profit maximisation.[30] The

Disassembly scheduling can be seen very similar to the disassembly-to-order planning as "problem of determining the order quantity of the used products to fulfil the demand for disassembled parts."[31] But scheduling should furthermore include a timing of disassembling.[32] This does not mean that scheduling is always a multi-period planning. It just means that, in addition to the quantities, the ordering is relevant for the decision maker. The

Disassembly line balancing solves the problem of assigning disassembly tasks to an order of stations such that the disassembly precedence relations are satisfied.[33] The optimisation criteria can be profit, cost, (cycle) time, number of workstations, levelled utilisation, etc. or combinations of these.[34] The

Flexible disassembly system planning is another relative big research area. It belongs to the field of automated disassembly and has a different (machine) layout than the disassembly line.[35] Nevertheless, the planning is somewhat similar to the line balancing with the exception of the layout and the focus on the automation, i.e., the aim is to plan the disassem-

[29] Cf. Taleb / Gupta (1997): *Disassembly of multiple products*, p. 950, Lambert / Gupta (2002): *Demand-driven disassembly optimization*, p. 123, and Lee et al. (2004): *Disassembly scheduling*, p. 1360.

[30] Cf. Lambert / Gupta (2002): *Demand-driven disassembly optimization*, p. 122, together with Ilgin / Gupta (2010): *ECMPRO: A review*, p. 579.

[31] Lee / Xirouchakis / Züst (2002): *Disassembly scheduling with capacity constraints*, p. 697.

[32] Cf. Kim / Lee / Xirouchakis (2006b): *Two-phase heuristic for disassembly scheduling*, p. 196.

[33] Cf. Altekin / Kandiller / Ozdemirel (2008): *Profit-oriented disassembly-line balancing*, p. 2677.

[34] Cf. McGovern / Gupta (2007): *Balancing method and GA for disassembly line balancing*, p. 693, Altekin / Kandiller / Ozdemirel (2008): *Profit-oriented disassembly-line balancing*, p. 2677, Koc / Sabuncuoglu / Erel (2009): *Disassembly line balancing using an AND/OR graph*, p. 870, and Agrawal / Tiwari (2008): *ACO to disassembly line balancing and sequencing*, p. 1414.

[35] Cf. Ilgin / Gupta (2010): *ECMPRO: A review*, p. 579, and Wiendahl et al. (2001): *Flexible disassembly systems*, p. 723.

bly with values (e.g., cost and revenues) and problem specific resources considered.[36]

There also exist approaches which combine, e.g., sequencing and scheduling aspects or sequencing and disassembly-to-order aspects,[37] without an established taxonomy. These are still subsumed as disassembly planning in the sequel. Another interesting albeit small research area is the active disassembly. Here, the focus is on the self-disassembly of a core or parts of it.[38]

In preparation of Table 2.1 containing the relevant literature, the used properties are discussed in the sequel. (The corresponding references to the properties can be found in Table 2.1.) When it comes to disassembly planning, one has to deal with uncertainties in general. Nevertheless, there exist **deterministic** models. Either because of valid information about the cores or the uncertainty is just neglected. For example, valid information can be gained by testing all incoming cores before the planning, which includes RFID, or by permanent maintenance of products by a company, which corporates with the disassembling facility. A second category is the one of **quasi-stochastic** models. Thereby, "quasi-stochastic" denotes a combination of deterministic and stochastic models in this work. This applies to planning situations, where uncertainty exists, but the probabilities, rates, and expectations of uncertain values are used in a deterministic style planning. The last type in this context is the **stochastic** modelling and planning. Models in this category explicitly incorporate distribution or density functions of stochastic variables into the model.

The considered uncertainties we find in the relevant literature regard the condition or quality of the cores and the quantities (i.e., yields or availability). In addition, the possible damaging during the disassembly process is another uncertainty to cope with. In this context, some articles differentiate between **destructive** and **non-destructive** disassembly.

Furthermore, a differentiation of planning situations covering just a single or multiple periods is useful, too. In this regard, the single-period planning is seen synonymously to the static planning, because within the single period (which could be infinitely long) no changes in data occur. For the **multi-**

[36] Cf. Tang / Zhou / Caudill (2001): *Integrated approach to disassembly planning and demanufacturing operation*, p. 778, and Williams (2007): *A review of research towards computer integrated demanufacturing*, p. 773.

[37] Cf. Santochi / Dini / Failli (2002): *Computer aided disassembly planning* and Xanthopoulos / Iakovou (2009): *On the optimal design of disassembly*.

[38] Cf. Ijomah / Chiodo (2010): *Application of active disassembly to extend profitable remanufacturing*.

period planning a change of data (quantities, limits, etc.) from period to period is assumed. Thus, a multi-period planning is assumed to always be dynamic in our overview. Moreover, the dynamic planning could further be clustered in planning with just different values of parameters for each period and planning with changing values of parameters. The latter could also be seen as dynamic planning with uncertainty, because of the fact that, when parameter changes within the overall planning occur, the parameters are not certain. For our consideration the distinction between multi-period (dynamic) and single-period (static) is sufficient.

The next aspects are concerned with the input side of the disassembly process. Here, **multi-core** and single-core approaches can be differentiated. Multi-core indicates the simultaneous consideration of more than one different core, e.g., a car and a truck. On the other hand, with a single core approach either the car or the truck is planned, but not both together. Thereby, it does not matter if only one unit or hundreds of units of the same core are considered. A further property tied to the cores is that of common parts, which is already discussed above. In addition, cores can contain **hazardous** parts. These can be as small as batteries or just material like lead. In this case a special treatment is necessary. Besides, for an economic consideration certain cost factors might be of interest, e.g., transportation and order cost, which are subsumed as **acquisition cost**.

In addition, the availability of cores might be limited, which is a supply limitation for the disassembly process. Besides this, further **limits** can exist—like distribution, (cycle) time, disassembly cell sizes, and storage space. Along with the storage space limitation the **inventory holding cost** can also be of interest. This cost component might be extended by **set-up cost** and **disassembly cost**. The latter accrue with almost every disassembly process—maybe not with active disassembly. When disassembling a core **completely** the disassembly cost is relatively high compared to an **incomplete** disassembly. Therefore, an incomplete disassembly planning is very promising. But when a core is literally not completely disassembled, it does not mean that the planning is an incomplete one. The differentiation is carried out by the number of relevant disassembly states. The disassembly state is the result of the disassembly process in terms of which items and modules are gained from the core. And if this set of items and modules is identical for all units of a core and a priori given, it is complete disassembly planning, because the modules can be seen as an (abstract) item. If, on the other hand, the disassembly state or states need to be determined with the planning, it is incomplete disassembly planning. Note that even for a complete disassembly many disassembly sequences might exist. Besides, the disassembly state corresponds to the disassembly depth or disassembly level.

After disassembling the cores the gained parts can be **reused** or resold, **recycled**, **disposed of**, etc. Which options are considered depends on the problem in focus. In some articles there exist no differentiation, e.g., in the case of non-destructive disassembly with the goal to minimise the disassembly time. In other publications all three of them are distinguished, especially when it comes to revenues and cost considerations. Moreover, for (material) recycling the **purity** of the material might be an issue.[39] But interestingly this is hardly to find in the literature. In addition, for every disassembly option a **demand** might be given.

The final properties regard the solving of the predominant quantitative approaches. The properties include whether an **optimal** or **heuristic** (i.e., near-optimal) solution is sought. If none of these two apply, one just tries to find a feasible solution. To define what is optimal for the focussed problem, the **objective** must be given. The most common are minimising a time or cost measure or maximising the profit (or something similar like the end-of-life value). In addition, the simultaneous consideration of more than one objective is denoted as **multi-criteria** optimisation. In relation to the proposed method in this work, the solution approaches based on some kind of linear programming are listed separately in the column **X-LP**. Possible entries are linear programming (LP), integer linear programming (ILP), binary integer linear programming (BILP), mixed integer linear programming (MILP), quadratic programming with linear constraints (QLP), and mixed integer quadratic programming with linear constraints (MIQLP). These can be combined with other **solution approaches** or other solution approaches are used alone (i.e., without any kind of LP). As with the linear programming, solution approaches based on an arbitrary **graph**—be it a simple disassembly tree or a complex network—is marked explicitly in relation to this work.

The relevant literature is summarised in Table 2.1. In addition to the summary, the thesis of Langella presents and compares several models for disassembly-to-order systems.[40] It includes deterministic and stochastic, single and multi-period, as well as single and multi-core models to find optimal solutions. Moreover, heuristics are presented, too. Even though the list contains quite a few entries, it could be extended by further publications from existing literature reviews and surveys. Reviews covering more than one research area are the ones from Santochi / Dini / Failli, Güngör /

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[39] Cf. Das / Naik (2002): *Process planning for product disassembly*, p. 1340.

[40] Cf. Langella (2007a): *Planning demand-driven diassembly for remanufacturing*.

Table 2.1 Literature summary

research area	publication key	deterministic	quasi-stochastic	stochastic	uncertainty	multi-period	multi-core	incomplete	commonality	multiplicity	reuse	recycling	disposal	demand	limits	hazardous	damaging	non-/destructive	condition	purity	acquisition cost	inventory cost	disassembly cost	disassembly lead time	set-up cost	objective	multi-criteria	X-LP	solution approach	optimal	heurisitc	graph	comment
planning	Behdad et al. (2010)	x					x	x			x	x	x		ot, su								x			p		MILP		x			
	Jorjani / Leu / Scott (2004)	x					x				x	x	x		lf								x			p		LP	PLO				
	Karakayali / Emir-Farinas / Akçali (2010)	x					x				x	x														p		QLP					
	Teunter (2006)			x				x			x	x	x					x	x				x			p			DP	x			
	Willems / Dewulf / Duflou (2006)		x					x			x	x	x													p		LP		x			
	Xanthopoulos / Iakovou (2009)	x				x	x	x		x				x							x	x	x		x	p	x		GP				1
sequencing	Adenso-Díaz / García-Carbajal / Gupta (2008)	x						x															x			c, pp			MH		x		2, 3
	Andrés / Lozano / Adenso-Díaz (2007)						x							x	cs						x					c, ie		MILP	MH		x		2, 4
	Behdad / Thurston (2012)				x			x																x		u		BILP		x		x	
	Chung / Peng (2006)	x						x																		nr			GA		x		
	Go et al. (2012)	x																								dt			GA		x		
	Grochowski / Tang (2009)		x					x			x		x													dv			PN, ES			x	
	Güngör / Gupta (1997)	x																								dt					x		
	Güngör / Gupta (1998)	x																								dt					x		
	Huang / Liao (2009)																									dc							
	Kang et al. (2010)	x						x			x	x	x										x			p						x	
	Kang et al. (2001)	x						x				x														p		MILP		x		x	
	Kara / Pornprasitpol / Kaebernick (2005)	x						x																		t				x		x	
	Kara / Pornprasitpol / Kaebernick (2006)	x																								dt						x	5
	Kongar / Gupta (2006a)	x									x	x	x	x				x								t			GA		x		
	Lambert (1999)	x						x															x			p		LP		x			
	Lambert (2002)	x																					x			p		LP		x		x	
	Lambert (2006)	x						x															x			p		ILP		x		x	6
	Lambert / Gupta (2008)	x																					x			p		ILP		x	x	x	6
	Li / Khoob / Tor (2006)	x						x																					DCG			x	
	Liu et al. (2012)	x																								c			ACO		x		
	Lu et al. (2008)	x																								dc, tc	x		ACO		x		
	McGovern / Gupta (2007)	x												x	t	x^e										sev.			GA		x		

Table 2.2 Literature summary (cont.)

research area	publication key	deterministic	quasi-stochastic	stochastic	uncertainty	multi-period	multi-core	incomplete	commonality	multiplicity	reuse	recycling	disposal	demand	limits	hazardous	damaging	non-/destructive	condition	purity	acquisition cost	inventory cost	disassembly cost	disassembly lead time	set-up cost	objective	multi-criteria	X-LP	solution approach	optimal	heurisitc	graph	comment
sequencing	Moore / Güngör / Gupta (2001)	x						x								x[e]							x			c			PN		x		
	Puente et al. (2010)	x						x				x														pp							
	Reveliotis (2007)			x	x						x	x	x						x							er			PN, DP	x		x	
	Sarin / Sherali / Bhootra (2006)	x						x					x										x		x	p			TSP	x	x		6
	Shimizu / Tsuji / Nomura (2007)	x																					x	x		t, c			GA		x		
	Shyamsundar / Gadh (1996)	x						x																									
	Smith / Smith / Chen (2012)	x						x																		nr			GA		x	x	
	Srinivasan / Gadh (2002)	x						x																		nr							7
	Tang (2009)		x		x			x			x		x						x				x	x		ev			FPN				8
	Tian et al. (2012a)			x	x																			x		t			MCS			x	
	Tian et al. (2012b)			x	x			x															x	x		c						x	
	Torres / Puente / Aracil (2003)	x						x																								x	
	Tripathi et al. (2009)	x						x					x										x		x	p			ACO		x		
	Tsai (2012)	x						x															x			c				x	x		
	Vinodh / Kumar / Nachiappan (2012)	x						x																		p						x	
	Wang / Xiang / Duan (2008)	x																					x			c			GA		x	x	
	Wan / Gonnuru (2012)	x						x			x		x	x		x			x				x		x	p			FGA		x	x	9
	Wu / Zuo (2009)	x																								dc, tc			GA, SA		x	x	10
	Yi et al. (2008)	x						x																		nr				x			
	Zhang / Zhang (2010)	x																								t			B&B		x!	x	
	Zha / Lim (2000)	x																								t			PN				
to order	Lambert / Gupta (2002)	x					x	x	x	x	x		x	x	su								x			c, p		LP		x		x	11
	Lee / Cho / Hong (2010)	x						x			x		x	x									x			p							
	Inderfurth / Langella (2006)			x			x		x				x	x							x		x			c					x		
	Kongar / Gupta (2002a)	x					x			x	x	x	x	x	st			x			x	x	x			p, etc.	x	MILP	LPP	x			
	Kongar / Gupta (2002b)		x		x		x			x	x	x	x	x	st		x	x	x		x	x	x			p, etc.	x	MILP	GP	x			3
	Kongar / Gupta (2006b)		x		x		x		x	x	x	x	x	x			x	x	x		x	x	x			p, etc.	x	MILP	FGP	x			12
	Langella (2007b)	x				x	x	x					x	x	su, st						x	x	x	x		c		MILP		x	x		

Table 2.3 Literature summary (cont.)

research area	publication key	deterministic	quasi-stochastic	stochastic	uncertainty	multi-period	multi-core	incomplete	commonality	multiplicity	reuse	recycling	disposal	demand	limits	hazardous	damaging	non-/destructive	condition	purity	acquisition cost	inventory cost	disassembly cost	disassembly lead time	set-up cost	objective	multi-criteria	X-LP	solution approach	optimal	heuristic	graph	comment
to order*	Ullerich (2011a)		x		x		x			x	x	x	x	x	su, di	x			x	x	x		x			p		MIQLP		x			
	Ullerich (2011b)	x				x	x			x	x	x	x	x	st, su, di	x				x	x	x	x			p		MILP	RP		x		13
	Ullerich / Buscher (2010)	x				x	x	x	x	x	x	x	x	x	su, di	x				x	x	x	x			p		MILP		x		x	
	Ullerich / Buscher (2011)	x				x	x			x	x	x	x	x	st, su, di	x				x	x	x				p		MILP	RP		x		13
scheduling	Barba-Gutiérrez / Adenso-Díaz (2009)				x	x								x										x					FRM				
	Brander / Forsberg (2005)	x																				x			x	st					x		6
	Barba-Gutierrez / Adenso-Diaz / Gupta (2008)	x				x								x										x	x	c					x		
	Kim / Lee / Xirouchakis (2006a)	x				x								x	t								x	x	x	c		MILP	LH	x	x		
	Kim / Lee / Xirouchakis (2006b)	x				x	x		x					x								x	x	x	x	c		MILP	REL		x		
	Kim et al. (2009)	x				x								x								x			x	c			B&B	x			
	Kim et al. (2003)	x				x								x								x	x	x	x	c			RO		x		
	Kim / Xirouchakis (2010)			x		x	x							x	t							x			x	c					x		14
	Kim et al. (2005)	x				x								x	t									x		nc		ILP		x			
	Kim et al. (2006)	x				x								x	t									x		nc				x	x		
	Lee / Xirouchakis / Züst (2002)	x				x				x				x	t						x	x	x	x		c		MILP		x			
	Lee / Xirouchakis (2004)	x				x				x			x	x							x	x	x	x	x	c					x		
	Lee et al. (2004)	x				x	x		x	x				x							x	x	x	x	x	c		MILP		x			
	Li et al. (2009)			x	x	x		x						x							x	x	x			c			DP	x			15
	Prakash / Ceglarek / Tiwari (2012)	x				x				x				x							x	x				c			SA		x		
	Tang / Zhou / Caudill (2001)	x																								ev			PN				
line balancing	Agrawal / Tiwari (2008)			x			x								ct											t, etc.	x		ACO		x		3
	Altekin / Kandiller / Ozdemirel (2008)	x						x						x	ct								x			p		MILP		x	x	x	
	Altekin / Akkan (2012)		x		x			x						x									x			p		MILP	DES	x	x		
	Ding et al. (2010)	x													t											nw, etc.	x		ACO		x		
	Duţă et al. (2008)	x						x			x		x										x			p/t			GA		x		
	Güngör / Gupta (2002)							x								x										t					x		
	Koc / Sabuncuoglu / Erel (2009)	x																								nw		BILP		x		x	
	McGovern / Gupta (2007)	x												x	t	x[e]										sev.			GA		x		

* In these publications the planning is more general than a pure to-order planning.

Table 2.4 Literature summary (cont.)

research area	publication key	deterministic	quasi-stochastic	stochastic	uncertainty	multi-period	multi-core	incomplete	commonality	multiplicity	reuse	recycling	disposal	demand	limits	hazardous	damaging	non-/destructive	condition	purity	acquisition cost	inventory cost	disassembly cost	disassembly lead time	set-up cost	objective	multi-criteria	X-LP	solution approach	optimal	heurisitc	graph	comment
flexible system	Gil et al. (2007)	x			x																												16
	Hsieh (2011)	x			x																								PN				
	Kim / Harms / Seliger (2007)	x			x		x																										
	Kuren (2006)	x																															
	Pisuchpen (2012)			x																									SI				
	Pisuchpen / Bohez / Janssens (2007)	x																											PN				
	Seeluangsawat / Bohez (2004)	x																											PN				
	Tang / Zhou / Caudill (2001)	x																								ev			PN				

Abbreviations and notations used in the table:

limits:

cs ... cell size
ct ... cycle time
di ... distribution
lf ... landfill
ot ... operation type
st ... storage
su ... supply
t ... time

hazardous:

x^e ... early removal

objective:

c ... cost
dc ... direction change
dt ... disassembly time
dv ... disassembly value
er ... expected return
etc. ... et cetera
ev ... end-of-life value
ie ... intercell exchanges
nc ... number of cores
nr ... number of removals
nw ... number of workstations
p ... profit
pp ... priority parts
sev. ... several
st ... set-up time
t ... time
tc ... tool change
u ... utility function

solution approach:

ACO ... ant colony optimisation
B&B ... branch and bound
DCG ... disassembly constraint graph
DES ... discrete event simulation
DP ... dynamic programming
ES ... expert system
FGA ... fuzzy genetic algorithm
FGP ... fuzzy goal programming
FPN ... fuzzy petri net
FRM ... fuzzy reverse material requirements planning (MRP)
GA ... genetic algorithm
GP ... goal programming
LH ... Langrange heuristic
LPP ... linear physical programming
MCS ... monte carlo simulation
MH ... metaheuristic
PLO ... piecewise linear objective
PN ... petri net
REL ... relaxation
RO ... rounding
RP ... rolling planning
SA ... simulated annealing
SI ... simulation
TSP ... travelling salesman problem

comments:

1 ... one state per core
2 ... two-phase approach
3 ... lexicographic
4 ... cell formation
5 ... some software used
6 ... sequence dependent
7 ... geometric constraint based
8 ... defective items
9 ... RFID
10 ... combined objective
11 ... two models
12 ... condition & damaging not core specific
13 ... incl. contractual penalties
14 ... backorder penalty cost
15 ... backlog cost
16 ... robots

Gupta, and Ilgin / Gupta.[41] Reviews with a focus on scheduling can be found from Lee / Kang / Xirouchakis and Kim / Lee / Xirouchakis.[42] As becomes clear in the literature summary table, quite a few articles about disassembly sequencing exist. Hence, the number of reviews is greater than for the other research areas. We find reviews from Tang et al., Dong / Arndt, Lambert, and Kang / Xirouchakis.[43] Lastly, two reviews in the field of flexible disassembly systems by Wiendahl et al. and Duţă / Filip shall be mentioned for the interested reader.[44]

In Table 2.1 exist entries with a combination of deterministic and uncertainty marked in the area of flexible disassembly systems. This indicates that the considered system reacts on unknown situations, but once the part is identified, it works deterministically. In addition, flexible disassembly systems are single core systems, because only one part at a time is disassembled in general. But when it comes to specific planning (way ahead of the disassembly) similar to scheduling, multiple cores and parts might be considered, likewise stochastic planning.

When taking a look at the summary table, we notice that the purity consideration is basically not present in the literature even though it is identified as relevant aspect.[45] To start the integration in the planning one should choose a superordinate planning like the disassembly-to-order planning. In addition, the fact of hazardous items and material as well as the condition of the cores are rarely to find.

There exist a few entries concerning the incomplete disassembly. Surely, they are diverse and cover aspects from determining just one disassembly state per core[46] up to several states per core.[47] But even the possibility of several states per core is rather limited, because a tree structure is assumed

[41] Cf. Santochi / Dini / Failli (2002): *Computer aided disassembly planning*, Güngör / Gupta (1999): *Issues in product recovery*, and Ilgin / Gupta (2010): *ECMPRO: A review*.

[42] Cf. Lee / Kang / Xirouchakis (2001): *Disassembly planning and scheduling* and Kim / Lee / Xirouchakis (2007): *Disassembly scheduling*.

[43] Cf. Tang et al. (2002): *Disassembly modeling, planning, and application*, Dong / Arndt (2003): *A review of current research on disassembly sequence generation*, Lambert (2003): *Disassembly sequencing: A survey*, and Kang / Xirouchakis (2006): *Disassembly sequencing for maintenance*.

[44] Cf. Wiendahl et al. (2001): *Flexible disassembly systems* and Duţă / Filip (2008): *Control and decision-making process in disassembling*.

[45] Cf. Das / Naik (2002): *Process planning for product disassembly*, p. 1340.

[46] Cf. Vinodh / Kumar / Nachiappan (2012): *Disassembly modeling, planning, and leveling*, p. 798.

[47] Cf. Lee / Cho / Hong (2010): *A hierarchical end-of-life decision model*.

in disassembly-to-order planning.[48] Of course, in disassembly sequencing a more detailed planning is done, which can be seen by the use of and/or graphs or other approaches that go beyond a disassembly tree structure. Here, a more detailed planning of the incomplete disassembly with regard to the optimal quantities to order, to disassemble, to distribute, to recycle, and to dispose of is necessary. In addition, this more detailed planning would also meet the existing flexibility of the today's disassembly practice, which is usually manual labour and highly flexible.[49] For this reason we call the incorporation of a more flexible disassembly process in the planning **flexible disassembly planning**. The supplement "to order" is neglected here, but a planning without considering information about demand or supply is not to favour.[50]

Note that the term flexible disassembly planning must not be mistaken for flexible disassembly system planning. The latter is the planning of a system that is the pendant to the flexible manufacturing system, only that it is used for disassembly instead of assembly. On the contrary, in this work the

> **Flexible Disassembly Planning** is concerned with the determination of quantities of multiple cores to acquire, to disassemble these into items and modules, and to distribute (i.e., reuse), recycle, or dispose of the gained items and modules while considering recycling purities, core conditions, hazardous items, item damaging (where applicable), and several capacities (supply, demand, labour time, storage, etc.). Thereby, each unit of a core can be disassembled into an individual disassembly state.

This again requires the representation of arbitrary product structures and not just tree-like disassembly structures. Furthermore, it is advisable to consider multiple cores in the planning, which includes commonality and multiplicity as inevitable aspects.

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48 Cf. Langella (2007b): *Heuristics for demand-driven disassembly planning*, p. 556.

49 Cf. Bley et al. (2004): *Human involvement in disassembly*.

50 The to-order planning is a special case of the planning considered in this work, because the distribution is going to be restricted by a lower and an upper limit. If these two limits are set to the same value it is equivalent to the given demand in the to-order planning. Thus, the planning in this work could be seen as a generalised disassembly-to-order planning. Therefore, the supplement to-order is neglected for the aspects developed in this work.

A favourable objective of the flexible disassembly planning is the profit maximisation, because it includes more than just the various costs. If a valid quantification, e.g., of time units into monetary units, is possible, even the trade-off between disassembly time to the cost and revenues can be incorporated. In any case, the planning can be extended to multi-period scenarios, too. In addition, further aspects from disassembly sequencing or scheduling could be integrated, but as we will see in Chap. 4 the incorporation of the core and item condition makes the planning complex.

The multi-period planning can be found in several disassembly scheduling publications. But in the disassembly-to-order planning not so many publications about multi-period planning exist. The deterministic ones that can be found in the literature use different values for the planning in subsequent periods which do not change. Of course, this is the nature of a deterministic planning, that the planning relevant data is known a priori. But what happens when uncertainties according to the supply or distribution occur? Two stochastic multi-period models for disassembly scheduling can be found in the summary table. On the other hand, no quasi-stochastic approach could be found, even though one might assume, that a quasi-stochastic approach is more intuitive for practitioners. One quasi-stochastic multi-period multi-core model is presented in this work. It can cope with highly dynamic data that not only differs from period to period, but also changes during the planning. Moreover, the decision maker can derive explicitly contracting recommendations for supply and distribution for future periods (see Sect. 3.4). To the best of our knowledge, this cannot be found in the literature.

Before we move on to a first disassembly-to-order planning as basis for the considerations in this work, a last term shall be discussed. It is the **selective disassembly**. Depending on the research area the term selective has different meanings. In the disassembly-to-order planning the term selective is used by Kongar / Gupta to indicate the existing choice between item reuse and material recycling.[51] On the contrary, in disassembly sequencing the term selective indicates the removal of only a necessary subset of parts of a core in order to retrieve a selected part.[52] From the definitions of the selective disassembly, we find that the latter (i.e., for the sequencing), the classification of an incomplete disassembly covers the selective disassembly totally. For the disassembly-to-order planning this distinction is not necessary, because if cores are disassembled, in most cases not all parts can

[51] Cf. Kongar / Gupta (2006b): *Disassembly to order*, p. 555.

[52] Cf. Srinivasan / Gadh (2002): *Selective disassembly sequence with geometric constraints*, p. 349, Shyamsundar / Gadh (1996): *Selective disassembly of virtual prototypes*, p. 3159, and Kara / Pornprasitpol / Kaebernick (2006): *Selective disassembly sequencing*, p. 37.

be reused. Presumably, there is always a fraction that has to be disposed of. Hence, here is the first selection necessary. But this selection is not considered selective disassembly. The extension by a recycling option does not—from the author's point of view—legitimate a special problem class, as every disassembly planning should be "selective" in this prospect.

After this overview of the relevant aspects of the disassembly planning, a first disassembly-to-order planning model by Kongar / Gupta is discussed. It contains important aspects, like the core condition, damaging, etc. It is a model considering complete disassembly, which facilitates the access to the matter. Unfortunately, the model makes some corrections necessary, but nevertheless it is a good access to the disassembly planning we focus on in this work.

2.3 A first disassembly-to-order planning model

Among the works mentioned in the literature review, the paper by Kongar / Gupta in 2006 is one that combines many aspects that need to be considered in the disassembly planning. These are acquisition, disassembly, transportation, and disposal cost, revenues, condition of the cores, as well as damaging. The approach is for complete disassembly, which is a good start into the disassembly planning. Furthermore, the approach is not limited to only a single core, which makes it possible to meet the demand for items and material by a mix of the available cores.[53] The model structure, i.e., an illustration of options to assign quantities to, is depicted in Fig. 2.1.

Unfortunately, the model shows some inconsistencies. These are marked by the grey boxes in the figure and are discussed in the sequel. The authors include two sources of revenues and in total eleven types of cost in their model. A simple example of 100 cores to be disassembled shall illustrate the model. For illustration purposes, we do neither consider multiple items nor material types. Each node of the tree in Fig. 2.1 holds information about the corresponding variable of the model (e.g., Y), the exemplary quantity (e.g., 100), and the corresponding revenues or cost (e.g., take-back cost). (The term "cost" is omitted in the figure for clear arrangement.)

Starting with 100 cores Y they might be disassembled for storage V, reuse X, recycling R, or disposal L. The sum of the cores in the four categories must equal 100. For the storage and the disposal the considerations end here. The items intended to be reused are separated in those that cannot

[53] Cf. Kongar / Gupta (2006b): *Disassembly to order*, pp. 552–559.

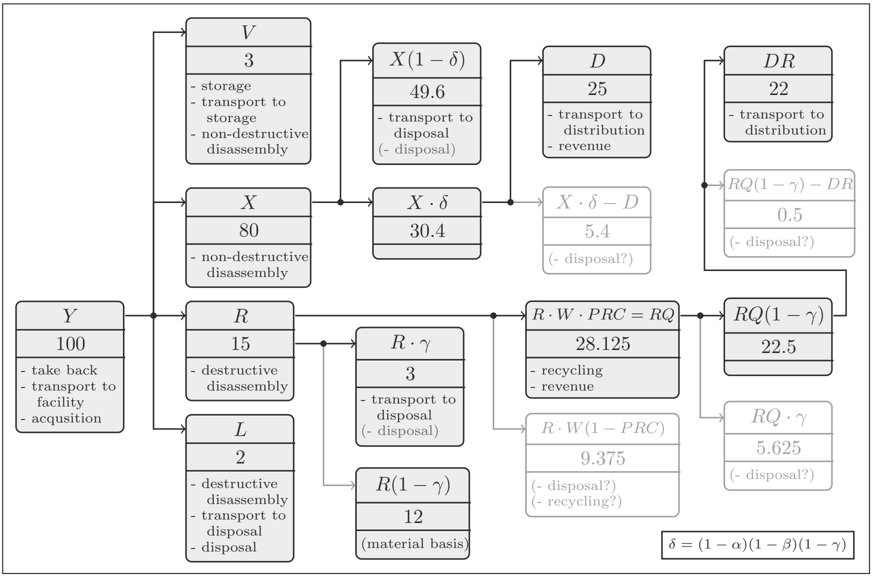

Fig. 2.1 Model structure

be reused ($X(1-\delta)$, with δ being the combined fraction of functioning, genuine, and undestroyed items) and the ones that can be reused (i.e., $X \cdot \delta$). Out of the reusable items the demand D is satisfied. The 15 destructively disassembled items intended for recycling are recycled within the facility. The resulting quantity of recycled material RQ depends on the weight W and a recyclable percentage PRC of the recycled items. From this recycled material only a fraction of $1-\gamma$ (γ is the fraction of non-genuine items) is of the correct material type and thus can be used to meet the given demand DR. Furthermore, a fraction of $R \cdot \gamma$ is transported to the disposal site.[54]

The five grey nodes in the figure are not considered in the model, but are important for illustrating missing item flow and absent cost considerations. Again, let us consider the items for reuse. The 80 items are non-destructively disassembled which leads to the corresponding cost. The unusable fraction of $1-\delta$ is transported to the disposal site, but no disposal cost are considered for this fraction. Out of the 30.4 usable items only 25 are demanded. These 25 items are transported to the distribution site and generate revenues. But, the exceeding 5.4 items are not considered in any way. The same applies

[54] In addition, the following values are used in the example: $\delta = 0.38$, $\gamma = 0.2$, $W = 2.5$, and $PRC = 0.75$.

to the recycling path of the tree. The fraction of $R \cdot \gamma$, that is transported to the disposal site, is not considered with disposal cost. The remaining fraction $R(1-\gamma)$ could be the basis for the material consideration, but is neglected here. This would be no problem, if the separation of γ and $1-\gamma$ was consequently applied after the material transformation.

Parallel to the just mentioned aspects of the recycling path, a given percentage of each item is the result of the recycling process. This quantity represents RQ which is basis for the recycling cost and the revenues, even though only DR are transported to the distribution site. The pendant to RQ, which is $R \cdot W(1 - PRC)$ is not considered by the authors. Interestingly, in the paper by Veerakamolmal / Gupta, that holds the data for the case example, it is stated with regard to PRC, that "the portion not recycled must be properly disposed of"[55]. In addition, the demand exceeding quantity of 0.5 is also not considered. A separate calculation of $RQ \cdot \gamma$ is not necessary, because it is part of the fraction of $R \cdot \gamma$.

The presented model includes a storage option, even though it does not consider multiple periods. It is also not a static or stochastic model that includes safety stock issues. Furthermore, there is no possibility to empty the storage, e.g., by selling the items. And lastly, the storage is not intended for material. As it seems from the model perspective the storage option equals the disposal and is therefore unnecessary.

In addition, the given optimal solution of their case example is not feasible (see appendix A). This alone is not a problem, but it decreases the understanding of their approach. In addition, missing information about the relationship between items and the material type makes it impossible to reproduce the results. In the paper the case example is based on (i.e., the paper by Veerakamolmal / Gupta)[56] no information can be found about material types, either.

With all these inconsistencies a sound modelling and application in practise is not possible. Therefore, a new model for planning complete disassembly with the aspects included in the above mentioned model is the start of the considerations in this work. In addition to the aspects above, a required purity level by the recycling company (or companies), labour time limitation, and special treatment of hazardous items is included. Thereby, the influence on the item usage caused by the expected conditions and the item and material flow inconsistencies are cleared.

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55 Veerakamolmal / Gupta (1999): *Design efficiency for disassembly*, p. 86.

56 Cf. Ibid..

Chapter 3
Complete disassembly planning

3.1 Basic model

3.1.1 Aspects to include in modelling complete disassembly

Next to KONGAR / GUPTA, the considerations are also motivated by the local city cleaning company SR-Dresden[1] in Dresden, Germany. This company takes back the cores from collection points and disassembles the cores for material recycling. The material recycling is conducted by other companies. The disassembly company does not decompose the cores totally. For example, only certain capacitors are removed from circuit boards, because of the hazardous material they consist of. But even though the disassembly is not complete it can be modelled as a complete disassembly by modelling modules as items. This is possible because they are never further disassembled. In addition, all cores are disassembled in the same way. The disassembly is conducted manually[2] and an important issue is the compliance with the required material purity by the recycling companies, which is not considered by KONGAR / GUPTA in their approach.[3]

KONGAR / GUPTA differentiate explicitly between non-destructive and destructive disassembly and base their cost calculation on the resulting item only. In addition, it is further assumed that when disassembling one item

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[1] `http://www.srdresden.de`.

[2] Manual disassembly is still state of the art. Cf. DUŢĂ / FILIP (2008): *Control and decision-making process in disassembling*, p. 25.

[3] Cf. KONGAR / GUPTA (2006b): *Disassembly to order*.

C. Ullerich, *Advanced Disassembly Planning*,
DOI 10.1007/978-3-658-03118-3_3, © Springer Fachmedien Wiesbaden 2014

destructively all other items are unaffected. There might exist cases where this is applicable, but in most cases this assumption cannot hold. To give an example let us consider the destructive disassembly of a car door. One destructive way of separating a door from a chassis of a car is just pulling hard enough. The result is a disconnected door and most likely a damaged chassis. Another destructive method is cutting the hinges which leads to a chassis with remains of the hinges. The point is that when disassembling an item destructively not only the item but the other items it is connected to are affected also.

Another problem occurs when referring the cost to the items. For instance, a nut and a bolt with the nut screwed on the bolt are two connected items. When unfastening the nut from the screw the core is completely disassembled. And the last item is automatically disassembled with the next to last. Thus, no work and no cost occur for the last item. One approach could be that the last item gets the cost factor zero. But, in a more complex core the last item is not always the same. This depends on the disassembly sequence. Thus, assigning the correct cost requires the determination of the optimal disassembly sequence for each possible disassembly result (including the items destroyed by the disassembly process) in advance and a subsequent cost assigning to the items. In theory this results in a determination of 2^n optima for a core consisting of n items in the worst case for complete disassembly. This can become too extensive to calculate. Therefore, if we assume that the resulting items after the disassembling can be used for reuse and/or recycling and that the cost does not differ,[4] the disassembly cost only depends on the number of cores and the complexity decreases significantly, because only one optimal disassembly sequence needs to be determined. For the example in Fig. 3.1 the optimal disassembly sequence is one of the ways through the graph from the node (AABC) to the node AABC. All other nodes of the lowest level are ignored.[5]

A basic model to determine the optimal quantities of cores to be disassembled into items to reuse, recycle, or dispose of is displayed in Fig. 3.2. Thereby, the Q^{C}, Q^{I}, Q^{R}, and Q^{D} denote the quantities of cores, items, recycling material, and waste, respectively. The X^{I}, X^{R}, and X^{D} represent the number of items assigned to reuse, recycling, and disposal, respectively. The X are the decision variables, because the Q can be easily calculated using

[4] Non-destructive disassembly is assumed.

[5] In Fig. 3.1 the "(...)" indicate modules with their constituent items and Ä indicates that item A is damaged during the destructive disassembly. An item outside the brackets is a single item. In total, 60 sequences are possible for a mix of non-destructive and destructive disassembly. On the contrary, if only non-destructive disassembly is conducted only three sequences are necessary to consider.

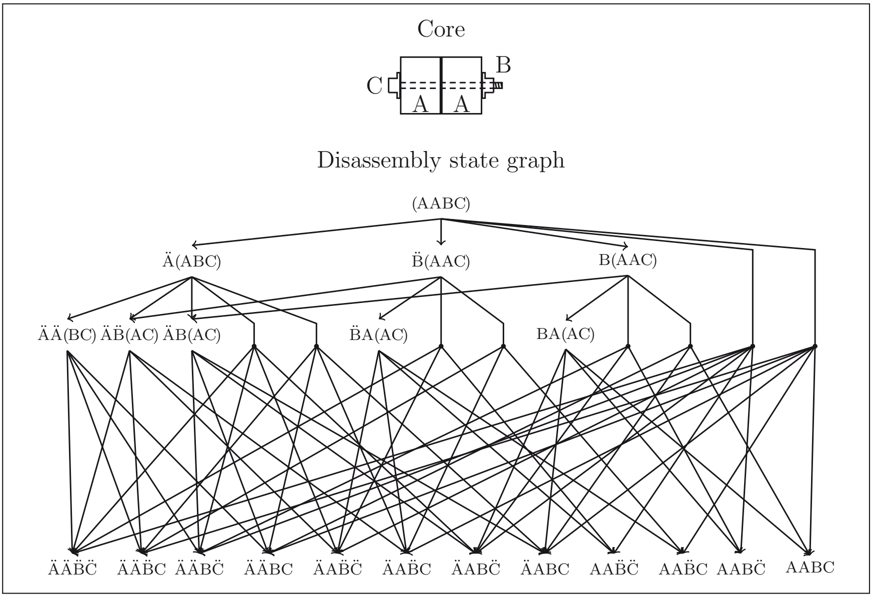

Fig. 3.1 Complexity of complete disassembly for illustrative example

the X. The quantity of cores to acquire Q^{C} in units determines the acquisition and disassembly cost. Note that the acquisition cost includes transport and take-back cost. On the other side, the quantity of items Q^{I} in units, material Q^{R} in kg, and disposal Q^{D} in kg are the basis for the revenues of items to reuse, revenues for material, and disposal cost, respectively. Note that the transportation cost is included in the revenues or disposal cost. The transformation from units to material is done by multiplication with the weight w in kg per unit.

The outgoing quantities are all limited by the demand D^{I} and D^{R} for items and material, respectively.[6] Further, the number of items to reuse is limited to genuine, functioning, and undamaged ones. The items of cores can be clustered according to their condition (see Fig. 3.3). Thereby, the condition is assumed to be unique for each item in a core. Items can be genuine or **non-genuine**. A genuine item is either an item that is in the core since the production of the core or a replaced item which is identical to the original item. Thus, a non-genuine item is a replaced item in the core that

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[6] Note that the demand is an upper limit. To fix the demand as in the to-order planning, the lower limit (introduced later) needs to be set to the same value.

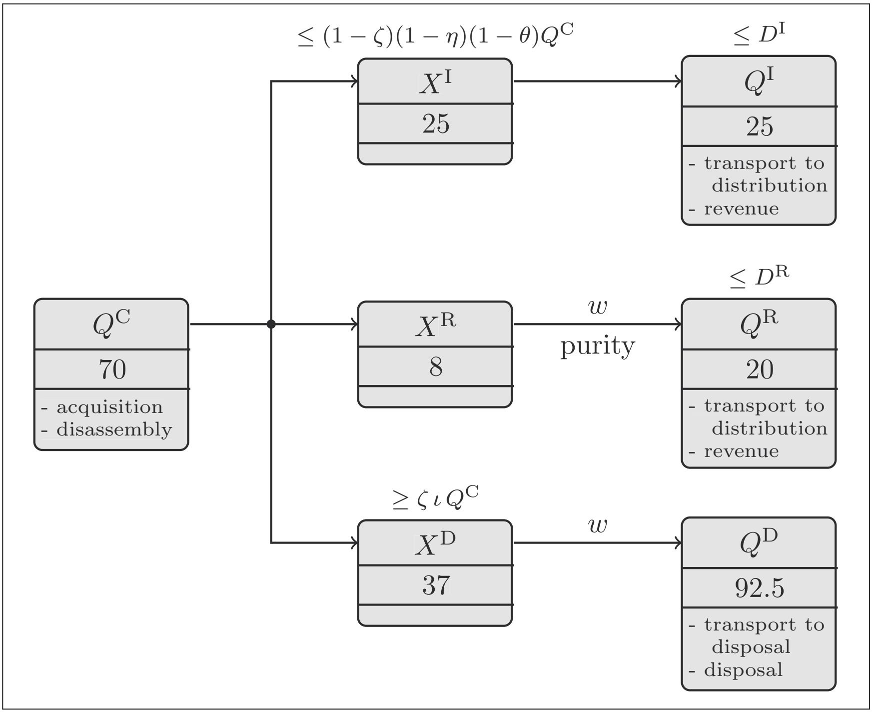

Fig. 3.2 Basic model structure for complete disassembly

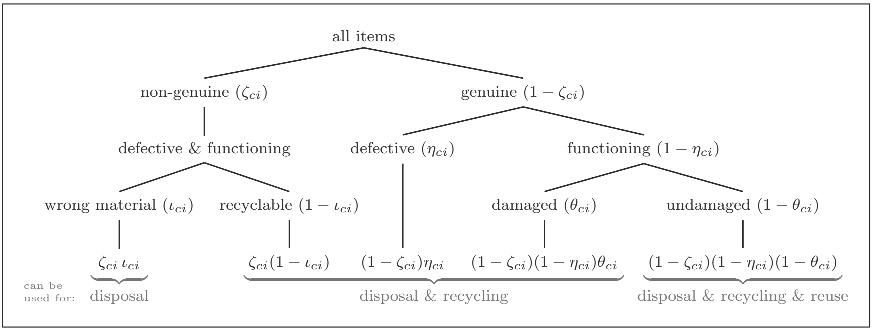

Fig. 3.3 Probability tree of core condition and damaging

is not identical to the item put in the product during production. Thereby, it is not important if the replaced item is even better than the original one. The probability that an item is non-genuine is denoted by ζ_{ci} for every core c and item i.[7] A second differentiation is if an item is functioning or **defective**. This differentiation is only necessary for genuine items, because non-genuine items have to be recycled or disposed of no matter if they are functioning or not. The probability for an item being defective is denoted by η_{ci}. A third differentiation regarding the condition can be done according to the material a non-genuine item consists of. Non-genuine items are either recycled or disposed of (see above). But the replaced non-genuine item can still be of the same material as the genuine one. If so, the non-genuine item can be used for recycling or disposal. If the non-genuine item is made of a different material, the non-genuine item must be disposed of, because it is assumed to be impossible to consider all possible materials non-genuine items consist of. The ι_{ci} denotes the probability that an item consists of the **wrong material**. With a probability of $(1 - \iota_{ci})$ it is recyclable.

A further influence on the usability of items for reuse is the **damage** during the disassembly. Of course, this is only relevant for genuine functioning items because all other items have to be recycled or disposed of anyway. The damaging is not a core condition because it is assumed that the damaging probability θ_{ci} only depends on the disassembly process, but it belongs to the determination of the probabilities an item has to be disposed of or can be recycled or reused.

As depicted in Fig. 3.3, with a probability of $\zeta_{ci}\,\iota_{ci}$ an item has to be disposed of because it is non-genuine and of the wrong material. Thereby, it is assumed that all mentioned probabilities are independent. On the other hand, an item can be reused with a probability of $(1-\zeta_{ci})(1-\eta_{ci})(1-\theta_{ci})$, because it is genuine, functioning, and undamaged. But a reusable item can also be recycled or disposed of. Thus, with the combined probabilities bounds of numbers of items can be identified.

To illustrate the above mentioned an example of disassembling 100 units of a core c shall be given. Let us further focus on just one item i of the core, such that 100 of these items in different conditions are gained. The probabilities for the three considered conditions and the damaging are $\zeta_{ci} = 0.2$, $\eta_{ci} = 0.5$, and $\iota_{ci} = 0.05$ and $\theta_{ci} = 0.1$, respectively. Given these probabilities we expect $100(1 - \zeta_{ci})(1 - \eta_{ci})(1 - \theta_{ci}) = 100 \cdot 0.8 \cdot 0.5 \cdot 0.9 = 36$ items to be reusable, i.e., items that are genuine, functioning, and undamaged. We further expect $100(1 - \zeta_{ci}\,\iota_{ci}) = 100 \cdot 0.99 = 99$ items to be recyclable. On the other hand, we expect $100\zeta_{ci}\,\iota_{ci} = 100 \cdot 0.01 = 1$ item to have to be disposed of because it is non-genuine and consists of the wrong material.

[7] In Fig. 3.2 the indices are neglected for a better overview.

Within these limits any allocation of items for reuse, recycling, and disposal is feasible. Hence, possible partitions are

- 36 items reuse, 63 items recycling, and one item disposal,
- zero items reuse, zero items recycling, and 100 items disposal,
- 25 items reuse, 50 items recycling, and 25 items disposal, etc.

Of course, the calculated values are expected values. This means that for a given batch the actual number of non-genuine items with the wrong material could be three instead of one. To avoid problems one could estimate the probabilities to the disadvantage of the company or consider a more thorough stochastic planning that includes, e.g., penalty costs for unmet demand as KIM / XIROUCHAKIS presented.[8] Eventually, the discrepancy between expected and actual numbers lessens with larger numbers. Thus, we assume this effect being negligible and the deterministic model is rather straight forward to facilitate the understanding of the problem.

As mentioned earlier, material purity is another key issue in disassembly planning. The recycling company can specify certain purity levels of the material they take for recycling. The purity property of an item is specific to the item of a core and the recycling process. We denote bins, lattice boxes, boxes, containers, etc., where the items are collected in for material recycling, as boxes r. Let us assume there exists a box for metal, a second for ceramics, and a third for plastics. Let us further assume that an exemplary item A consists of 40 g steel, 15 g glass, and 45 g plastics. In total the item weighs 100 g. Putting the item in the box for metal results in a purity of 40 %. On the contrary, putting it in the box for plastics 45 % purity of plastics is gained. Depending on the required purity, putting certain items in boxes is not allowed. But there exists the possibility to balance the impurity with other items of higher purity. An example could be the adding of one item B containing just 50 g steel. Putting both items in the box leads to 90 g steel and 150 g material in total. Thus, a purity of $\frac{90}{150} = 60\,\%$ of metal is achieved. The beneficial fractions π_{cir} of the two items are displayed in Table 3.1. The glass is undesirable, because no glass box exists. Derived from this, the sum of the beneficial fractions per item does not have to be 100 %. Furthermore, this approach also allows several quality levels for the same material (e.g.,

[8] Cf. KIM / XIROUCHAKIS (2010): *Capacitated disassembly scheduling*. For stochastic planning in disassembly the reader might find information in: TEUNTER (2006): *Optimal disassembly and recovery strategies*, INDERFURTH / LANGELLA (2006): *Heuristics for disassemble-to-order problems*, TIAN et al. (2012b): *Probability evaluation models of product disassembly cost*, LI et al. (2009): *Stochastic dynamic programming based model for re-manufacturing system*, and AGRAWAL / TIWARI (2008): *ACO to disassembly line balancing and sequencing*.

Table 3.1 Beneficial fractions for the exemplary items

		beneficial fraction for material box		
item	weight	metal	ceramics	plastics
A	100 g	40 %	0 %	45 %
B	50 g	100 %	0 %	0 %

steel and metal mix, instead of just metal). Having more than one box the material is beneficial for, the sum of all beneficial fractions per item might exceed the 100 %.

The disposal bins d are another output that is based on weight units. Generally, for disposal no limitations exist. But in the model not only one kind of disposal is considered. A separate treatment of, for instance, regular and hazardous disposal is accounted for. Rios / Stuart as well as Ferguson / Browne consider contaminating or hazardous material, too.[9] It is assumed that including one piece of hazardous waste (e.g., a battery) in any other box or disposal bin leads to a contaminated box or disposal bin. Thus, hazardous items can only be distributed (if demanded) or put into the hazardous disposal.

Because of considering the core condition each individual item of a core is denoted by an index. An approach with a quantity matrix as in Kongar / Gupta, Langella, Lambert / Gupta, and Vadde / Zeid / Kamarthi cannot be used, because two general identical items in different positions in the core might show different conditions caused by wearing etc.[10] In addition, for the incomplete disassembly (see Chap. 4) the position of an item in relation to other items is important such that two identical items cannot be seen as just an item that appears twice per core. Therefore, each item must be considered individually, which leads to a more comprehensive model.

[9] Cf. Rios / Stuart (2004): *Scheduling selective disassembly for plastics recovery*, pp. 188–189, and Ferguson / Browne (2001): *Issues in EOL recovery*, p. 540.

[10] For quantity matrix approaches cf. Kongar / Gupta (2006b): *Disassembly to order*, Langella (2007b): *Heuristics for demand-driven disassembly planning*, pp. 560 et seq., Lambert / Gupta (2002): *Demand-driven disassembly optimization*, p. 131, and Vadde / Zeid / Kamarthi (2011): *Pricing decisions for product recovery*, pp. 188 et seq.

Many papers focus on cost minimisation.[11] Others focus on profit maximisation.[12] And again others include multiple criteria.[13] In this work the focus is on the profit maximisation, because it includes more than just the cost. On the other hand, it includes all that is necessary in multiple criteria decision making with the exception of the trade-off between the single criteria. For example, a profit maximisation model includes the number of cores acquired to calculate the corresponding cost. If the workload is capacitated, an equation exists in the model to calculate the required workload. Taking these two exemplary included aspects an enhancement to a multiple criteria approach is straightforward.

The presented approach is explicitly multi-core. This requires in general a consideration of commonality and multiplicity. The latter occurs in single-core approaches, too. Commonality and multiplicity are usually issues in relation to items and modules. But for material it exists as well, because several items of a core or across cores can contain the same material. Hence the presented approach includes commonality and multiplicity for all types of output.

3.1.2 Model formulation

In the sequel the model is developed. The objective function is the profit P, which is calculated by subtracting the cost C from the revenues R. The profit is to be maximised.

$$\text{Maximise} \quad P = R - C \tag{3.1}$$

The revenues are the sum of all received payments for demanded items for reuse Q_e^{I} and demanded items for material recycling Q_r^{R}. The Q_e^{I} are in units so that the prices r_e^{I} are in €/unit. Analogically, the prices r_r^{R} in €/kg for material recycling are based on the Q_r^{R}, which are measured in weight

[11] Cf., e.g., Langella (2007b): *Heuristics for demand-driven disassembly planning*, Gharbi / Pellerin / Sadr (2008): *Production rate control for stochastic remanufacturing systems*, and Lambert / Gupta (2002): *Demand-driven disassembly optimization*.

[12] Cf., e.g., Johnson / Wang (1998): *Economical evaluation of disassembly operations*, Lambert / Gupta (2002): *Demand-driven disassembly optimization*, and Veerakamolmal / Gupta (1998): *Optimal analysis of lot-size balancing for multiproducts selective disassembly*.

[13] Cf., e.g., Kongar / Gupta (2006b): *Disassembly to order*, Kongar / Gupta (2002b): *A multi-criteria decision making approach*, and Kongar / Gupta (2002a): *Disassembly-to-order system using linear physical programming*.

units, i.e., kg. The index r denotes the boxes the items for material recycling are collected in and e denotes the demanded items. Note that not all items of a core have to be demanded.

$$R = \sum_e r_e^{\mathrm{I}} Q_e^{\mathrm{I}} + \sum_r r_r^{\mathrm{R}} Q_r^{\mathrm{R}} \tag{3.2}$$

In the case of complete disassembly the disassembly cost is identical for each unit of core c. Thus, the disassembly cost for the complete core c_c^{J} and the acquisition cost c_c^{A} occur for each core.[14] The quantity of cores Q_c^{C} is measured in units and the quantity of items disposed Q_d^{D} of is measured in kg. The corresponding disposal cost are denoted by c_d^{D}.

$$C = \sum_c \left(c_c^{\mathrm{A}} + c_c^{\mathrm{J}}\right) Q_c^{\mathrm{C}} + \sum_d c_d^{\mathrm{D}} Q_d^{\mathrm{D}} \tag{3.3}$$

The constraints of the model can be structured in the four groups

- item flow,
- core condition,
- purity, and
- limits,

which is illustrated in the sequel.

Item flow constraints

A multi-core approach has to deal with cores of different numbers of containing items. This information is stored in $\bar{I}_c$ and so the indexing of the items in each core starts with $c = 1$ and ends with $\bar{I}_c$, i.e., $i \in \{1, \ldots, \bar{I}_c\}$. The items of a core have different utilisation (see Fig. 3.2). One is for reuse. Thereby, X_{ci}^{I} denotes the number of item i of core c intended for item reuse. The second utilisation is material recycling, which is denoted by X_{cir}^{R}. For these items and the ones for disposal X_{cid}^{D} a third index is introduced that represents the assignment to the recycling box r or disposal bin d.[15]

[14] For an approach with a differentiated disassembly cost consideration with regard to destructive and non-destructive disassembly the reader is referred to appendix B.1.

[15] For a model with explicit modelling of non-destructive and destructive disassembly the reader is referred to appendix B.1. The differences are mainly in the item flow constraints and the objective function.

$$Q_c^{\mathrm{C}} = X_{ci}^{\mathrm{I}} + \sum_r X_{cir}^{\mathrm{R}} + \sum_d X_{cid}^{\mathrm{D}} \quad \forall\, c, i \in \{1, \ldots, \bar{I}_c\} \tag{3.4}$$

The transformation of the items into weight units is achieved by the multiplication of the corresponding weights w_{ci}. In each box or bin only the accumulated weight is of interest so that the sum over all cores and items is applied.

$$Q_r^{\mathrm{R}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{cir}^{\mathrm{R}} \quad \forall\, r \tag{3.5}$$

$$Q_d^{\mathrm{D}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{cid}^{\mathrm{D}} \quad \forall\, d \tag{3.6}$$

One last flow of items remains. It is the connection between the disassembled items for reuse X_{ci}^{I} and the distributed quantity Q_e^{I}. Let us assume there exists a demand for an item that appears thrice in core $c = 1$ and once in core $c = 2$. The corresponding indices shall be $i = 1$, $i = 5$, and $i = 17$ for core 1 and $i = 2$ for the item in core 2. All these four items accommodate the demand. The demand index for this item is $e = 1$ and in a set $\mathcal{P}_e$ all core item combinations (c, i) that accommodate the demand are stored, e.g., $\mathcal{P}_1 = \{(1, 1), (1, 5), (1, 17), (2, 2)\}$. The resulting quantity is the sum of all these items.

$$Q_e^{\mathrm{I}} = \sum_{(c,i) \in \mathcal{P}_e} X_{ci}^{\mathrm{I}} \quad \forall\, e \tag{3.7}$$

Obviously, all items that cannot be used to accommodate the demand must not be considered for reuse. Hence, the numbers of items to reuse of all core item combinations not in any demand set $\mathcal{P}_e$ are zero.

$$X_{ci}^{\mathrm{I}} = 0 \quad \forall\, (c, i) \notin \bigcup_e \mathcal{P}_e \tag{3.8}$$

Condition constraints

As explained above the condition of cores restricts the usability of the items for reuse and recycling. At least all non-genuine items with the wrong material have to be disposed of, regardless which disposal bin they are assigned to (see Fig. 3.3). The probabilities of an item being non-genuine is denoted by ζ_{ci} and consisting of the wrong material is denoted by ι_{ci}. Thus, the restriction for items assigned to disposal can be expressed as follows.

$$\sum_d X^{\mathrm{D}}_{cid} \geq \zeta_{ci} \iota_{ci} Q^{\mathrm{C}}_c \quad \forall\ c, i \in \{1, \ldots, \bar{I}_c\} \tag{3.9}$$

On the other hand, only items that are genuine, functioning, and do not get damaged during the disassembly can be reused. Thereby, the probabilities are denoted by $(1-\zeta_{ci})$, $(1-\eta_{ci})$, and $(1-\theta_{ci})$. This restriction only applies to items with a demand because otherwise the numbers are zero anyway (see Eq. (3.8)).

$$X^{\mathrm{I}}_{ci} \leq (1-\zeta_{ci})(1-\eta_{ci})(1-\theta_{ci}) Q^{\mathrm{C}}_c \quad \forall\ (c, i) \in \bigcup_e \mathcal{P}_e \tag{3.10}$$

Expressing the same restriction from a different point of view leads to the following equation. All items that are not genuine, functioning, and undamaged must either be recycled or disposed of.

$$\sum_r X^{\mathrm{R}}_{cir} + \sum_d X^{\mathrm{D}}_{cid} \geq (1-(1-\zeta_{ci})(1-\eta_{ci})(1-\theta_{ci}))\, Q^{\mathrm{C}}_c \quad \forall\ (c, i) \in \bigcup_e \mathcal{P}_e \tag{3.11}$$

Purity constraints

The required purity level of a recycling box is given by the external recycling company and denoted with ω_r. The beneficial fraction of an item for a specific box is given with the parameter π_{cir}. And the cumulated beneficial weight of all items in a box must exceed the required level.

$$\omega_r Q^{\mathrm{R}}_r \leq \sum_c \sum_{i=1}^{\bar{I}_c} \pi_{cir} w_{ci} X^{\mathrm{R}}_{cir} \quad \forall\ r \tag{3.12}$$

The aspect of hazardous items requires a little bit more modelling. For a first understanding, it is assumed that only one regular and one hazardous disposal bin exist in the company. Without loss of generality the regular disposal bin is the first one, i.e., $d = 1$. Thus, the second disposal bin is reserved for hazardous disposal. Furthermore, putting any hazardous item in a recycling box and disposal bin leads to a complete contamination of the box and bin, respectively. Thus, allocating a hazardous item to a box or regular waste bin is prohibited. The result is that hazardous items can only be distributed (if a demand exists) or disposed of in the hazardous disposal bin. Hazardous items are denoted by core item combinations (c, i) in the set $\mathcal{H}$.

$$X^{\mathrm{R}}_{cir} = 0 \quad \forall\ (c,i) \in \mathcal{H}, r \tag{3.13}$$

$$X^{\mathrm{D}}_{cid} = 0 \quad \forall\ (c,i) \in \mathcal{H}, d \in \{1\} \tag{3.14}$$

Consequently, if no hazardous items exist, the Eqs. (3.13) and (3.14) can be neglected. Or if more regular disposal bins exist, the set {1} in Eq. (3.14) just needs to be extended by the corresponding indices.

Limits constraints

As fourth group of constraints the limits and domain of the variables are listed. Generally, all interfaces to the outside of the model scope can be limited by lower and/or upper values. These variables are Q^{C}_c, Q^{I}_e, Q^{R}_r, and Q^{D}_d. The most important (upper) limits are the demand for items D^{I}_e and material D^{R}_r. Core availability $\overline{Q}^{\mathrm{C}}_c$ and a disposal quantity limitation $\overline{Q}^{\mathrm{D}}_d$ could be relevant, too. If contracts with business partners or legislative guidelines exist, these can be included with the lower limits $\underline{Q}^{\mathrm{C}}_c$, $\underline{Q}^{\mathrm{I}}_e$, $\underline{Q}^{\mathrm{R}}_r$, and $\underline{Q}^{\mathrm{D}}_d$, for cores, items, material, and disposal, respectively. If no such commitments exist, these parameters have a value of zero.

$$\underline{Q}^{\mathrm{C}}_c \leq Q^{\mathrm{C}}_c \leq \overline{Q}^{\mathrm{C}}_c \quad \forall\ c \tag{3.15}$$

$$\underline{Q}^{\mathrm{I}}_e \leq Q^{\mathrm{I}}_e \leq D^{\mathrm{I}}_e \quad \forall\ e \tag{3.16}$$

$$\underline{Q}^{\mathrm{R}}_r \leq Q^{\mathrm{R}}_r \leq D^{\mathrm{R}}_r \quad \forall\ r \tag{3.17}$$

$$\underline{Q}^{\mathrm{D}}_d \leq Q^{\mathrm{D}}_d \leq \overline{Q}^{\mathrm{D}}_d \quad \forall\ d \tag{3.18}$$

A further limitation of the disassembly process is the available labour time. The time needed to disassembly one core is given by t^{J}_c in hours per unit. The available labour time is denoted by $\bar{L}$ in hours.

$$\sum_c t^{\mathrm{J}}_c Q^{\mathrm{C}}_c \leq \bar{L} \tag{3.19}$$

The domain of the relevant variables is given by

$$X^{\mathrm{I}}_{ci}, X^{\mathrm{R}}_{cir}, X^{\mathrm{D}}_{cid} \in \mathbb{Z}^* \quad \forall\ c, i \in \{1, \ldots, \bar{I}_c\}, r, d\,. \tag{3.20}$$

With these variables the remaining variables (Q^{C}_c, Q^{I}_e, Q^{R}_r, Q^{D}_d, P, R, C) automatically are in the correct domain. Because of the Eqs. (3.2)–(3.7), where variables are set equal to other terms. The corresponding variables can be substituted with a term and thus the model can be reduced by these variables. This model formulation can be found in appendix B.2, but for a

Table 3.2 Number of decision variables and constraints

integer variables	$\left\|\bigcup_e \mathcal{P}_e\right\| + \left(\sum_c \bar{I}_c - \|\mathcal{H}\|\right)(r+d) + \|\mathcal{H}\|$
constraints	$2\sum_c \bar{I}_c + c + \left\|\bigcup_e \mathcal{P}_e\right\| + 3r + 2e + 2d + 1$

better understanding the variables are kept in the model formulation in the sequel.

To illustrate the model size the number of decision variables and constraints are listed in Table 3.2. The basis of the determination of the numbers is the compact model, i.e., the model formulation in appendix B.2. To avoid confusion with existing variables the number of indices is denoted by the index itself. This means, that the c in the table must be read as $\sum_c 1$. Hence, when writing $c \cdot r$ the interpretation is to calculate the number of cores times the number of recycling boxes. The X^{I}_{ci} only have values different than zero for elements of the set $\bigcup_e \mathcal{P}_e$, which leads to the entry $|\bigcup_e \mathcal{P}_e|$ in the table. The variables X^{R}_{cir} and X^{D}_{cid} occur $\sum_c \bar{I}_c$ times r and d, respectively. But, Eqs. (3.13) and (3.14) set the value of core item combinations of hazardous items equal to zero. Thus, these variables are excluded from the consideration. The resulting number of decision variables for a model with one hazardous disposal bin is depicted in the table. Note that for every index the number of variables and constraints increases linearly.

The number of constraints is determined by taking a look at each constraint and especially with a focus on the right side of the universal quantifier ($\forall$). Taking for example Eq. (3.9) or (B.25): this formulation represents $\sum_c \bar{I}_c$ constraints and not just a single one. For the determination only one set of constraints described by Eqs. (3.10) and (3.11) (see Eq. (B.26)) is included, because only one set is necessary. Adding all the constraints leads to the term in Table 3.2. As can be seen, the number of variables and constraints changes linearly with a variation of a single parameter, e.g., the number of cores c.

To illustrate the model size calculation a small example shall serve. Let us assume that two cores with four and six items, three recycling boxes, two disposal bins, one hazardous disposal bin, two hazardous items, two demand positions, and in total five items to accommodate the demand for items are considered. The number of decision variables results in $5 + ((4+6)-2)(3+2)+2 = 47$ and the number of constraints equals $2\cdot(4+6)+2+5+3\cdot 3+2\cdot 2+2\cdot 2+1 = 45$.

Summarising the above mentioned, the presented model maximises the profit of revenues from distributed items for reuse and items for material

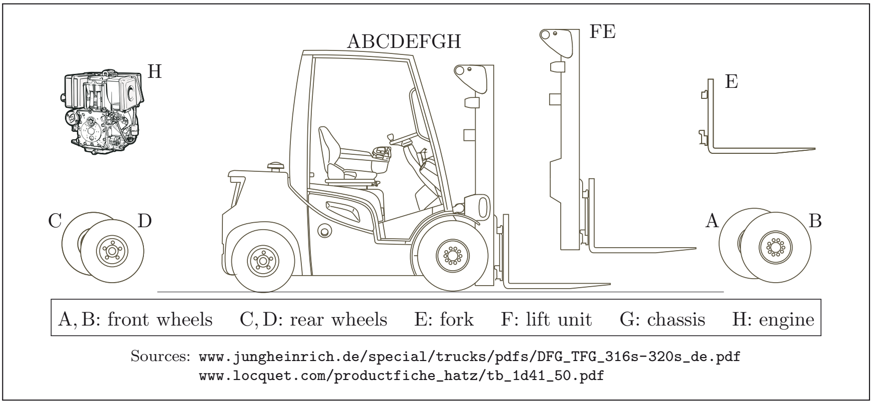

Fig. 3.4 Bill of materials of forklift truck

recycling and costs. The costs include core acquisition, disassembly, and disposal cost. The transport unit cost for cores, items (for reuse, recycling, and disposal) is included in the corresponding price or per unit cost. Furthermore, several material types with specific purity requirements and disposal types are modelled. The demand of items for reuse or material recycling forms the upper limit and does not necessarily has to be met. But, if a certain demand must be distributed, it is easily achieved by setting the lower distribution limit to the same values as the demand. Moreover, the condition of the core and thus the condition of the items within the cores form further restrictions on the use of an item. The three considered conditions correspond to the case that an item is non-genuine, defective, and of wrong material. In addition, the possibility of damage during the disassembly process is included, too.

3.1.3 Numerical example

3.1.3.1 Data

To illustrate the use of the model a first numerical example is presented in the sequel. It is a simplified case of three types of forklift trucks. One is powered by diesel, a second by gas, and the third by electricity. The general construction of these three is identical and is depicted in Fig. 3.4. The (abstract) eight items A through H may be identical across the three

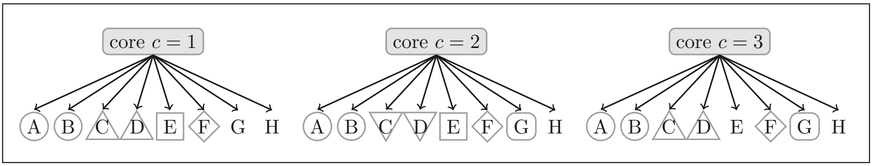

Fig. 3.5 Structure of cores

Table 3.3 Item commonalities and multiplicities

		item i							
core c	type	A	B	C	D	E	F	G	H
1	diesel	(1, A)	(1, A)	(1, C)	(1, C)	(1, E)	(1, F)		
2	gas	(1, A)	(1, A)	(2, C)	(2, C)	(1, E)	(1, F)	(2, G)	
3	electricitiy	(1, A)	(1, A)	(1, C)	(1, C)		(1, F)	(2, G)	

types or not. This information is displayed in Fig. 3.5 and Table 3.3. All cores (1–3) and items (A–H) are listed. Item A (a front wheel) is identical in all three cores and exists twice in each core, which can be identified by the tuple $(1, \mathrm{A})$ in all three rows of column $i = \mathrm{A}$ and $i = \mathrm{B}$. In the figure this information is depicted with the same shape around the label of the item, e.g., a circle. In addition, the chassis of the gas and electricity powered trucks are identical, too. This can be seen by the tuple $(2, \mathrm{G})$ and the rounded rectangle. On the contrary, the engines of all three cores are unique and therefore not listed in the table. An example of pure multiplicity is the back wheels of the gas powered truck $(2, \mathrm{C})$. Two of them exist in just one core. Each tuple in the table is a reference to a specific item in a core—a representative item.

The order the data is given follows more or less the flow in the disassembly process, i.e., from incoming cores to outgoing items and material. All three cores consist of eight items each, thus $\bar{I}_c = 8 \; \forall \, c$ (see Table 3.4). For each core certain quantities of cores to acquire are already fix and given by $\underline{Q}_c^{\mathrm{C}}$. An upper limit does not exist in this example. The cost per core for acquisition c_c^{A}, disassembly c_c^{J}, and the weights w_{ci} are given in the table. Moreover, the time for disassembling t_c^{J} a unit is assumed to be ten, nine, and eight hours for the cores. The labour time for the planning period is limited to $\bar{L} = 2{,}200\,\mathrm{h}$.

When the core is completely disassembled into its items, hazardous items need to be handled in a special way. The diesel engine shall be the only hazardous item in the numerical example. Thus, the set $\mathcal{H}$ contains only one

Table 3.4 Item weight, core acquisition cost and limit, disassembly cost, number of items per core

	w_{ci} item i												
c	A	B	C	D	E	F	G	H	$\underline{Q}_c^{\mathrm{C}}$	c_c^{A}	c_c^{J}	$\bar{I}_c$	t_c^{J}
1	11	11	8	8	40	180	950	200	30	2,300	300	8	10
2	11	11	7	7	40	180	900	150	50	2,600	280	8	9
3	11	11	8	8	36	180	900	100	25	2,900	260	8	8

Table 3.5 Probability of item being non-genuine, defective, of wrong material, and getting damaged

	$c=1$ item i								$c=2$ item i								$c=3$ item i							
	A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H
ζ_{ci}	0.1	0.1	0.15	0.15	0	0	0	0	0.1	0.1	0.1	0.1	0	0	0	0	0.1	0.1	0.15	0.15	0	0	0	0
η_{ci}	0.5	0.5	0.5	0.5	0.01	0.05	0.01	0.05	0.5	0.5	0.5	0.5	0.01	0.05	0.01	0.05	0.5	0.5	0.5	0.5	0.01	0.05	0.01	0.01
ι_{ci}	0	0	0.01	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0.01	0	0	0	0
θ_{ci}	0	0	0	0	0	0.01	0	0.02	0	0	0	0	0	0.01	0	0	0	0	0	0	0	0.01	0	0

Table 3.6 Demand limits, prices, demand position

e	$\underline{Q}_e^{\mathrm{I}}$	D_e^{I}	r_e^{I}	$\mathcal{P}_e$
1	20	250	30	$\{(1,\mathrm{A}),(1,\mathrm{B}),(2,\mathrm{A}),(2,\mathrm{B}),(3,\mathrm{A}),(3,\mathrm{B})\}$
2	0	300	300	$\{(1,\mathrm{E}),(2,\mathrm{E})\}$
3	10	300	2,400	$\{(2,\mathrm{G}),(3,\mathrm{G})\}$

core item combination and that is (1, H): $\mathcal{H} = \{(1,\mathrm{H})\}$. Further important core item related information is the condition of the acquired cores and its constituent items. The probabilities of items in cores being non-genuine, defective, of the wrong material, and become damaged during the disassembly process are ζ_{ci}, η_{ci}, ι_{ci}, and θ_{ci}, respectively. These are given in Table 3.5. For example, 15 % of all cores $c = 1$ contain a non-genuine item C and D and in every second core item A, B, C, and D are defective. Note that the probabilities are independent of each other.

Once the cores are disassembled the resulting items are either sold for reuse or for material recycling or are disposed of. Out of the possible 11 items (three cores times eight item minus the common and multiple items, see Table 3.3) three are demanded for reuse. These are item A of core 1 (1, A), item E of core 1 (1, E), and item G of core 2 (2, G) including the identical items in other cores. For each demand position e the core item combinations that meet the demand are stored in the set $\mathcal{P}_e$ (see Table 3.6).

Table 3.7 Quantity limits and cost information for material selling and disposal

material recycling				disposal		
r	$\underline{Q}_r^{\mathrm{R}}$	D_r^{R}	r_r^{R}	d	$\underline{Q}_d^{\mathrm{D}}$	c_d^{D}
1	2,000	1,000,000	1.35	1	0	0.2
2	0	1,000,000	0.95	2	0	0.4
3	0	1,000,000	0.75			
4	0	1,000,000	0.45			

Table 3.8 Minimum purity requirement and beneficial fractions

		beneficial fraction π_{cir}																							
		$c = 1$ i								$c = 2$ i								$c = 3$ i							
r	ω_r	A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H
1 steel	0.90	0.5	0.5	0.5	0.5	1	0.99	0	0.1	0.5	0.5	0.5	0.5	1	0.99	0	0.15	0.5	0.5	0.5	0.5	1	0.99	0	0.1
2 metal	0.85	0.5	0.5	0.5	0.5	1	0.99	0.7	0.97	0.5	0.5	0.5	0.5	1	0.99	0.7	1	0.5	0.5	0.5	0.5	1	0.99	0.7	1
3 rubber	0.50	0.5	0.5	0.5	0.5	0	0	0	0	0.5	0.5	0.5	0.5	0	0	0	0	0.5	0.5	0.5	0.5	0	0	0	0
4 plastics	0.95	0	0	0	0	0	0.01	0.3	0	0	0	0	0	0	0.01	0.3	0	0	0	0	0	0	0.01	0.3	0

Furthermore, the lower limits $\underline{Q}_e^{\mathrm{I}}$ to distribute, the demand, as well as the prices r_e^{I} are listed in the same table.

The quantities, prices and unit cost for the material distribution and disposal are listed in Table 3.7. The disposal bin $d = 2$ holds the hazardous items. Therefore, the unit cost is higher than the one of the regular disposal. An upper limit for disposal is not assumed.

Lastly, the purity requirements for the distribution need to be considered. In total four categories of boxes for collecting material exist: steel, metal, rubber, and plastics. The minimum purity ω_r of a steel box is 90 % and of a metal mix box 85 % (see Table 3.8). Each item contributes more or less to this purity and this is expressed by the beneficial fraction π_{cir} for each item i of core c in regard to box r. For example, item A consists of 50 % steel and 50 % rubber. Thus, the beneficial fraction of the weight of this item is 50 % for the steel and 50 % for the rubber box. But also for the metal mix box the steel percentage of the item benefits to it. So, 50 % is also beneficial to box $r = 2$. On the contrary, item E consists of metal (e.g., aluminium) and plastics. The beneficial fraction for the steel box is zero, because aluminium is no steel. For the metal mix box the complete fraction of aluminium is beneficial. (Even though the minimum purity for plastics is not achievable, the box $r = 4$ is listed for illustration.)

Table 3.9 Optimal solution of the basic model

variables representing the interfaces							
Q_1^C	30	Q_1^I	216	Q_1^R	52,610	Q_4^R	0
Q_2^C	188	Q_2^I	215	Q_2^R	60,500	Q_1^D	32
Q_3^C	25	Q_3^I	210	Q_3^R	0	Q_2^D	6,000

integer variables																					
	X_{ci}^I			X_{cir}^R												X_{cid}^D					
				$r=1$			$r=2$			$r=3$			$r=4$			$d=1$			$d=2$		
	c			c			c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13	84	11	17	104	14	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
B	13	84	11	17	104	14	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
C	.	.	.	29	188	24	.	.	.	.	.	.	.	.	.	1	.	1	.	.	.
D	.	.	.	29	188	24	.	.	.	.	.	.	.	.	.	1	.	1	.	.	.
E	29	186	.	1	2	25	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
F	.	.	.	30	188	25	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
G	.	186	24	1	.	.	29	2	1	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	3	.	.	185	25	.	.	.	.	.	.	.	.	.	30	.	.

A dot denotes a value of zero.

3.1.3.2 Results

According to the data (three cores of eight items each, three demand positions, in total $|\bigcup_e \mathcal{P}_e| = 10$ demanded items, four recycling boxes, two disposal bins and one hazardous item) the resulting model size is $10 + (24 - 1)(4+2)+1 = 149$ integer variables and $2\cdot 24+3+10+3\cdot 4+2\cdot 3+2\cdot 2+1 = 84$ constraints (see Table 3.2). Solving this model—in a fraction of a second with GUROBI 5.0—with the given (fictive) data, results in a maximal profit of $P = 2{,}632.1$ €.[16] The revenues and cost are 703,478.5 € and 700,846.4 €, respectively. The values of remaining variables are displayed in Table 3.9.

This profit is achieved by disassembling 30, 188, and 25 units of core 1, 2, and 3, respectively (see Table 3.9). In the lower section of Table 3.9 (integer variables) it is given how the cores should be treated. For example, 30 units of core $c = 1$ are completely disassembled. Thereby, 13 units of item $i = \text{A}$, 13 units of item $i = \text{B}$, and 29 units of item $i = \text{E}$ are appointed for item reuse. According to Table 3.5, 10 % and 50 % of the items $i = \text{A}$ of core $c = 1$ are expected to be non-genuine and defective, respectively. This means that only 45 % (90 % times 50 %) of the items are expected to be applicable for

[16] The computer used for the calculation is one with two AMD Opteron 6282SE 2.6 GHz and eight threads used for the GUROBI 5.0 solver.

item reuse, i.e., $\lfloor 13.5 \rfloor = 13$ of the 30 units.[17] This limitation is considered. The remaining 17 units of item $i = \text{A}$ must be recycled or disposed of. As can be seen in Table 3.9, 17 units are intended for material recycling for box $r = 1$. All together the 30 units of item A of core 1 are reused or recycled.

Another limiting aspect is the purity requirement.[18] 29 units of item G of core 1 are allocated into recycling box $r = 2$, i.e., the metal box. One item weighs 950 kg. Thus, 29 units make 27,550 kg. But only 70 % are beneficial for metal. (The remaining 30 % are plastics.) This alone would not be sufficient for the purity requirement of 85 % (see Table 3.8). Therefore, further (more pure) metal is needed, e.g., two, one, 185, and 25 units of items $i = \text{G}$ of core $c = 2$, $i = \text{G}$ of core $c = 3$, $i = \text{H}$ of core $c = 2$, and $i = \text{H}$ of core $c = 3$, respectively, (see Table 3.9). Taking the weights of the items and the units $950 \cdot 29 + 900 \cdot 2 + 900 \cdot 1 + 150 \cdot 185 + 100 \cdot 25 = 60{,}500$ kg material are in recycling box $r = 2$ (see Q_2^{R} in Table 3.9). The beneficial weight results in $(950 \cdot 29 + 900 \cdot 2 + 900 \cdot 1)0.7 + (150 \cdot 185 + 100 \cdot 25)1 = 51{,}425$ kg (see Table 3.8), which is 85 % of the material weight of 60,500 kg. Hence, the purity limitation $\omega_2 = 0.8$ is satisfied. Item $i = \text{H}$ of core $c = 1$ is the only hazardous item. Thus, it can only be reused or disposed of as hazardous waste. As can be seen, it is optimal to dispose of all 30 items, because no demand exists for reusing these hazardous items.

Furthermore, the lower core acquisition limits of 30, 50, and 25 units are considered and two of them are limiting, i.e., of core 1 and 3 (see Table 3.9). On the contrary, the lower limits for demanded items as well as the one for material recycling are not binding (apart from the non-negativity constraint). The same applies to the demand because the Q_e^{I} are less than the D_e^{I}. For the material recycling the demand is big enough so that a lot more material could be distributed. But this would lead to less profit, which would be suboptimal. For disassembling the units according to the solution $10 \cdot 30 + 9 \cdot 188 + 8 \cdot 25 = 2{,}192$ h are needed (see Eq. (3.19)). This is slightly less than the available $\bar{L} = 2{,}200$ h, which might be caused by the integrality constraints.

With the presented approach the disassembling companies are now able to determine the profit maximal quantities of cores to acquire, items to distribute, material to recycle, and waste to dispose of, when prices and unit cost are quantity independent. For larger quantities a dependency between the quantities and the prices and/or unit cost might occur, which is focused on in the following section.

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[17] The rounding down is caused by the integrality constraint for variables X_{cir}^{R}.

[18] It is limiting because a marginal decrease of ω_2 results in a different solution with a higher profit.

3.2 Linear price-quantity dependencies

3.2.1 Applicable quantity and price dependencies

In approaches of optimising the cost or the profit in disassembly planning usually no quantity dependent unit cost and prices are considered.[19] Of course, such an approach without quantity dependency allows an analysis of properties in disassembly planning already. But, when obvious dependencies exist, a thorough analysis is to favour. And disassembly companies do have interfaces to other members on markets, e.g., suppliers and customers. Thus, the modelled acquiring of cores, distributing of items, recycling of material, and the disposal are aspects with possible quantity dependencies.

A price-quantity dependency can occur in arbitrary forms. This could be discrete, discontinuous, and continuous combined with arbitrary, progressive, linear, or degressive and increasing, decreasing, or constant progress. However, in the literature one can find continuous linear, isoelastic, exponential, algebraic, and several other non-linear price-consumption or price-demand functions, i.e., the modelled dependency between price and quantity.[20] Determining the correct dependency is not an easy task, which usually results in an approximation using somewhat adequate and easy to handle functions. One of these is surely a linear function, which shall be focused

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[19] Cf. Kim / Lee / Xirouchakis (2007): *Disassembly scheduling*, pp. 4468 et seq., Langella (2007b): *Heuristics for demand-driven disassembly planning*, pp. 559 et seqq., Lee / Xirouchakis (2004): *Two-stage heuristic for disassembly scheduling*, pp. 289 et seq., Kongar / Gupta (2006b): *Disassembly to order*, pp. 552 et seqq., as well as Inderfurth (2004): *Optimal policies in hybrid manufacturing systems*, p. 329.

[20] For linear functions cf. Choi (1991): *Price competition in a channel structure*, p. 275, Choi (1996): *Price competition*, p. 122, Ingene / Parry (2007): *Bilateral monopoly*, p. 589, Berndt / Cansier (2007): *Produktion und Absatz*, p. 177, Xie / Neyret (2009): *Co-op advertising and pricing models*, p. 1376, Xie / Wei (2009): *Coordinating advertising and pricing*, p. 787, Zhang / Liu / Wang (2012): *Pricing decisions*, p. 524, and Wu / Chen / Hsieh (2012): *Competitive pricing decisions*, p. 267; for isoelastic functions cf. Yue et al. (2006): *Coordination of cooperative advertising*, p. 68, Szmerekovsky / Zhang (2009): *Pricing and two-tier advertising*, p. 906, and Esmaeili / Zeephongsekul (2010): *Seller-buyer models*, p. 147; for algebraic functions cf. Mesak / Mayyasi (1995): *Simple model of international joint venture distributorships*, p. 527 and SeyedEsfahani / Biazaran / Gharakhani (2011): *Coordinate pricing*, p. 265; for exponential functions cf. Mesak / Mayyasi (1995): *Simple model of international joint venture distributorships*, p. 527; for arbitrary non-linear cf. Amir / Stepanova (2006): *Second-mover advantage and price leadership*, p. 8, and Ding / Ross / Rao (2010): *Price as an indicator*, pp. 69 et seqq.

on in the sequel.[21] Thereby, three cases are distinguished, i.e., an increasing, a constant, and a decreasing progress of a linear function. An extension of the linear function can be found in Sect. 3.3 using piecewise linear functions.

In general, two points of view can be differentiated. One is the price being the exogenous variable and the quantity the endogenous one and the second possibility is the quantity being the exogenous and the price the endogenous variable. In the case of a linear dependency both points of view can easily be converted into another, because no ambiguities occur when inverting the corresponding functions. (A counter example is, for instance, a quadratic function, where two different exogenous values lead to the same endogenous value, e.g., $(x-2)^2 = y$.) Thus, for modelling purposes it is irrelevant which of the two variables is the exogenous and which the endogenous as long as the functional dependency is clear.

Focusing back on the disassembling company the first price-quantity dependency is the one for cores. W.l.o.g. the quantity is assumed to be the exogenous variable, i.e., the one that is set. The price (or unit cost) is the endogenous variable, i.e., the price levels according to the quantity comparable to quantity discounts. What effects, in terms of price and unit cost, depend on the quantity of incoming cores? The first aspect is the limit of cores on the market. Since the availability of cores depends on the return of used products by consumers an infinite availability of cores does not exist. Thus, when increasing the quantity a competitive situation occurs that leads to increasing prices of cores. This behaviour can also be motivated by the fact that with higher prices, because of the competition, consumers are willing to give away their used products earlier for higher return prices, which in turn leads to higher quantities of cores on the market. (This behaviour could be monitored with the so-called "Umweltprmie" in Germany for used cars.)[22]

A second aspect is the transport distance. Again, caused by the limited availability of cores near by the disassembling company it is assumed that the transport distances increase with increasing quantities of cores acquired. Of course, there might exist levels where, for instance, cheaper transport facilities can be used or other economies of scale might apply, but the general tendency is an increasing unit cost. A third aspect could be a quality reduction of the cores, because the company cannot choose the best picks anymore. Worse quality cores might have a lower price but might lead to higher cleaning, testing, and disassembly cost. These effects are difficult to

[21] The most commonly used functions are linear and isoelastic functions, cf. Lau / Lau (2003): *Effects of a demand-curve's shape*, pp. 530 et seq.

[22] Cf. BAFA (2009): *Richtlinie zur Förderung des PKW-Absatzes*.

quantify and therefore it is assumed that the quality reduction is negligible. (A detailed consideration beyond the scope of this work would surely be imaginable.) Summarising the above mentioned, on the market of cores a positive correlation between quantity and price and unit cost is assumed for the disassembly company.

The distribution of items and material recycling are discussed jointly as they follow the same rules with only differing in the distribution target. With increasing quantities of items and material being distributed the price of a unit decreases. Of course, for low quantities this might not apply, because a certain critical mass needs to be reached to make the usage of used items and material economically beneficial, but firstly we assume that the disassembling company already is running—and thus the critical mass is exceeded—and secondly legal might apply that assure a minimal quantity of distribution. An example is the end-of-life vehicles regulation in Germany where on average at least 85 % of the weight of the cars per year must be reused or recycled (incl. energetic recycling) and at least 80 % have to be reused or material recycled. In 2015 the targets raise to 95 and 85 %, respectively.[23] All in all, we assume that if quantities increase the price decreases.

The last customer relationship is the one to the disposal companies. Let us assume that no demand exists for waste, because otherwise it could be sold as material. Though, one tangible aspect is the limited space for landfilling. But not only this; in general, the resource earth is limited and thus every additional unit of waste should lead to increasing disposal cost. Reasons could be increasing distances, legal regulations, etc. Even though the ultimate goal is avoiding any waste, we know that it is practically not realisable, yet.

Even though, cross dependencies among the cores or items etc. exist (e.g., transporting different cores from one supplier with the same truck or picking up cores from and delivering items to the same company) they shall be neglected in the sequel. Furthermore, all suppliers of one core are aggregated to one supplier, i.e., the market. The same applies to the distribution, recycling, and disposal. In addition, the company in focus does not consider other disassembling companies directly and their effects on each other. Thus, the disassembling company is faced with independent suppliers and customers and has no competitor, which makes this consideration comparable to a simple game theoretic approach. Nonetheless, it is a first inclusion of quantity dependent pricing in disassembly planning to the best of our knowledge. An extended game theoretic examination of the business

[23] Cf. 5 (1) Verordnung ber die berlassung, Rcknahme und umweltvertrgliche Entsorgung von Altfahrzeugen (AltfahrzeugV) [=end-of-life vehicles regulation].

relations and the behaviour of the player might be of interest, but would go beyond the scope of this work.

3.2.2 Proof of concavity for the objective function

The general linear function is $y = mx + n$. For monotone increasing linear functions the slope m is positive and for decreasing functions negative. If the slope equals zero, the function is constant, i.e., neither increasing nor decreasing. To introduce the slope and the fixed term into the optimisation model additional parameters are necessary. These are based on the original ones (e.g., c_c^{A}) and get accents. The fixed term is marked with a bar (e.g., $\bar{c}_c^{\mathrm{A}}$) and the slope with a hat (e.g., $\hat{c}_c^{\mathrm{A}}$). Thus, the according price and cost parameters in the revenue and cost functions in Eqs. (3.2) and (3.3) are substituted by a linear expression depending on the quantity—just like: $\bar{c}_c^{\mathrm{A}} + \hat{c}_c^{\mathrm{A}}\, Q_c^{\mathrm{C}}$ replaces c_c^{A}. This substitution leads to

$$R = \sum_e \left(\bar{r}_e^{\mathrm{I}} + \hat{r}_e^{\mathrm{I}}\, Q_e^{\mathrm{I}}\right) Q_e^{\mathrm{I}} + \sum_r \left(\bar{r}_r^{\mathrm{R}} + \hat{r}_r^{\mathrm{R}}\, Q_r^{\mathrm{R}}\right) Q_r^{\mathrm{R}} \tag{3.21}$$

and

$$C = \sum_c \left(\bar{c}_c^{\mathrm{A}} + \hat{c}_c^{\mathrm{A}}\, Q_c^{\mathrm{C}} + c_c^{\mathrm{J}}\right) Q_c^{\mathrm{C}} + \sum_d \left(\bar{c}_d^{\mathrm{D}} + \hat{c}_d^{\mathrm{D}}\, Q_d^{\mathrm{D}}\right) Q_d^{\mathrm{D}}\,. \tag{3.22}$$

According to the argumentation above, the prices of items and material decrease with increasing quantities, i.e., the slope of the price function is negative. Hence, $\hat{r}_e^{\mathrm{I}}$ and $\hat{r}_r^{\mathrm{R}}$ are less than or equal zero. On the contrary, unit costs for acquisition and disposal increase with increasing quantities, which means that the slope of the unit cost function is positive. Therefore, $\hat{c}_c^{\mathrm{A}}$ and $\hat{c}_d^{\mathrm{D}}$ are greater than or equal zero.

The resulting objective function $P = R - C$ is a quadratic polynomial, because when expanding the function not only decision variables are multiplied with parameters (e.g., $\bar{r}_e^{\mathrm{I}}\, Q_e^{\mathrm{I}}$), but also two decision variables are multiplied (e.g., $\hat{r}_e^{\mathrm{I}}\, Q_e^{\mathrm{I}}\, Q_e^{\mathrm{I}}$). In this context, the parameters are not relevant. Just the decision variables determine the model (and polynomial) class. Hence, the model is no longer a mixed integer linear problem (MILP). Now it belongs to the class of mixed integer quadratic problems with linear constraints (MIQLP). Usually the "L" in the abbreviation is neglected, but here it shall emphasise that the problem only contains linear constraints. Obviously, this problem cannot be solved with a linear solver anymore.

Solving an arbitrary problem of this class requires solution approaches of general non-linear problems. But with particular properties a subset of this problem class can be solved fairly easily. The required property is that the objective function is concave (for maximisation problems) and continuous and that the first order derivative is continuous (i.e., degree 1 continuity), too. The property of being concave assures that the optimisation direction (towards the optimum) is clear. The other two properties (summarised as: G^1 continuous) are fulfilled by quadratic functions.

In order to apply a solution algorithm that requires concave quadratic function, concavity of the objective function needs to be proved. One possibility is the transformation of the objective function into a matrix and vector notation. The basic form is $P(\mathbf{x}) = \mathbf{c}^{\mathrm{T}}\mathbf{x} + \mathbf{x}^{\mathrm{T}}\mathbf{D}\mathbf{x}$. In this case the matrix $\mathbf{D}$ must be negative semi-definite in order to be a concave function $P(\mathbf{x})$.[24] The variables Q_e^{I}, Q_r^{R}, Q_c^{C}, Q_d^{D}, X_{ci}^{I}, X_{cir}^{R}, and X_{cid}^{D} form the vector $\mathbf{x}$. Vector $\mathbf{c}$ and matrix $\mathbf{D}$ contain the corresponding parameters such that the functions are identical.[25]

$$P(\mathbf{x}) = \mathbf{c}^{\mathrm{T}}\mathbf{x} + \mathbf{x}^{\mathrm{T}}\mathbf{D}\mathbf{x} \tag{3.23}$$

$$= \begin{pmatrix} \bar{r}_e^{\mathrm{I}} \\ \bar{r}_r^{\mathrm{R}} \\ -\bar{c}_c^{\mathrm{A}} - c_c^{\mathrm{J}} \\ -\bar{c}_d^{\mathrm{D}} \\ 0 \\ 0 \\ 0 \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} Q_e^{\mathrm{I}} \\ Q_r^{\mathrm{R}} \\ Q_c^{\mathrm{C}} \\ Q_d^{\mathrm{D}} \\ X_{ci}^{\mathrm{I}} \\ X_{cir}^{\mathrm{R}} \\ X_{cid}^{\mathrm{D}} \end{pmatrix} + \begin{pmatrix} Q_e^{\mathrm{I}} \\ Q_r^{\mathrm{R}} \\ Q_c^{\mathrm{C}} \\ Q_d^{\mathrm{D}} \\ X_{ci}^{\mathrm{I}} \\ X_{cir}^{\mathrm{R}} \\ X_{cid}^{\mathrm{D}} \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \hat{r}_e^{\mathrm{I}} & 0 & 0 & 0 & 0\,0\,0 \\ 0 & \hat{r}_r^{\mathrm{R}} & 0 & 0 & 0\,0\,0 \\ 0 & 0 & -\hat{c}_c^{\mathrm{A}} & 0 & 0\,0\,0 \\ 0 & 0 & 0 & -\hat{c}_d^{\mathrm{D}} & 0\,0\,0 \\ 0 & 0 & 0 & 0 & 0\,0\,0 \\ 0 & 0 & 0 & 0 & 0\,0\,0 \\ 0 & 0 & 0 & 0 & 0\,0\,0 \end{pmatrix} \begin{pmatrix} Q_e^{\mathrm{I}} \\ Q_r^{\mathrm{R}} \\ Q_c^{\mathrm{C}} \\ Q_d^{\mathrm{D}} \\ X_{ci}^{\mathrm{I}} \\ X_{cir}^{\mathrm{R}} \\ X_{cid}^{\mathrm{D}} \end{pmatrix} \tag{3.24}$$

The $\hat{r}_e^{\mathrm{I}}$ and $\hat{r}_r^{\mathrm{R}}$ are less than or equal zero. On the contrary, the $\hat{c}_c^{\mathrm{A}}$ and $\hat{c}_d^{\mathrm{D}}$ are greater than or equal zero. Thus, all elements of the main diagonal of matrix D are negative or zero. All other elements of the matrix are zero, because the quadratic terms only result of multiplications with the same variable (e.g., $\left(Q_c^{\mathrm{C}}\right)^2$) and not with another variable (e.g., $Q_c^{\mathrm{C}}\, Q_e^{\mathrm{I}}$). Therefore, matrix $\mathbf{D}$ is negative semi-definite, which means that the objective function is concave. Hence, the problem belongs to the class of relatively easy to solve quadratic problems. In the sequel, a numerical example illustrates this.

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[24] Cf. Ellinger / Beuermann / Leisten (2003): *Operations Research*, pp. 205–209.

[25] The arrays X_{ci}^{I}, X_{cir}^{R}, and X_{cid}^{D} must be vectorised to be put into the vector $\mathbf{x}$ and $\mathbf{c}$ or the matrix $\mathbf{D}$.

Table 3.10 Data for linear price-quantity dependent function

c	$\bar{c}_c^{\mathrm{A}}$	$\hat{c}_c^{\mathrm{A}}$	e	$\bar{r}_e^{\mathrm{I}}$	$\hat{r}_e^{\mathrm{I}}$	r	$\bar{r}_r^{\mathrm{R}}$	$\hat{r}_r^{\mathrm{R}}$	d	$\bar{c}_d^{\mathrm{D}}$	$\hat{c}_d^{\mathrm{D}}$
1	2,220	2.2	1	31	−0.004	1	1.43	−0.0000014	1	0.2	0.0000002
2	2,095	2.7	2	350	−0.22	2	1.00	−0.0000009	2	0.4	0.0000004
3	2,900	2.8	3	2,500	−0.49	3	0.75	−0.0000007			
						4	0.45	0			

3.2.3 Numerical example

Basically all data is identical to the one given in the section before (see Tables 3.3–3.8). Only the parameters c_c^{A}, c_d^{D}, r_e^{I}, and r_r^{R} are substituted by their pendants $\bar{c}_c^{\mathrm{A}}$ and $\hat{c}_c^{\mathrm{A}}$ etc. The fixed terms as well as the slopes are chosen in a way that the profit more or less equals the one of the former model when applying the solution of Table 3.9. The data is listed in Table 3.10.

Using the solution in Table 3.9 a profit of 2,614.41 € results and this almost equals the 2,631.1 € of the example in the preceding section. Solving the model results in an increased profit of $P = 10{,}897.13$ € with revenues $R = 564{,}735.23$ € and cost $C = 553{,}838.10$ €. The remaining values of the variables are depicted in Table 3.11. The solution is significantly different than the preceding one. 45 units less of core 2 are acquired. This leads to less workload, less items for reuse (on average by 130/3), less material for steel recycling (by 17,082 kg), a bit more material for metal recycling (by 590 kg), and material for rubber recycling of $Q_3^{\mathrm{R}} = 562$ kg. In addition, the allocation of the items represented by the X_{ci}^{I} and X_{cir}^{R} have changed accordingly. The exemplary problem is solved to optimum in 14 s with GUROBI 5.0.[26] The number of variables and constraints is identical to that of the preceding model.

This section represents a first step in including market dependencies. In the following section the approach is expanded in order to allow a more detailed modelling of the existing price-quantity dependencies. Nevertheless, drawbacks still exists because the focus is still on approximations and not on arbitrary dependency functions.

[26] From the longer solving time of this example one cannot derive a general worse solving time for quadratic problems compared to linear ones.

Table 3.11 Optimal solution of the quadratic model

variables representing the interfaces							
Q_1^{C}	30	Q_1^{I}	176	Q_1^{R}	35,528	Q_4^{R}	0
Q_2^{C}	143	Q_2^{I}	170	Q_2^{R}	60,990	Q_1^{D}	32
Q_3^{C}	25	Q_3^{I}	165	Q_3^{R}	562	Q_2^{D}	6,000

integer variables																					
	X_{ci}^{I}			X_{cir}^{R}												X_{cid}^{D}					
				$r=1$			$r=2$			$r=3$			$r=4$			$d=1$			$d=2$		
	c			c			c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13	64	11	5	42	14	.	.	.	12	37	.	.	.	.	.	.	.	.	.	.
B	13	64	11	17	79	14	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
C	.	.	.	29	142	24	.	.	.	.	1	.	.	.	.	1	.	1	.	.	.
D	.	.	.	29	143	22	.	.	.	.	.	2	.	.	.	1	.	1	.	.	.
E	29	141	.	1	.	.	.	2	25	.	.	.	.	.	.	.	.	.	.	.	.
F	.	.	.	30	136	.	.	7	25	.	.	.	.	.	.	.	.	.	.	.	.
G	.	141	24	.	1	.	30	1	1	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	143	25	.	.	.	.	.	.	.	.	.	30	.	.

A dot denotes a value of zero.

3.3 Piecewise linear price-quantity dependencies

3.3.1 Objective function and its properties

The actual price-quantity dependency may be a non-linear arbitrary function. But, the determination of the correct function might be too difficult or impossible, e.g., because of missing information for every possible quantity price combination. Thus, an approximation is advisable. One approach that extends the pure linear dependency is the approximation by piecewise linear functions. The parameters for the particular sections (pieces) are fairly easy to determine and by increasing the number of sections the approximation becomes better. This also assures a quadratic objective function for which solver software exists. Nonetheless, in the end the resulting objective function of the optimisation problem must be concave. Only then, the determination of the optimum ends definitively in the global optimum.

Exemplary piecewise linear functions are depicted in Fig. 3.6. The left case represents an anonymous market, i.e., the basic price $\bar{r}$ does not depend on the quantity. Hence, the slope of the function is $\hat{r}_1 = 0$. The middle case is an extension by a part with a negative slope $\hat{r}_2$ where the price

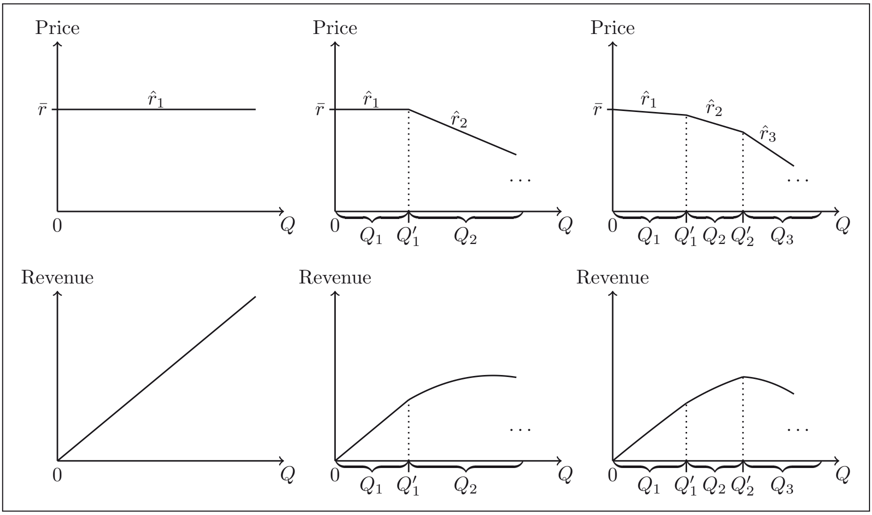

Fig. 3.6 Piecewise linear price-quantity dependency and resulting revenue functions

decreases with an increasing quantity Q (see argumentation above). Here, the quantity $\check{Q}_1$ is a limit up to which the price is inelastic. If $\hat{r}_1$ is set to a value smaller than zero the price already decreases with the very first unit. It is assumed (as in the section above), that the price is identical for the whole quantity. This means that only one price exists and thus no differentiation of markets is made. The right case extends the middle case by another linear section of the curve. This can be continued even further, but it increases the number of variables Q_s to model this function. Each variable Q_s denotes the quantity allocated in the section s. In terms of a good approximation of the actual price-quantity function one might want to add many sections but this has a trade-off in adding extra decision variables and constraints as is illustrated in the following. The price-quantity dependency function has to be continuous, because the objective function shall be continuous. A jump discontinuity in the quantity dependency function leads to a jump discontinuity in the objective function, because the quantity-dependency function is just multiplied with the quantity.

The price $r(Q)$ for a given quantity Q of the right case (see Fig. 3.6) can be determined by the partially defined function

$$r(Q) = \begin{cases} \bar{r} + \hat{r}_1\, Q & 0 \le Q \le \check{Q}_1 \\ \bar{r} + (\hat{r}_1 - \hat{r}_2)\, \check{Q}_1 + \hat{r}_2\, Q & \check{Q}_1 < Q \le \check{Q}_2 \\ \bar{r} + (\hat{r}_1 - \hat{r}_2)\, \check{Q}_1 + (\hat{r}_2 - \hat{r}_3)\, \check{Q}_2 + \hat{r}_3\, Q & \check{Q}_2 < Q \end{cases} \tag{3.25}$$

or by using the section variables Q_s

$$r = \bar{r} + \hat{r}_1\, Q_1 + \hat{r}_2\, Q_2 + \hat{r}_3\, Q_3 = \bar{r} + \sum_{s=1}^{3} \hat{r}_s\, Q_s\,. \tag{3.26}$$

Thereby, the distributed quantity Q is split into Q_1, Q_2, and Q_3, i.e., $Q = Q_1 + Q_2 + Q_3$ and the first two variables are limited by

$$0 \le Q_1 \le \check{Q}_1 - 0 \tag{3.27}$$

$$0 \le Q_2 \le \check{Q}_2 - \check{Q}_1, \tag{3.28}$$

whereas variable Q_3 must not be negative and has not necessarily an upper limit. With this the generating of the price function with more sections should be comprehensible. The revenues $r(Q)\,Q$ that result from a distributed quantity Q are

$$r(Q)\,Q = \left(\bar{r} + \sum_{s} \hat{r}_s\, Q_s\right) Q, \tag{3.29}$$

where s denotes the index of the section. The result is a quadratic function, because of the multiplication of two decision variables. For a maximisation the objective function should be concave with respect to the feasible values of the decision variables. This can be checked by the second order derivative of the function.

For showing that the revenue function is concave we derive from the general revenue function a function for an arbitrary section $\tilde{s}$ with $Q = \sum_s Q_s$ between $\check{Q}_{\tilde{s}-1}$ and $\check{Q}_{\tilde{s}}$

$$\begin{aligned} & r_{\tilde{s}}(Q)\,Q \\ &\quad = \left(\bar{r} + \sum_{s=1}^{\tilde{s}} \hat{r}_s\, Q_s\right) Q = \left(\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right) + \hat{r}_{\tilde{s}} \left(Q - \check{Q}_{\tilde{s}-1}\right)\right) Q \end{aligned} \tag{3.30}$$

with $\check{Q}_0$ being zero. The first order derivative of this function is

$$\left.\frac{\partial}{\partial Q}\right|_{\check{Q}_{\tilde{s}-1}\le Q\le \check{Q}_{\tilde{s}}} = \bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s(\check{Q}_s - \check{Q}_{s-1}) + \hat{r}_{\tilde{s}}(2\,Q - \check{Q}_{\tilde{s}-1}) \tag{3.31}$$

and the second order derivative is

$$\left.\frac{\partial^2}{\partial Q^2}\right|_{\check{Q}_{\tilde{s}-1}\le Q\le \check{Q}_{\tilde{s}}} = 2\,\hat{r}_{\tilde{s}} \le 0. \tag{3.32}$$

Thus, in each section the function is concave as long as $\hat{r}_s$ is less or equal than zero. In addition, the first order derivative of the objective function not only has to decrease within the sections but also on the section borders. Hence, the first order derivative of section $\tilde{s}$ must be greater or equal than the one in section $\tilde{s}+1$ for $Q \to \check{Q}_{\tilde{s}}$, i.e., the quantity Q where the change from one to the next section appears.

$$\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s(\check{Q}_s - \check{Q}_{s-1}) + \hat{r}_{\tilde{s}}(2\,Q - \check{Q}_{\tilde{s}-1}) \\ \ge \bar{r} + \sum_{s=1}^{\tilde{s}} \hat{r}_s(\check{Q}_s - \check{Q}_{s-1}) + \hat{r}_{\tilde{s}+1}(2\,Q - \check{Q}_{\tilde{s}}) \tag{3.33}$$

$$\hat{r}_{\tilde{s}}(2\,Q - \check{Q}_{\tilde{s}-1}) \ge \hat{r}_{\tilde{s}}(\check{Q}_{\tilde{s}} - \check{Q}_{\tilde{s}-1}) + \hat{r}_{\tilde{s}+1}(2\,Q - \check{Q}_{\tilde{s}}) \tag{3.34}$$

$$\hat{r}_{\tilde{s}}(2\,Q - \check{Q}_{\tilde{s}}) \ge \hat{r}_{\tilde{s}+1}(2\,Q - \check{Q}_{\tilde{s}}) \tag{3.35}$$

For $Q \to \check{Q}_{\tilde{s}}$ the inequality reduces to

$$\hat{r}_{\tilde{s}}\,\check{Q}_{\tilde{s}} \ge \hat{r}_{\tilde{s}+1}\,\check{Q}_{\tilde{s}} \tag{3.36}$$

and since $\check{Q}_{\tilde{s}} > 0$ we get

$$\hat{r}_{\tilde{s}} \ge \hat{r}_{\tilde{s}+1}\,. \tag{3.37}$$

This inequality shows that, if the slope of the price function of the succeeding section is lower than the considered section, the first order derivative of the revenue function of the succeeding section is also lower, which assures concavity for the revenue function on the section changes. Furthermore, the price-quantity dependency function is continuous and therefore the revenue function, too. Consequently, the revenue function is concave, because it has no saltus, is concave within the sections, and is concave at the section changes.

The price development for every demanded item is assumed to be independent (in terms of market, i.e., no cannibalisation) of the other items and, therefore, the revenues are independent also. Thus, for every demanded item an individual independent concave revenue function exists and adding those for all distributed items results in an overall concave revenue function.[27] The same applies to the remaining three functions. Adding all four concave functions and the linear disassembly cost function an overall concave quadratic objective function results.

3.3.2 Model formulation

The general formulation of the model is identical to the one in Sect. 3.2. Only the price and cost functions are modified and constraints for splitting of the quantities into the sections are introduced. The revenues of items are denoted by $r_e^{\mathrm{I}}(Q_e^{\mathrm{I}})Q_e^{\mathrm{I}}$, where $r_e^{\mathrm{I}}(Q_e^{\mathrm{I}})$ is a function of Q_e^{I}. The revenue function is derived from Eq. (3.29), i.e.,

$$r_e^{\mathrm{I}}(Q_e^{\mathrm{I}})Q_e^{\mathrm{I}} = \left(\bar{r}_e^{\mathrm{I}} + \sum_s \hat{r}_{es}^{\mathrm{I}}\, Q_{es}^{\mathrm{I}}\right) Q_e^{\mathrm{I}}\,. \tag{3.38}$$

The variable $\bar{r}_e^{\mathrm{I}}$ denotes the price for distributing just one item, $\hat{r}_{es}^{\mathrm{I}}$ denotes the slope of the price development in section s, and Q_{es}^{I} denotes the quantity within the section. Analogically, the revenues of material, the acquisition cost, and the disposal cost are formulated. This leads to the already known objective function

$$P = R - C \tag{3.39}$$

with the revenues R

$$R = \sum_e \left(\bar{r}_e^{\mathrm{I}} + \sum_s \hat{r}_{es}^{\mathrm{I}}\, Q_{es}^{\mathrm{I}}\right) Q_e^{\mathrm{I}} + \sum_r \left(\bar{r}_r^{\mathrm{R}} + \sum_s \hat{r}_{rs}^{\mathrm{R}}\, Q_{rs}^{\mathrm{R}}\right) Q_r^{\mathrm{R}} \tag{3.40}$$

and the cost C

[27] A function $f(\mathbf{x})$ is concave if the linear interpolation of two function values of the points $\mathbf{x}$ and $\mathbf{y}$ is less or equal than the function value of the interpolated points, i.e., $\lambda\, f(\mathbf{x}) + (1-\lambda)f(\mathbf{y}) \leq f(\lambda\,\mathbf{x} + (1-\lambda)\mathbf{y})$. Cf. Zangwill (1967): *Non-linear programming*, p. 345, and Beale (1955): *Minimizing a convex function*, p. 173. Thus, adding two functions $f(\mathbf{x})$ and $g(\mathbf{x})$ with this property results in $\lambda\, f(\mathbf{x}) + (1-\lambda)f(\mathbf{y}) + \lambda\, g(\mathbf{x}) + (1-\lambda)g(\mathbf{y}) \leq f(\lambda\,\mathbf{x} + (1-\lambda)\mathbf{y}) + g(\lambda\,\mathbf{x} + (1-\lambda)\mathbf{y})$. If $h(\mathbf{x}) = f(\mathbf{x}) + g(\mathbf{x})$ we can summarise to $\lambda\, h(\mathbf{x}) + (1-\lambda)h(\mathbf{y}) \leq h(\lambda\,\mathbf{x} + (1-\lambda)\mathbf{y})$ which shows that $h(x)$ is concave, too.

$$C = \sum_c \left(c_c^{\mathrm{J}} + \bar{c}_c^{\mathrm{A}} + \sum_s \hat{c}_{cs}^{\mathrm{A}}\, Q_{cs}^{\mathrm{C}} \right) Q_c^{\mathrm{C}} + \sum_d \left(\bar{c}_d^{\mathrm{D}} + \sum_s \hat{c}_{ds}^{\mathrm{D}}\, Q_{ds}^{\mathrm{D}} \right) Q_d^{\mathrm{D}} \,. \quad (3.41)$$

Furthermore, the sum of the section variables equals the quantity.

$$Q_e^{\mathrm{I}} = \sum_s Q_{es}^{\mathrm{I}} \quad \forall\ e, \qquad Q_r^{\mathrm{R}} = \sum_s Q_{rs}^{\mathrm{R}} \quad \forall\ r \quad (3.42)$$

$$Q_c^{\mathrm{C}} = \sum_s Q_{cs}^{\mathrm{C}} \quad \forall\ c, \qquad Q_d^{\mathrm{D}} = \sum_s Q_{ds}^{\mathrm{D}} \quad \forall\ d \quad (3.43)$$

According to the Eqs. (3.27) and (3.28) the section variables need to be limited. Assuming that the first section starts at zero, e.g., $\check{Q}_{e0}^{\mathrm{I}} = 0$, and the last section S is an open interval, e.g., $\check{Q}_{eS}^{\mathrm{I}} = \infty$, the section limitations are

$$0 \le Q_{es}^{\mathrm{I}} \le \check{Q}_{es}^{\mathrm{I}} - \check{Q}_{e,s-1}^{\mathrm{I}} \quad \forall\ e, s, \quad 0 \le Q_{rs}^{\mathrm{R}} \le \check{Q}_{rs}^{\mathrm{R}} - \check{Q}_{r,s-1}^{\mathrm{R}} \quad \forall\ r, s \quad (3.44)$$

$$0 \le Q_{cs}^{\mathrm{C}} \le \check{Q}_{cs}^{\mathrm{C}} - \check{Q}_{c,s-1}^{\mathrm{C}} \quad \forall\ c, s, \quad 0 \le Q_{ds}^{\mathrm{D}} \le \check{Q}_{ds}^{\mathrm{D}} - \check{Q}_{d,s-1}^{\mathrm{D}} \quad \forall\ d, s \,. \quad (3.45)$$

The variables Q_e^{I}, Q_r^{R}, Q_c^{C}, and Q_d^{D} can be substituted throughout the model with the corresponding sums, e.g., $\sum_s Q_{es}^{\mathrm{I}}$. This saves variables and the constraints (3.42) and (3.43), but is worse for understanding it.

Ordering constraints are unnecessary. Ordering constraints assure that at first the quantity of a section needs to be at its upper limit before the quantity of the next section can be greater than zero. The reason why this is not necessary is that the slope of a lower section is greater (in absolute terms smaller) than the one of an upper section. Hence, the incentive is always to put as much "quantity" into the lowest possible section, which is exactly the intention of ordering constraints. For instance, let us consider the following. Three sections exist with a basic price $\bar{r} = 10$ and the slopes $\hat{r}_1 = -0.1$, $\hat{r}_2 = -0.2$, and $\hat{r}_3 = -0.3$. Each section variable must be in the interval $Q_s \in [0, 10]\ \forall\ s$. Assuming the total quantity equals 15, i.e., $\sum_{s=1}^{3} Q_s = 15$, there exist many combinations of Q_s that fulfil this equation. For the solution $Q_1 = Q_2 = Q_3 = 5$ the resulting price is seven and the revenue 105. Increasing the last section and decreasing the second one by one unit results in a decreased price of 6.9 and therefore a smaller revenue of 103.5, because the quantity is equal. The reason for the lower price is obviously the smaller slope of $\hat{r}_3$ compared to $\hat{r}_2$. Thus, the marginal decrease of the price is smaller in the first section compared to the following sections. Therefore, when maximising the revenue the incentive is to have a high price, which is realised by a least possible decrease of the basic price. And this is achieved by choosing the greatest (or absolute smallest) slope possible, i.e., starting with $\hat{r}_1$. Following this, the optimal ordering of the sections is starting from the lowest and going to the upper ones, which would equal the behaviour of ordering constraints. For the given example the solution $Q_1 = 10$, $Q_2 = 5$ and $Q_3 = 0$ results in the highest price of eight with a revenue of 120.

3.3.3 Solution finding

3.3.3.1 Unconstrained optimum

Finding the optimal solution of the given problem is not as easy as with just linear price-quantity dependencies. For standard quadratic solvers that are also relatively fast for MILP the model formulation with the section variables Q^{C}_{cs}, Q^{I}_{es}, Q^{R}_{rs}, and Q^{D}_{ds} must be used, because a partial defined objective function cannot be used. The problem using section variables is that for the solver these variables are assumed to be independent of each other, even though they are not independent, and thus the objective function is not concave anymore. To avoid this, an existing solution algorithm could be modified to include partially defined concave objective functions. Candidates would be gradient based algorithms or Newton based solution algorithm. The latter has been successfully used to solve such a problem.[28] The benefit of a Newton based solution algorithm is the generally better convergence than with gradient based methods.[29] But a more promising way with just using standard LP solvers is presented in the sequel.

To illustrate the procedure a small example shall be used. The quadratic maximisation problem with an unconstrained optimum (QPUO) is defined by a concave objective function

$$\text{(QPUO):} \qquad \text{Maximise } z = -x_1^2 - x_2^2 - \frac{3}{2} x_1 x_2 + 4x_1 + 4x_2 + 12 \tag{3.46}$$

and four constraints

$$\text{s.t.} \qquad 2x_1 + 5x_2 \geq 5 \tag{3.47}$$

$$2x_1 - 3x_2 \leq 5 \tag{3.48}$$

[28] Cf. Ullerich (2011a): *Disassembly planning with linear PCF*.

[29] Cf. Alt (2002): *Nichtlineare Optimierung*.

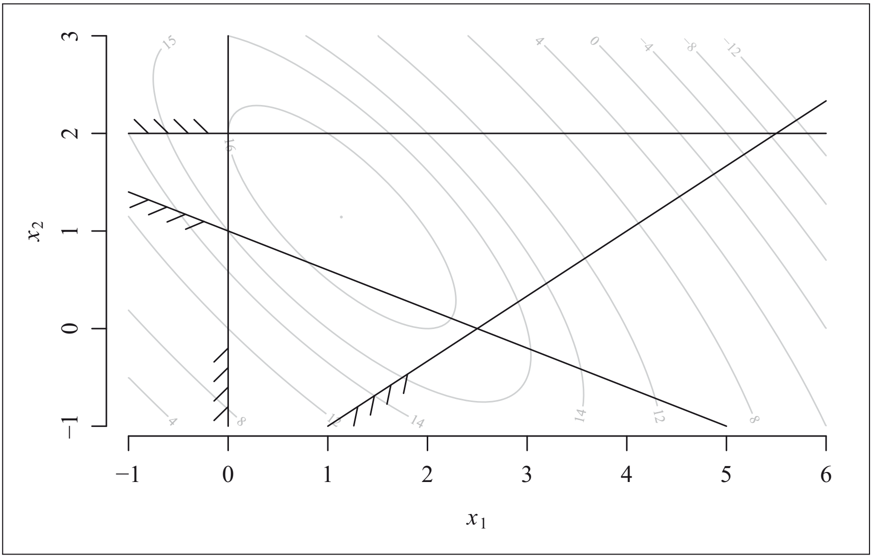

Fig. 3.7 Optimisation problem with unconstrained optimum

$$x_1 \geq 0 \tag{3.49}$$

$$x_2 \leq 2\,. \tag{3.50}$$

The graphical interpretation of this problem with various levels of the objective function (i.e., $z = -16, -12, \ldots, 8, 12, 14, 15, 16$) is depicted in Fig. 3.7. For the beginning, the objective function is not partially defined yet. This will come later. The four constraints are straight lines with the infeasible side marked by the four short lines coming off the constraint. Thus, the feasible solution space is located within the quadrangle and is convex. Every feasible solution space limited by linear constraints is convex. This property and that the solution space is completely bounded are basic requirements for the following solution procedure. Bounding the solution space completely for the given problems in disassembly planning is no restriction, because an arbitrary large limit for every decision variable can be found easily and might be given by workload capacity or the like.

The objective function levels in Fig. 3.7 are helpful to identify the gradient at the corresponding solution points. The gradient $\nabla z(\mathbf{x})$ is directed towards the greatest increase of the objective function at a given point and can be calculated by the first order partial derivatives of the objective function with respect to the decision variables.

$$\nabla z(\mathbf{x}) = \begin{pmatrix} \frac{\partial z(\mathbf{x})}{\partial x_1} \\ \frac{\partial z(\mathbf{x})}{\partial x_2} \end{pmatrix} = \begin{pmatrix} -2x_1 - \frac{3}{2}x_2 + 4 \\ -2x_2 - \frac{3}{2}x_1 + 4 \end{pmatrix} \tag{3.51}$$

Furthermore, the gradient is right-angled to the tangent on the corresponding level curve. The gradient is of major importance, because the coefficients of the objective function of the used LP equal the gradient at a given solution point. To illustrate this, let us choose the point $(2.5, 0)$ as starting point.[30] The gradient at this point is $(-1, 1/4)^{\mathrm{T}}$, which results from the partial derivatives with respect to x_1 and x_2 at the point $(2.5, 0)$. These two values -1 and $1/4$ are the coefficients of the objective function of the LP. The constraints are initially identical to the ones of the (QPUO). Thus the first LP to solve is:

$$\text{Maximise } z = -x_1 + \frac{1}{4}\, x_2 \tag{3.52}$$

$$\text{s.t.} \qquad 2\,x_1 + 5\,x_2 \geq 5 \tag{3.53}$$

$$2\,x_1 - 3\,x_2 \leq 5 \tag{3.54}$$

$$x_1 \geq 0 \tag{3.55}$$

$$x_2 \leq 2\,. \tag{3.56}$$

The solution of the LP is the point $(0, 2)$. Using this point the coefficients of the objective function of the LP are updated with the gradient at this point, which is $(1, 0)^{\mathrm{T}}$. Hence, the objective function of the LP changes to

$$\text{Maximise } z = x_1\,. \tag{3.57}$$

Solving this again results in the solution $(5.5, 2)$. Updating the coefficients and solving it again leads to the solution $(0, 1)$. Again, updating the coefficients and solving results in the already known solution $(5.5, 2)$. At this moment a cycle is detected, which is an oscillation between the points $(0, 1)$ and $(5.5, 2)$. Thus, the solution is "somewhere" in between these two points. But, the solution we are looking for does not have to be on the line segment between these two points. Therefore, further steps are required. These steps are motivated by the solution algorithms for QLP by Beale (adding assisting constraints) and Rosen (determining the step width).[31]

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[30] An infeasible point like $(0, 0)$ can be used as well.

[31] Cf. Beale (1959): *Quadratic programming*, p. 232, and Rosen (1960): *Gradient projection method*, p. 203.

Starting from one of the oscillation points (e.g., $x_{i-1} = (0,1)^{\mathrm{T}}$) a vector towards another oscillation point[32] (e.g., $x_i = (5.5,2)^{\mathrm{T}}$) is determined. This vector is called direction vector $\mathbf{s}$. Along this vector a point $\hat{\mathbf{x}}$ exists where the gradient $\nabla z(\hat{\mathbf{x}})$ is orthogonal to the vector $\mathbf{s}$, i.e., the direction vector is the tangent of the objective function at point $\hat{\mathbf{x}}$. At this point the highest objective value is achieved along the vector $\mathbf{s}$. This point $\hat{\mathbf{x}}$ is the next iterative solution point and it can be found using the step-size ρ calculation according to Rosen

$$\rho = \frac{\mathbf{s}^{\mathrm{T}} \nabla z(\mathbf{x}_{i-1})}{\mathbf{s}^{\mathrm{T}} \left(\nabla z(\mathbf{x}_{i-1}) - \nabla z(\mathbf{x}_i) \right)} \,. \tag{3.58}$$

The step size is illustrated in Fig. 3.8 with a direction vector from an arbitrary $\mathbf{x}_{i-1}$ to another $\mathbf{x}_i$. The gradient at $\mathbf{x}_{i-1}$ forms an acute angle with the vector $\mathbf{s}$, which is marked with a "+", because the vector product of $\nabla z(\mathbf{x}_{i-1})$ and $\mathbf{s}$ is greater than zero. This also means that along the direction vector the objective value increases. At the end of $\mathbf{s}$, i.e., at $\mathbf{x}_i$, the gradient $\nabla z(\mathbf{x}_i)$ and $\mathbf{s}$ form an obtuse angle, which results in a negative vector product. This indicates that a better objective value is reached from $\mathbf{x}_i$ by moving in the reverse direction of $\mathbf{s}$. Therefore, there exists a point $\hat{\mathbf{x}}$ in between $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$ with the highest objective value along $\mathbf{s}$ starting at $\mathbf{x}_{i-1}$. And the step size ρ as fraction of the length of $\mathbf{s}$ is the one to find $\hat{\mathbf{x}}$.

If ρ equals zero, the actual solution $\mathbf{x}_{i-1}$ is the one with the highest objective value along the vector $\mathbf{s}$ and this means that $\mathbf{x}_{i-1}$ is the optimal solution. On the other hand, if ρ is greater than one, $\hat{\mathbf{x}}$ would be outside the feasible area. In this case the solution on the border of the feasible space $\mathbf{x}_i$ is checked whether a further improvement with an updated objective function under the same constraints is possible. If no improvement is possible, $\mathbf{x}_i$ is the next iterative solution, i.e., $\hat{\mathbf{x}} = \mathbf{x}_i$. Otherwise, the procedure is continued with the solution $\mathbf{x}_i$ and the new found one by determining a new direction vector $\mathbf{s}$. Note that the cases $\rho \leq 0$ and $\rho \geq 1$ cannot appear in the first run of this part of the solution procedure, because of the oscillating solutions of the preceding LP. If ρ is greater than or equals one, the LP with updated objective function coefficients and the constraints of the (QPUO) is solved to identify possible further improvement. Else, if $0 < \rho < 1$, the iterative solution point is determined by the linear combination

$$\hat{\mathbf{x}} = \mathbf{x}_{i-1} + \rho \, \mathbf{s} \,. \tag{3.59}$$

[32] There could be more than two oscillation points.

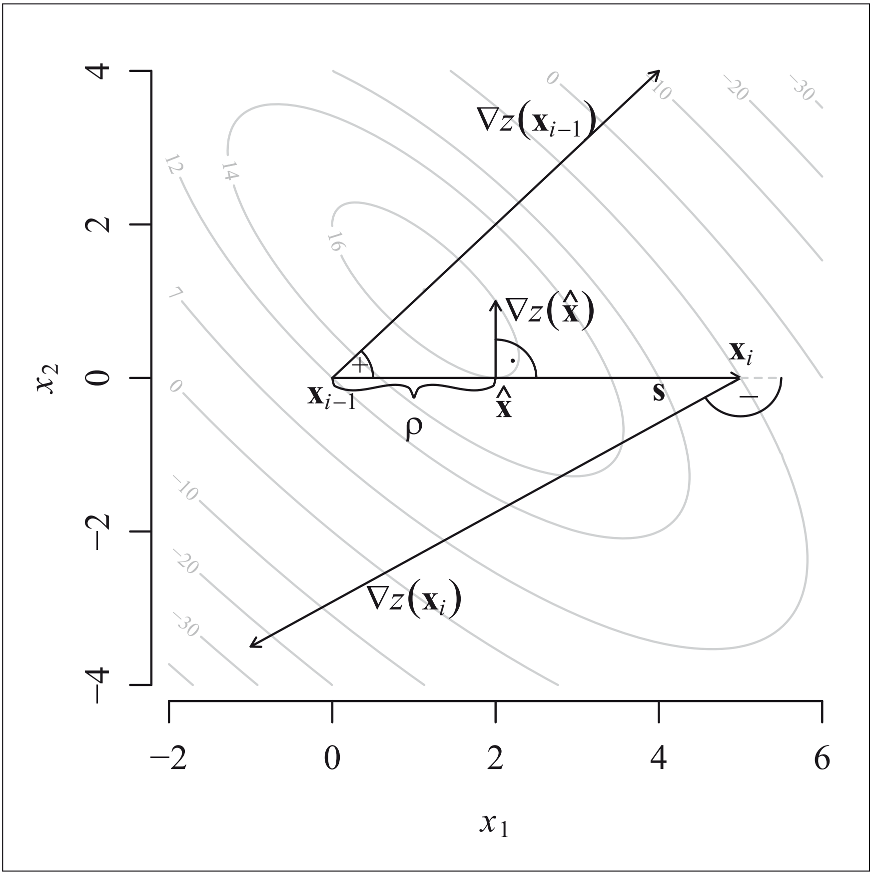

Fig. 3.8 Visualisation of step-size determination

Since the solution of a LP is always situated on an edge of the feasible solution space, at least one assisting constraint needs to be added (in the case $0 < \rho < 1$) such that the subsequent LP solving can result in this point $\hat{\mathbf{x}}$. Otherwise, it would always end in the known oscillation points from the beginning. In general, up to the dimension of the solution space (i.e., the number of decision variables) many assisting constraints can be necessary to determine the optimal solution. But this presented approach works with a direction vector and one assisting constraint to keep the flexibility of the solution finding of standard LP solvers at the cost of worse convergence.

Since the algorithm also stops at a step width of $\rho = 0$ an extra edge of the solution space does not have to be generated by extra constraints.

The assisting constraint should allow the best improvement of the objective value from the point $\hat{\mathbf{x}}$. Hence, $\hat{\mathbf{x}}$ as well as the gradient at $\hat{\mathbf{x}}$ should be "within" the hyperplane that forms the assisting constraint. But with these two elements a hyperplane is not completely defined, yet. To illustrate this, the hyperplane could be any hyperplane rotated around the gradient at point $\hat{\mathbf{x}}$. Thus, we look for a normal vector $\mathbf{n}$ of the hyperplane that is orthogonal to the gradient at point $\hat{\mathbf{x}}$. One such vector is the direction vector $\mathbf{s}$, i.e., $\mathbf{n} = \mathbf{s}$. To completely define the assisting constraint the right hand side value is missing, which assures that $\hat{\mathbf{x}}$ is element of the hyperplane. This value is easily calculated by $\mathbf{s}^{\mathrm{T}}\hat{\mathbf{x}}$. Now the assisting constraint is completely defined and can be added to the existing LP in the form

$$\mathbf{s}^{\mathrm{T}}\mathbf{x} = \mathbf{s}^{\mathrm{T}}\hat{\mathbf{x}}\,. \tag{3.60}$$

Updating the coefficients or the objective function and solving the LP leads to a (possibly new) solution. If the solution is the same as the last iterative solution point or the step width equals zero the optimal solution is found. Otherwise, the procedure repeats until the above mentioned conditions are fulfilled. The complete algorithm is depicted in Fig. 3.9.

Our small example shall illustrate the algorithm. Our last solution $\mathbf{x}_4 = (5.5, 2)^{\mathrm{T}}$ is a solution that already appeared, which answers the question in the diamond on the bottom left with "yes". Therefore, we continue the flowchart on the top of the right side. Starting at $\mathbf{x}_3 = (0, 1)^{\mathrm{T}}$ the direction vector equals

$$\mathbf{s} = \mathbf{x}_4 - \mathbf{x}_3 = \begin{pmatrix} \frac{11}{2} \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{11}{2} \\ 1 \end{pmatrix}. \tag{3.61}$$

The step size ρ results in

$$\begin{aligned} \rho &= \frac{\begin{pmatrix} \frac{11}{2} \\ 1 \end{pmatrix}^{\mathrm{T}} \nabla z\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)}{\begin{pmatrix} \frac{11}{2} \\ 1 \end{pmatrix}^{\mathrm{T}} \left(\nabla z\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) - \nabla z\left(\begin{pmatrix} \frac{11}{2} \\ 2 \end{pmatrix}\right)\right)} = \frac{\begin{pmatrix} \frac{11}{2} \\ 1 \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \frac{5}{2} \\ 2 \end{pmatrix}}{\begin{pmatrix} \frac{11}{2} \\ 1 \end{pmatrix}^{\mathrm{T}} \left(\begin{pmatrix} \frac{5}{2} \\ 2 \end{pmatrix} - \begin{pmatrix} -10 \\ -\frac{33}{4} \end{pmatrix}\right)} \\ &= \frac{63}{316}\,. \end{aligned} \tag{3.62}$$

Since $0 < \rho < 1$ is valid, the iteration point $\hat{\mathbf{x}}$ is

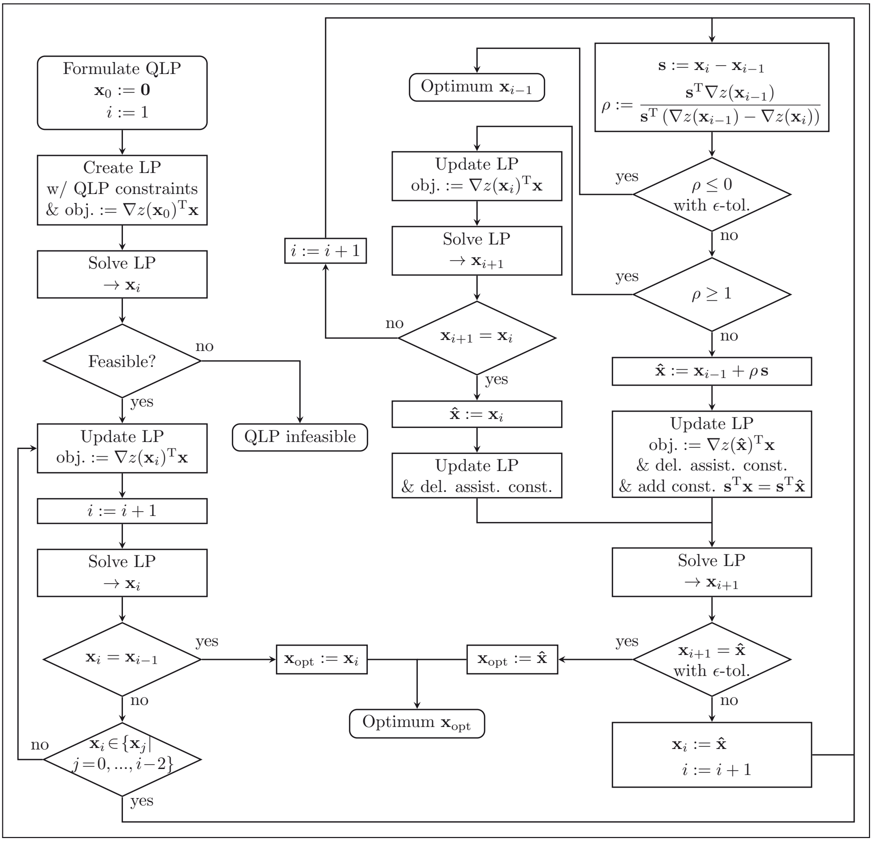

Fig. 3.9 Flowchart of QLP solution algorithm

$$\hat{\mathbf{x}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{63}{316} \begin{pmatrix} \frac{11}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{693}{632} \\ \frac{379}{316} \end{pmatrix}. \tag{3.63}$$

Consequently to Eq. (3.60), the assisting constraint to be added for the next solution step is

$$\begin{pmatrix} \frac{11}{2} & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{11}{2} & 1 \end{pmatrix} \begin{pmatrix} \frac{693}{632} \\ \frac{379}{316} \end{pmatrix} = \frac{9139}{1264} \tag{3.64}$$

or in “standard” polynomial equation writing

$$\frac{11}{2}\, x_1 + 1\, x_2 = \frac{9139}{1264}\,. \tag{3.65}$$

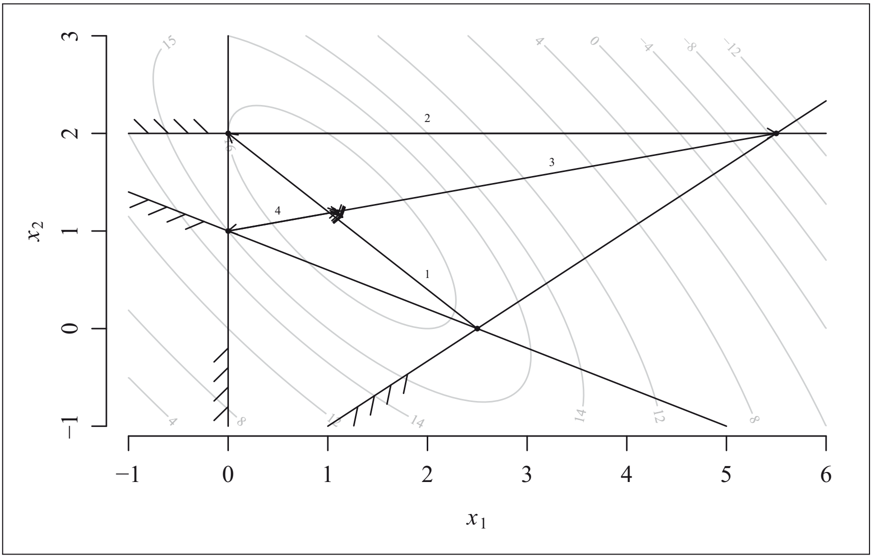

Fig. 3.10 Solution path of illustrative example

The objective coefficients are updated to the gradient at point $\hat{\mathbf{x}}$ (i.e., $^{5}/_{632}$ and $-^{55}/_{1264}$) and the LP is solved anew. The next solution $\mathbf{x}_5$ would be the point $(^{13125}/_{10744}, {}^{2747}/_{5372})$ which is the edge of the assisting constraint and the first constraint (3.47). This solution does not equal the last iterative solution point $\hat{\mathbf{x}}$. Thus, the optimal solution is not found yet. The value of $\mathbf{x}_4$ is updated by the last iterative solution, i.e., $\mathbf{x}_5 = \hat{\mathbf{x}}$, and the procedure is repeated with the updated iteration solution point.

This new solution together with the last solution determines the direction vector $\mathbf{s}$ for the new iteration. Since $\mathbf{s}$ is not zero the optimum is not found yet. A new iteration point $\hat{\mathbf{x}}$ is calculated and a new assisting constraint is added. Before adding the new constraint the old assisting constraint is deleted. Therefore, only one assisting constraint exists in each further iteration. The coefficients of the objective function of the LP are updated and the solving is repeated. This procedure iterates until the iteration point of the actual iteration equals the one of the preceding iteration. Thereby, equal means that they are identical to a given ϵ of for example $\epsilon = 10^{-9}$. The solution path is displayed in Fig. 3.10 and 3.11, with the latter being a magnification for emphasising on the convergence. The necessary iterations to find the optimum are 68 (see Table 3.12). This illustrates the rather bad convergence of algorithms solely based on the gradient.

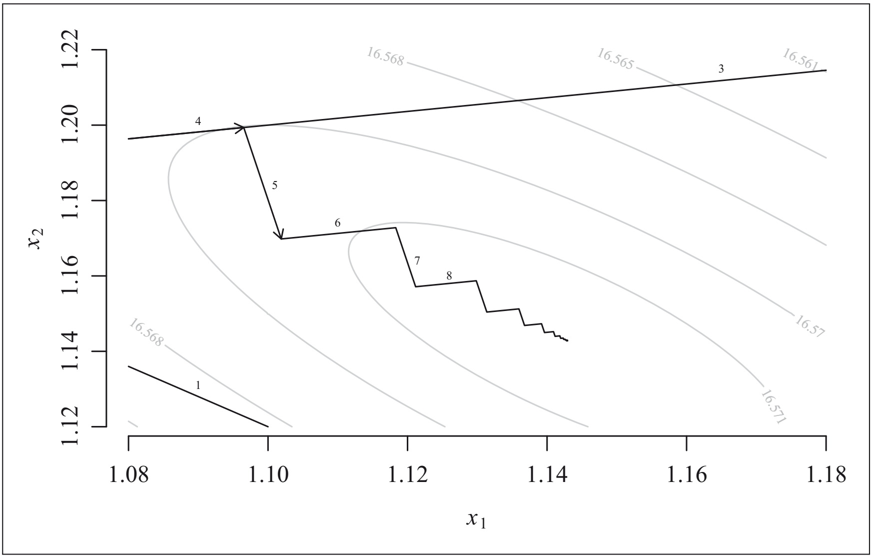

Fig. 3.11 Zoom in on solution path of illustrative example

Table 3.12 Iterative solution points of illustrative example

iteration	solution	iteration	solution	iteration	solution	iteration	solution
0	(2.5, 0)	4	(1.0965, 1.1994)	8	(1.1299, 1.1587)	12	(1.1392, 1.1473)
1	(0, 2)	5	(1.1019, 1.1698)	9	(1.1314, 1.1504)	13	(1.1396, 1.1450)
2	(5.5, 2)	6	(1.1183, 1.1728)	10	(1.1360, 1.1513)	⋮	⋮
3	(0, 1)	7	(1.1212, 1.1571)	11	(1.1368, 1.1469)	68	(1.1429, 1.1429)

The values are rounded to four digits. Values with less than four post decimal digits indicate an exact value without rounding being necessary.

An improved version of the algorithm in terms of convergence is the following. This version is confined to objective functions with a known unconstrained optimum. For any (concave or convex) quadratic objective function the unconstrained optimum can easily be determined by solving the equation system of the first order partial derivatives of the objective functions with respect to the decision variables equalling zero. This optimal solution can then be tested on feasibility according to the given constraints. If it is feasible the optimal solution is found. Otherwise, the above mentioned algorithm is applied with the following modification. Instead of using the direction vector $\mathbf{s}$ as normal vector of the assisting constraint a different normal vector is constructed. The desired normal vector $\mathbf{n}$ should be orthogonal to the vector $\mathbf{o} = \mathbf{x}_{\text{opt}} - \hat{\mathbf{x}}$ from the iteration point $\hat{\mathbf{x}}$ towards the unconstrained optimum $\mathbf{x}_{\text{opt}}$. The equation that represents this is

$$\mathbf{n}^{\mathrm{T}} \mathbf{o} = 0\,. \tag{3.66}$$

But many vectors comply with this property so that a further property is necessary. We assume that the normal vector $\mathbf{n}$ is a linear combination of the direction vector $\mathbf{s}$ and the vector towards the unconstrained optimum.[33] This property can be expressed by the equation

$$\alpha\,\mathbf{s} + \beta\,\mathbf{o} = \mathbf{n}\,. \tag{3.67}$$

The length of $\mathbf{n}$ is not of interest so that β can be set to the value one. Substituting $\mathbf{n}$ in Eq. (3.66) by Eq. (3.67) leads to

$$(\alpha\,\mathbf{s} + \mathbf{o})^{\mathrm{T}}\,\mathbf{o} = 0\,. \tag{3.68}$$

Solving this equation with respect to α results in

$$\alpha = -\frac{\mathbf{o}^{\mathrm{T}}\mathbf{o}}{\mathbf{s}^{\mathrm{T}}\mathbf{o}}\,. \tag{3.69}$$

Hence the desired normal vector is

$$\mathbf{n} = \mathbf{o} - \frac{\mathbf{o}^{\mathrm{T}}\mathbf{o}}{\mathbf{s}^{\mathrm{T}}\mathbf{o}}\mathbf{s}\,. \tag{3.70}$$

In the case that the vector $\mathbf{o}$ is orthogonal to the direction vector $\mathbf{s}$ the direction vector is the wanted normal vector $\mathbf{n}$, i.e., $\mathbf{n} = \mathbf{s}$. In an additional step the normal vector can be normalised to the length one, if desired. The assisting constraint is not completely defined yet, because the right hand side value is missing. This value is easily calculated by $\mathbf{n}^{\mathrm{T}}\hat{\mathbf{x}}$. Now the assisting constraint is completely defined and can be added to the existing LP in the form

$$\mathbf{n}^{\mathrm{T}}\mathbf{x} = \mathbf{n}^{\mathrm{T}}\hat{\mathbf{x}}\,. \tag{3.71}$$

Another special case is present when $\mathbf{s}$ and $\mathbf{o}$ are linear dependent. This happens when the new solution is located on the vector from $\hat{\mathbf{x}}$ towards $\mathbf{x}_{\text{opt}}$. In this case the vector $\mathbf{n}$ equals the zero vector. This is comparable with not adding a constraint at all, because $\mathbf{0}^{\mathrm{T}}\mathbf{x} = 0$ is true for all $\mathbf{x}$. The remaining steps of the algorithm equal the ones from the preceding approach.

[33] The property of being a linear combination might not be the best one, but the determination of the normal vector is straightforward.

Table 3.13 Iterative solution points of illustrative example without gradient

iteration	solution	iteration	solution	iteration	solution
0	(2.5, 0)	2	(5.5, 2)	4	(1.0965, 1.1994)
1	(0, 2)	3	(0, 1)	5	(1.1429, 1.1429)

The values are rounded to four digits. Values with less than four post decimal digits indicate an exact value without rounding being necessary.

The different calculation shall be illustrated by the same small example as above. Again, we start our consideration at the point $(0, 1)$. The direction vector is $\mathbf{s} = (11/2, 1)^{\mathrm{T}}$ and the iteration point is $\hat{\mathbf{x}} = (693/632, 379/316)^{\mathrm{T}}$. The feasible unconstrained optimum is $\mathbf{x}_{\mathrm{opt}} = (8/7, 8/7)^{\mathrm{T}}$, which can be determined by solving the equation system

$$\frac{\partial z(\mathbf{x})}{\partial x_1} = -2x_1 - \frac{3}{2}x_2 + 4 = 0 \tag{3.72}$$

$$\frac{\partial z(\mathbf{x})}{\partial x_2} = -2x_2 - \frac{3}{2}x_1 + 4 = 0\,. \tag{3.73}$$

According to Eq. (3.70) and the vector $\mathbf{o} = \left(\frac{205}{4424} \ -\frac{125}{2212}\right)^{\mathrm{T}}$ the normal vector is

$$\mathbf{n} = \mathbf{o} - \frac{\mathbf{o}^{\mathrm{T}}\mathbf{o}}{\mathbf{s}^{\mathrm{T}}\mathbf{o}}\mathbf{s} = \begin{pmatrix} -\frac{250}{2457} \\ -\frac{205}{2457} \end{pmatrix}. \tag{3.74}$$

Obtaining the right hand side value for the assisting constraint $(-40/189)$, updating the objective function coefficients of the LP with the values $\nabla z(\hat{\mathbf{x}})$, and solving the LP results in the solution $\mathbf{x}_4 = (15/8, 1/4)^{\mathrm{T}}$. The next iterative solution point is $\hat{\mathbf{x}} = (8/7, 8/7)^{\mathrm{T}}$, which equals the optimal solution. Nonetheless, another assisting constraint is added, the LP is solved again, and a step width of $\rho = 0$ signals that the last iterative solution point is the optimal one. Hence, the algorithm stops.

The solution finding with this version is depicted in Fig. 3.12. As it is easily recognisable the convergence is much better, because after six iterations the optimal solution is found (see Table 3.13). Of course, the optimum is known beforehand, but for cases with constrained optima the convergence should be better, too, especially when the first direction vector goes through the solution space (and not along just one edge of the solution space) and the unconstrained optimum is relatively close to the solution space. An example is depicted in Fig. 3.13.

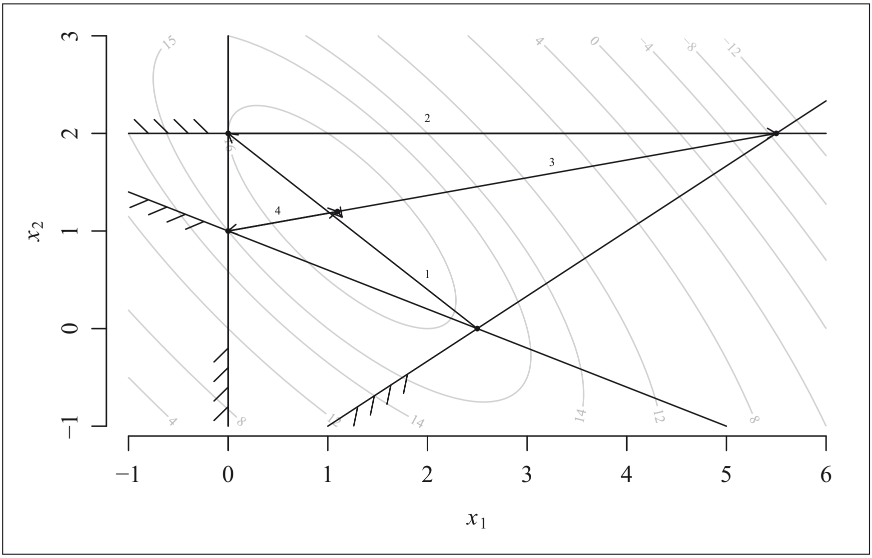

Fig. 3.12 Solution path of illustrative example not using the gradient

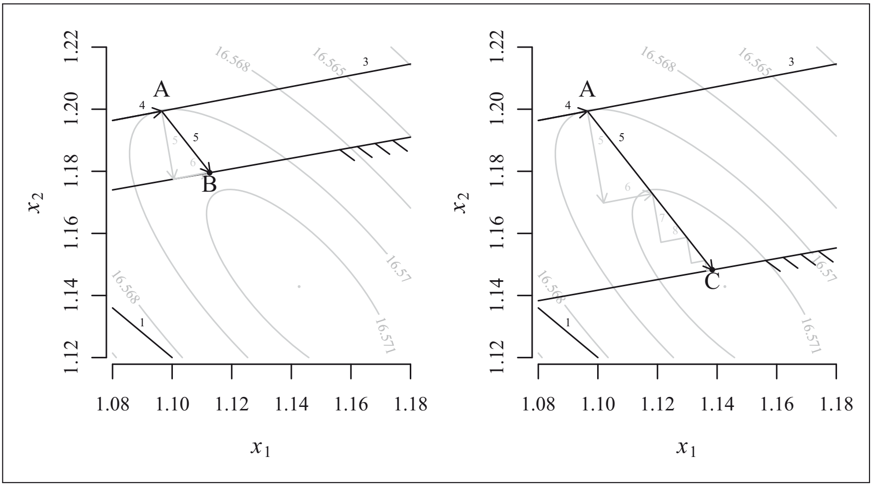

Fig. 3.13 Comparison of gradient and direct version

The solution path of the version with the gradient is marked by the solid grey line/arrow (see Fig. 3.11 to compare) and the path with vector **o** is marked by the solid black arrow. Additionally, a constraint is added—in the left part one that is further away from the unconstrained optimum and in the right part one that is closer to the optimum. In the left part the number of iterations for both versions is almost identical. From point A the constraint is reached within one step (5) and the next step (6) is along the constraint or just one step (5) to the constrained optimum B. In the right part, the version with vector **o** reaches the constrained optimum C in less steps (i.e., one step: 5) than the gradient version (eight steps: 5 through 12). The modified algorithm flow is depicted in Fig. 3.14.

3.3.3.2 Constrained optimum

To illustrate the case with a constrained optimum the constraints of our small example from above (QPUO) are slightly modified so that the quadratic problem with a constrained optimum (QPCO) results.

$$\text{(QPCO):} \qquad \text{Maximise } z = -x_1^2 - x_2^2 - \frac{3}{2}\, x_1\, x_2 + 4\, x_1 + 4\, x_2 + 12 \tag{3.75}$$

$$\text{s.t.} \qquad 2\, x_1 + 5\, x_2 \geq 5 \tag{3.76}$$

$$2\, x_1 - 3\, x_2 \leq 5 \tag{3.77}$$

$$-x_1 + 10\, x_2 \leq 10 \tag{3.78}$$

$$x_1 + 10\, x_2 \leq 14 \tag{3.79}$$

The first step according to the algorithm (see Fig. 3.14) is to determine the unconstrained optimum, which is still $\mathbf{x}_{\text{opt}} = (8/7, 8/7)^{\text{T}}$. This solution is not feasible, because of constraint (3.78). Therefore a LP is generated with the same constraints as the (QPCO) and the two coefficients of the linear objective function are 4 and 4 (i.e., the gradient), assuming that we start with $\mathbf{x}_0 = (0, 0)^{\text{T}}$.

The problem and its solution path is visualised in Fig. 3.15 and the solution points are listed in Table 3.14. Starting the solution finding with the gradient at point $(0, 0)$ results in an oscillation between the points $(4, 1)$ and $(0, 1)$. This means that the optimal solution should be found "somewhere" in between these two points. The direction vector from $(0, 1)$ to $(4, 1)$ is $\mathbf{s} = (4, 0)^{\text{T}}$. The step width for the highest objective value along the vector $\mathbf{s}$ is $\rho = 5/16$. Thus, the iteration point equals $\hat{\mathbf{x}} = \left(\frac{5}{4}, 1\right)^{\text{T}}$. The normal vector of the assisting constraint is orthogonal to the vector $\mathbf{o}$ from $\hat{\mathbf{x}}$ towards $\mathbf{x}_{\text{opt}}$. Updating the coefficients of the LP with the gradient $\nabla z(\hat{\mathbf{x}})$ and solving

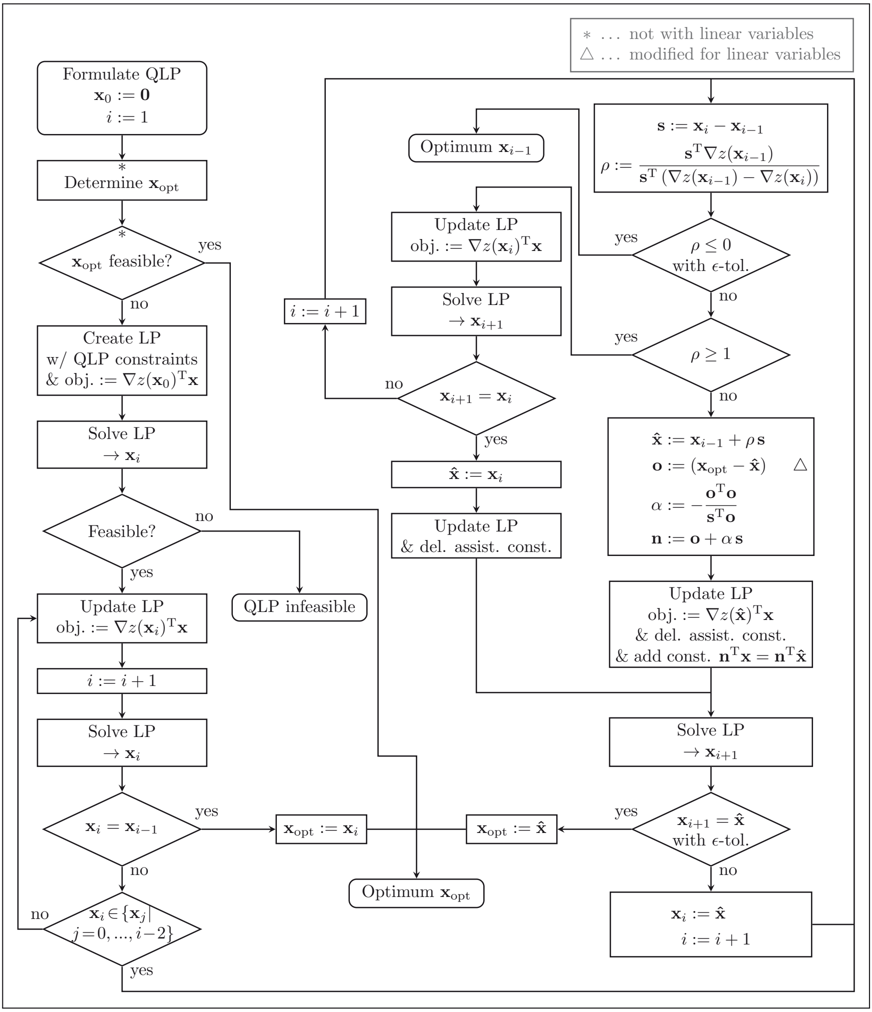

Fig. 3.14 Flowchart of modified QLP solution algorithm

it leads to the solution $(50/43, 48/43)$. The resulting step width is greater than one which leads us to an update of the objective function with the gradient at $(50/43, 48/43)$ and solving it again. The resulting solution is the same, which means that this is the next iterative solution. The assisting constraint is removed and the LP is solved anew. The solution is $(2, 6/5)$. Continuing this procedure according to the algorithm leads to the optimal

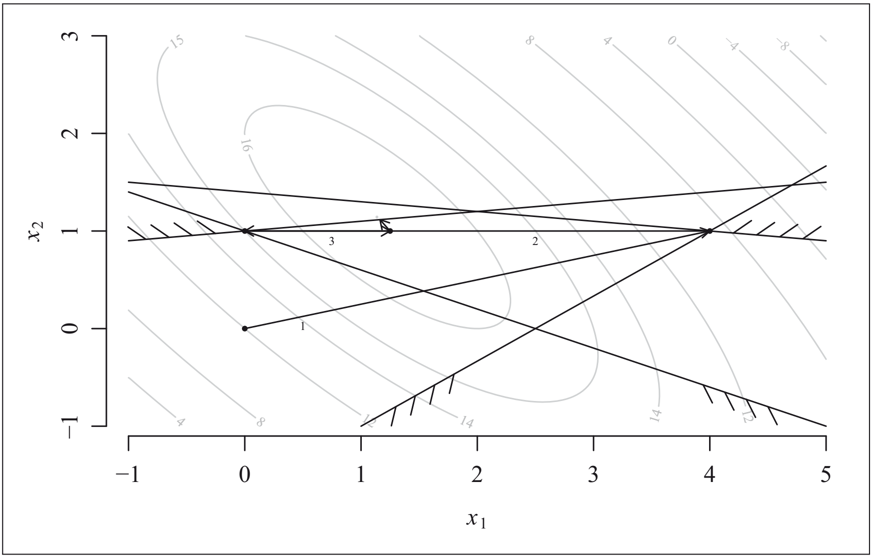

Fig. 3.15 Solution path of illustrative example (QPCO)

Table 3.14 Iterative solution points of illustrative example (QPCO)

iteration	solution	iteration	solution	iteration	solution
0	(0, 0)	2	(0, 1)	4	(1.1628, 1.1163)
1	(4, 1)	3	(1.25, 1)	5	(1.1638, 1.1164)

The values are rounded to four digits. Values with less than four post decimal digits indicate an exact value without rounding being necessary.

solution $\mathbf{x}_5 = (^{135}/_{116}, ^{259}/_{232})$ with an objective value of the (QPCO) of $^{7689}/_{464} \approx 16.5711$.

3.3.3.3 Section inclusion

So far, a solution algorithm is presented that can be substituted by the standard QLP solver, because we did not consider the sections of the objective function, i.e., the saltuses of the first order partial derivatives. But this is changed in the following. Therefore, the objective function of the small example (QPCO) is changed to a quadratic problem with constrained optimum and sections (QPCOS). For each variable (x_1 and x_2) three sections exist. For variable x_1 the section borders are $x_1 = 1$ and $x_1 = 2$. For variable x_2 the borders are $x_2 = 1/2$ and $x_2 = 3/2$. The three sections regarding x_1 are labelled with A, B, and C and regarding x_2 with I, II, and III, respectively. In total nine section combinations exist and for each combination the objective function is slightly changed to demonstrate the changes caused by the piecewise linear structure of the quantity-price dependency. Note that the resulting objective function is concave and continuous, even though it is partially defined. For the case that $x_1 \leq 1$ and $x_2 \leq 1/2$, which is denoted by the section combination "A I", the objective function is the already known of the (QPCO). The objective function with all nine cases is

$$z = \begin{cases} -x_1^2 - x_2^2 - \frac{3}{2}x_1x_2 + 4x_1 + 4x_2 + 12 & \text{for} \quad x_1 \leq 1, \quad x_2 \leq \frac{1}{2} \quad \text{(A I)} \\ -\frac{11}{10}x_1^2 - x_2^2 - \frac{3}{2}x_1x_2 + 4x_1 + 4x_2 + \frac{121}{10} & \text{for } 1 < x_1 \leq 2, \quad x_2 \leq \frac{1}{2} \quad \text{(B I)} \\ -\frac{6}{5}x_1^2 - x_2^2 - \frac{3}{2}x_1x_2 + 4x_1 + 4x_2 + \frac{25}{2} & \text{for } 2 < x_1 \quad , \quad x_2 \leq \frac{1}{2} \quad \text{(C I)} \\ -x_1^2 - \frac{11}{10}x_2^2 - \frac{3}{2}x_1x_2 + 4x_1 + 4x_2 + \frac{481}{40} & \text{for} \quad x_1 \leq 1, \frac{1}{2} < x_2 \leq \frac{3}{2} \quad \text{(A II)} \\ -\frac{11}{10}x_1^2 - \frac{11}{10}x_2^2 - \frac{3}{2}x_1x_2 + 4x_1 + 4x_2 + \frac{97}{8} & \text{for } 1 < x_1 \leq 2, \frac{1}{2} < x_2 \leq \frac{3}{2} \quad \text{(B II)} \\ -\frac{6}{5}x_1^2 - \frac{11}{10}x_2^2 - \frac{3}{2}x_1x_2 + 4x_1 + 4x_2 + \frac{501}{40} & \text{for } 2 < x_1 \quad , \frac{1}{2} < x_2 \leq \frac{3}{2} \quad \text{(C II)} \\ -x_1^2 - \frac{6}{5}x_2^2 - \frac{3}{2}x_1x_2 + 4x_1 + 4x_2 + \frac{49}{4} & \text{for} \quad x_1 \leq 1, \frac{3}{2} < x_2 \quad \text{(A III)} \\ -\frac{11}{10}x_1^2 - \frac{6}{5}x_2^2 - \frac{3}{2}x_1x_2 + 4x_1 + 4x_2 + \frac{247}{20} & \text{for } 1 < x_1 \leq 2, \frac{3}{2} < x_2 \quad \text{(B III)} \\ -\frac{6}{5}x_1^2 - \frac{6}{5}x_2^2 - \frac{3}{2}x_1x_2 + 4x_1 + 4x_2 + \frac{51}{4} & \text{for } 2 < x_1 \quad , \frac{3}{2} < x_2 \quad \text{(C III)} \end{cases} . \tag{3.80}$$

This partially defined objective function with the nine cases can also be written as a function with two partially defined terms with three cases each.

$$\text{(QPCOS):} \qquad \text{Maximise } z = \left(\begin{cases} -x_1^2 & x_1 \leq 1 \quad \text{(A)} \\ -\frac{11}{10}\,x_1^2 + \frac{1}{10} & 1 < x_1 \leq 2 \quad \text{(B)} \\ -\frac{6}{5}\,x_1^2 + \frac{1}{2} & 2 < x_1 \quad \text{(C)} \end{cases} \right) + \left(\begin{cases} -x_2^2 & x_2 \leq \frac{1}{2} \quad \text{(I)} \\ -\frac{11}{10}\,x_2^2 + \frac{1}{40} & \frac{1}{2} < x_2 \leq \frac{3}{2} \quad \text{(II)} \\ -\frac{6}{5}\,x_2^2 + \frac{1}{4} & \frac{3}{2} < x_2 \quad \text{(III)} \end{cases} \right) - \frac{3}{2}\,x_1\,x_2 + 4\,x_1 + 4\,x_2 + 12 \tag{3.81}$$

$$\text{s.t.} \qquad 2\,x_1 + \;\; 5\,x_2 \geq 5 \tag{3.82}$$

$$2\,x_1 - \;\; 3\,x_2 \leq 5 \tag{3.83}$$

$$-x_1 + 20\,x_2 \leq 20 \tag{3.84}$$

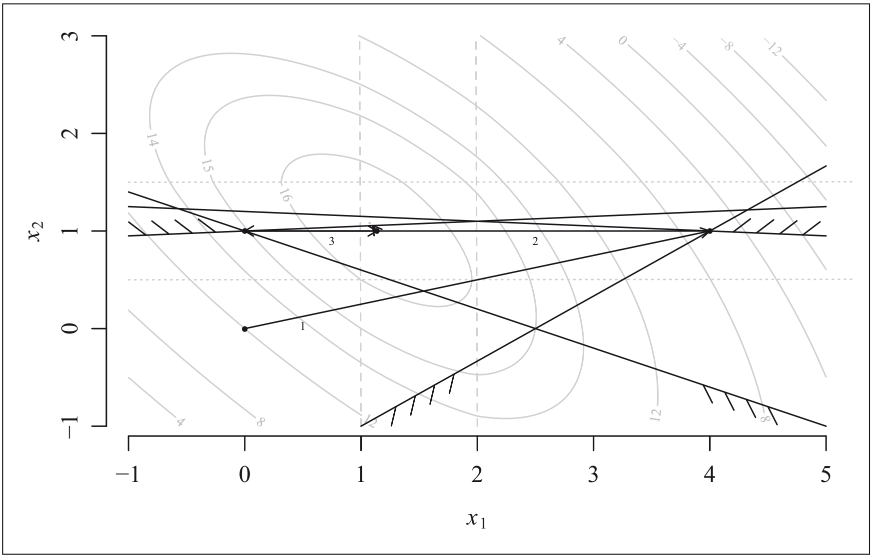

Fig. 3.16 Solution path of illustrative example with sections

$$x_1 + 20\,x_2 \leq 24 \tag{3.85}$$

As can be seen by the partially defined function in Eq. (3.81) the decision variables and its section borders are treated individually. The constraints are slightly modified so that the unconstrained optimum is not within the feasible solution space. The graphical representation of the problem is depicted in Fig. 3.16. The kinks in the objective level curves appear at the section borders, which are marked with the dotted and dashed grey lines.

The first step according to the solution algorithm (see Fig. 3.14) is to determine the unconstrained optimum. This is a bit more extensive than with an objective function that is not partially defined. Firstly, the first order partial derivatives need to be calculated.

$$\frac{\partial z(\mathbf{x})}{\partial x_1} = \begin{cases} -2\,x_1 - \frac{3}{2}\,x_2 + 4 & x_1 \leq 1 \qquad \text{(A)} \\ -\frac{11}{5}\,x_1 - \frac{3}{2}\,x_2 + 4 & 1 < x_1 \leq 2 \qquad \text{(B)} \\ -\frac{12}{5}\,x_1 - \frac{3}{2}\,x_2 + 4 & 2 < x_1 \qquad \text{(C)} \end{cases} \tag{3.86}$$

Table 3.15 Optimal solutions of section combinations

sections	x_1	x_2	sections	x_1	x_2	sections	x_1	x_2
A I	$8/7$ ×	$8/7$ ×	B I	$40/43$ ×	$56/43$ ×	C I	$40/51$ ×	$24/17$ ×
A II	$56/43$ ×	$40/43$ ✓	B II	$40/37$ ✓	$40/37$ ✓	C II	$280/303$ ×	$120/101$ ✓
A III	$24/17$ ×	$40/51$ ×	B III	$120/101$ ✓	$280/303$ ×	C III	$40/39$ ×	$40/39$ ×

The "✓" indicates that the value is within the corresponding section and "×" if not.

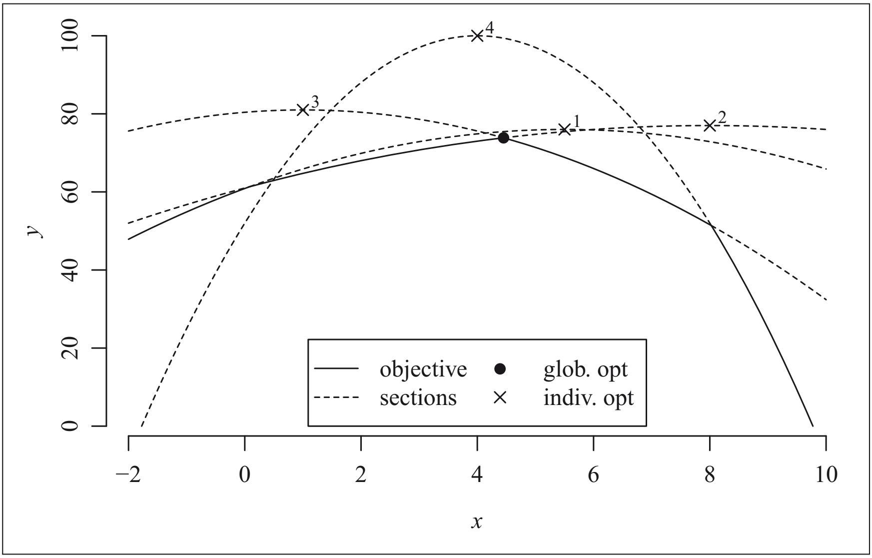

Fig. 3.17 Determining unconstrained optimum with partially defined objective function

$$\frac{\partial z(\mathbf{x})}{\partial x_2} = \begin{cases} -2\,x_2 - \frac{3}{2}\,x_1 + 4 & x_2 \leq \frac{1}{2} & \text{(I)} \\ -\frac{11}{5}\,x_2 - \frac{3}{2}\,x_1 + 4 & \frac{1}{2} < x_2 \leq \frac{3}{2} & \text{(II)} \\ -\frac{12}{5}\,x_2 - \frac{3}{2}\,x_1 + 4 & \frac{3}{2} < x_2 & \text{(III)} \end{cases} \tag{3.87}$$

For each combination of sections A through C and I through III the optima of the corresponding objective functions are listed in Table 3.15. Only the combination of the sections B and II leads to an optimal solution that is located within the corresponding section. This procedure to find the optimum is rather extensive and does not directly account for the case that the optimum is located exactly on a section border.

Before we consider multiple variables let us take a look at the finding process with just one variable. In Fig. 3.17 four sections with their individual

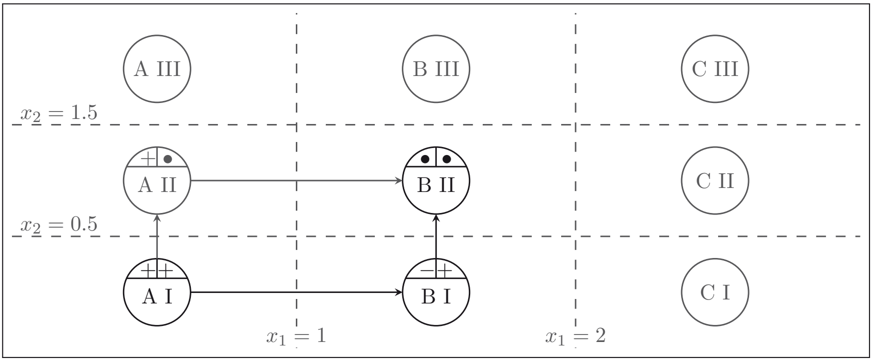

Fig. 3.18 Selective determination of the unconstrained optimum

functions are displayed. The resulting concave function is represented by the solid line. Note that in general it is not the minimum of all section functions.[34] For example, starting from the smallest value (i.e., from the left most section) the optimum of this section is located right of the section. Since, the objective function is concave the optimum must be right of the first section. Thus, the optimum of the second section is determined. It also is located right of the corresponding section. Again, the optimum must be right of the section. Taking a look at the third section shows that the optimum of this section is located left of it. Therefore, the optimum is between section two and three, i.e., on the section border. A consideration of the fourth section is not necessary. The same result is obtained if the procedure is started from the other side, i.e., from the greatest to the smallest value.

This consideration can be transferred to the case with more variables. When we start with the section combination "A I" we find that both values (x_1 and x_2) of the optimum are greater than the section limits allow. Thus, it is expected that either the next section for greater x_1 or x_2 or both might contain the optimum. This information is represented by the "+" in Fig. 3.18. (The left symbol indicates the position of the optimal value x_1 in regard to the section and the right symbol the one for the value x_2.)

[34] The problem specific individual functions are more restricted in their course of the function than displayed here. The individual optima of all succeeding sections (i.e., for greater values) are left (i.e., have smaller values) of the one of the actual section (see appendix B.3). Furthermore, the individual objective functions of all preceding and succeeding sections have greater values in the actual section than the individual objective function of the section (see appendix B.4). Hence, for our problem the resulting concave objective function is the minimum of all individual objective functions. This would make the considerations easier, but we try to keep it as general as possible.

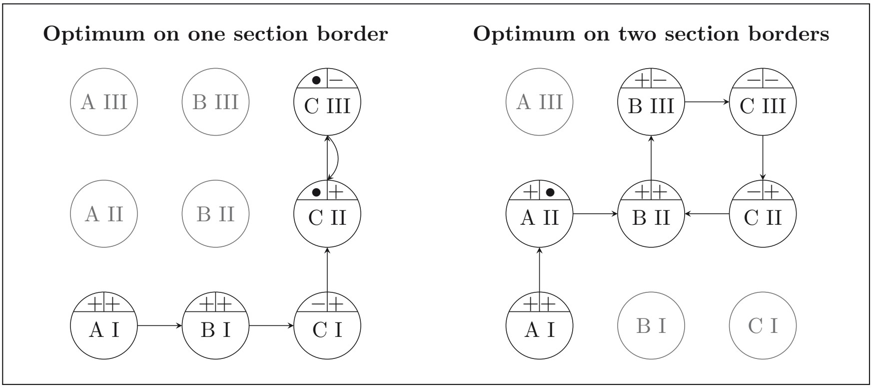

Fig. 3.19 Selective determination of the unconstrained optimum on section border

The "–" would indicate that a section with lower values might contain the optimum. If the optimal value is within the section a "•" is used.

So, starting at "A I", the next section with regard to x_1 and x_2 might contain the optimum, because we have two "+". Which one we consider first does not matter. An increase of x_1 leads to the section combination "B I". The optimal value is not in this section, because of the "+" for x_2. Thus, with respect to x_2 the optimal solution is expected to be in a higher section. On the contrary, the optimal value of x_1 is located in a lower section. But, we just came this way. Therefore, the unexplored way is taken to the section combination "B II". Solving the objective function of this combination, results in a solution that is within the corresponding section limits. Hence, the optimal solution is found and no further optima of section combinations need to be determined. Note that the basic requirements are a concave and continuous objective function. According to that, the unconstrained optimum of the partially defined objective function is $(^{40}/_{37}, ^{40}/_{37})$ (see Table 3.15). Thereby, we needed only three optima calculations and not all nine. The grey path from "A I" to "B II" is an alternative if we had chosen an increase of x_2 instead of x_1 in the first place. Note that it is advisable to start the process of finding the optimum in the middle (e.g., "B II"), because from there the maximal path to each node is two. When starting at "A I", the maximal path is four.

Figure 3.19 shows exemplary situations of the optimum finding with the optimum lying on one section border or two section borders. These cases can be identified by circles within the finding path. On the left side the optimal value for x_2 is the section border between sections "II" and "III".

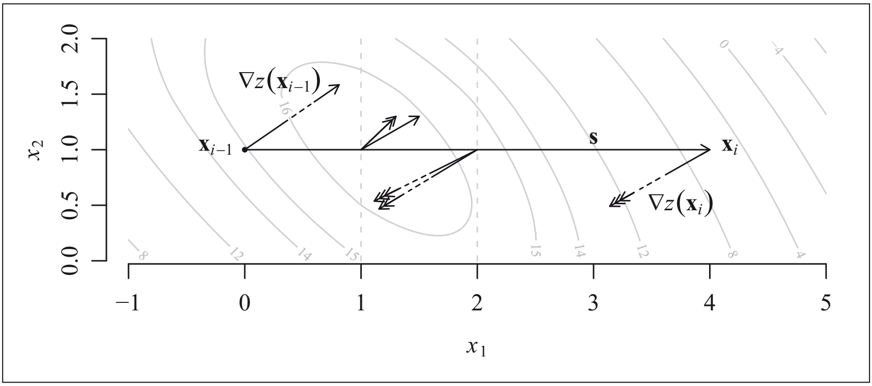

Fig. 3.20 Gradients at section borders along direction vector

The value for x_1 is in the section “C”. In the example on the right side of the figure the optimal value is situated on the borders between sections “B” and “C” as well as “II” and “III”.

In the above mentioned, the general case of interdependent optimal values is described, i.e., the optimal value of x_1 depends on the value x_2 and vice versa (see Eqs. (3.86) and (3.87) with x_1 and x_2 being in the first order derivatives). An effect that is based on this interdependence is that in Fig. 3.18 the “–” changes to a “•” (from “B I” to “B II”) for x_1 even though only the section with regard to x_2 is changed. If this interdependence between the variables does not exist, the solution finding is simpler. Each variable is considered separately (see Sect. 3.3.4).

Now that we have the unconstrained optimum of our problem (QPCOS), we find that it is infeasible. Hence, a LP is created and solved—according to the above developed solution algorithm. After two LP updates and solvings an oscillation is detected. This leads to the second part of the algorithm. The direction vector is calculated with $\mathbf{s} = (4, 0)$. Starting from the point $(0, 1)$ the direction vector crosses two section borders at $x_1 = 1$ and $x_1 = 2$ (see Fig. 3.16). This needs to be considered in the determination of the step width ρ.

Figure 3.20 illustrates the step width determination. (The gradients with the dotted parts are shortened to fit in the figure.) The gradient at $\mathbf{x}_{i-1}$ and $\mathbf{s}$ form an acute angle, i.e., the scalar product is greater than zero. (A scalar product less than zero would indicate that the vector $\mathbf{s}$ points in the direction of a decrease of the objective.) The gradient of the first section on the section border $x_1 = 1$ still forms an acute angle with $\mathbf{s}$. The gradients of the first section are marked with one, the ones of the second section with two, and the ones of the third section with three arrow heads. As can be seen, the gradient of $\mathbf{x}_i$ forms an obtuse angle with $\mathbf{s}$. This means, that a point $\hat{\mathbf{x}}$ with a better objective value is located in between $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$.

For quadratic continuously defined objective functions (i.e., without the sections) the calculation in Eqs. (3.58) and (3.59) leads to $\hat{\mathbf{x}}$. But, because of the abrupt change of the gradient at the section borders, this calculation cannot be used directly. The abrupt change is observable for example at section border $x_1 = 1$ where the gradient of objective function

$$-x_1^2 - \frac{11}{10}\,x_2^2 + \frac{1}{40} - \frac{3}{2}\,x_1\,x_2 + 4\,x_1 + 4\,x_2 + 12 \tag{3.88}$$

(i.e., section combination "A II") is $\nabla z((1,1)) = \left({}^{1}\!/_{2}, {}^{3}\!/_{10}\right)^{\mathrm{T}}$ and not identical with the gradient of the objective function

$$-\frac{11}{10}\,x_1^2 + \frac{1}{10} - \frac{11}{10}\,x_2^2 + \frac{1}{40} - \frac{3}{2}\,x_1\,x_2 + 4\,x_1 + 4\,x_2 + 12 \tag{3.89}$$

(i.e., section combination "B II") with $\nabla z((1,1)) = \left({}^{3}\!/_{10}, {}^{3}\!/_{10}\right)^{\mathrm{T}}$. The gradient with one arrow head forms a smaller angle with $\mathbf{s}$ than the gradient of the second section with two arrow heads. This can also be observed at the section border $x_1 = 2$. The gradients at the section border of the higher section point further towards smaller values of x_1, because the first order derivative of the higher section is smaller than the one of the lower section. (The same applies to section borders with respect to x_2.) This is a property of the required concavity (see Sect. 3.3.1).

To find the point $\hat{\mathbf{x}}$ where the gradient is orthogonal to $\mathbf{s}$ the sections need to be considered separately. The procedure of determining the step width ρ to calculate $\hat{\mathbf{x}}$ according to Eq. (3.59) is depicted in Fig. 3.21. The content of this flowchart substitutes the top right box of the flowchart in Fig. 3.9 and 3.14. Firstly, we check whether a point with an orthogonal gradient on $\mathbf{s}$ is overrun and whether there exists a section change in between $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$. Only if both properties are true a consideration section by section is necessary. Otherwise, the known determination of ρ can be used. If we need to do it section by section the first step is to determine the relevant sections and sort them in order of passing. To stick with our example of three sections with respect to x_1 and three to x_2 we note them like constraints:

$$\mathbf{B}\,\mathbf{x} - \mathbf{h} = \mathbf{b} \tag{3.90}$$

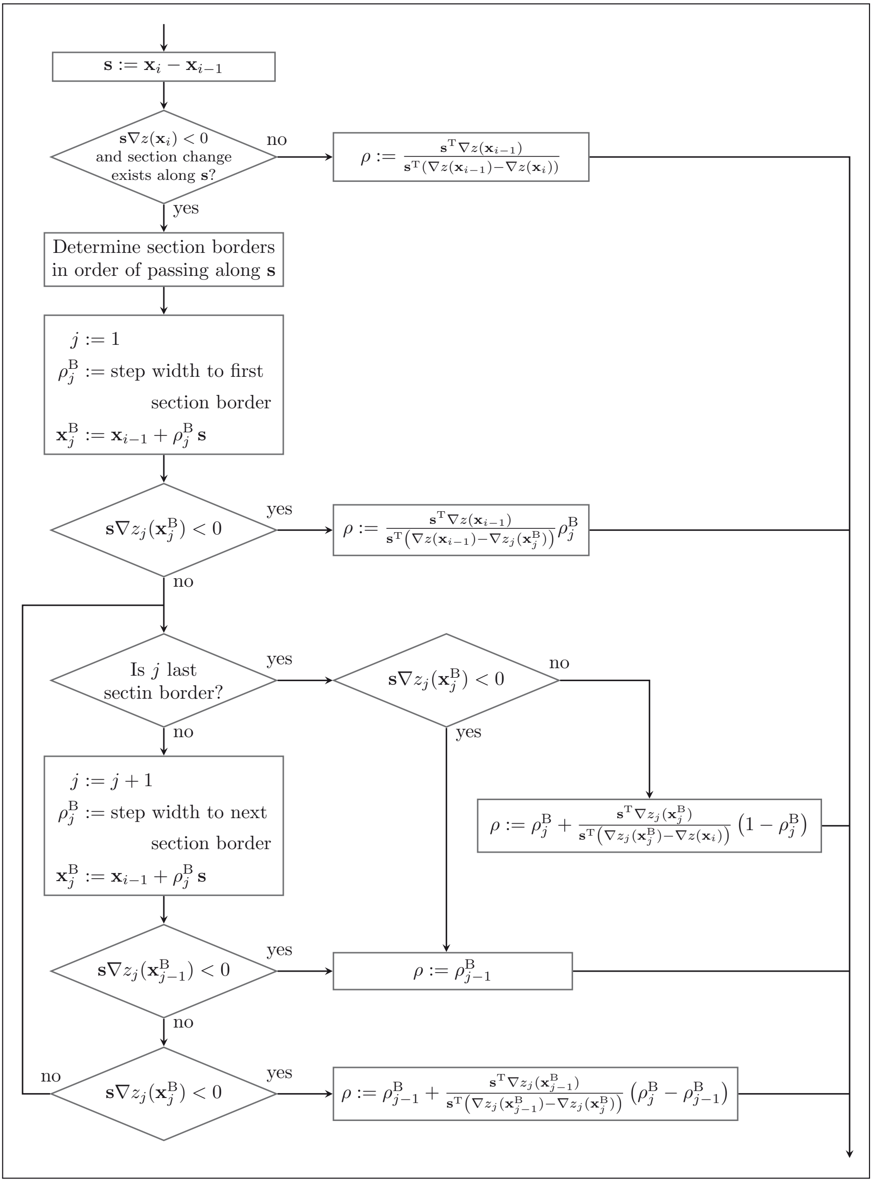

Fig. 3.21 Flowchart of step width calculation

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \mathbf{h} = \begin{pmatrix} 1 \\ 2 \\ \frac{1}{2} \\ \frac{3}{2} \end{pmatrix} . \tag{3.91}$$

Substituting $\mathbf{x}$ by $\mathbf{x}_{i-1}$ and $\mathbf{x}_i$ and comparing the results of $\mathbf{h}_{i-1}$ and $\mathbf{h}_i$, respectively,

$$\mathbf{h}_{i-1} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ \frac{1}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, \quad \mathbf{h}_i = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ \frac{1}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, \tag{3.92}$$

we notice a change of signs in the first two rows of the vectors. This indicates a section change at these two values, i.e., the path along $\mathbf{s}$ starting at $\mathbf{x}_{i-1}$ crosses these two section borders. And since $\mathbf{s}\nabla z((4,1)^{\mathrm{T}}) = -{}^{142}/_{5} < 0$ a section by section consideration is necessary, because a point with an orthogonal gradient is passed. We further see the step width until we reach the section borders when we divide the first and second element of $\mathbf{h}_{i-1}$ by the corresponding scalar products of the first and second row of $\mathbf{B}$ with the vector $\mathbf{s}$. Doing this we get

$$\left| \frac{-1}{(1\ 0) \begin{pmatrix} 4 \\ 0 \end{pmatrix}} \right| = \frac{1}{4} \quad \text{and} \quad \left| \frac{-2}{(1\ 0) \begin{pmatrix} 4 \\ 0 \end{pmatrix}} \right| = \frac{1}{2} . \tag{3.93}$$

The first section border is reached after a quarter of the direction vector and the second section border after a half of $\mathbf{s}$. Hence, $\rho_1^{\mathrm{B}} = {}^1/_4$ and $\rho_2^{\mathrm{B}} = {}^1/_2$ are the values required for the section borders $j = 1$ and $j = 2$, respectively. The points on the direction vector and the section borders are

$$\mathbf{x}_1^{\mathrm{B}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{1}{4} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{x}_2^{\mathrm{B}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} . \tag{3.94}$$

The gradient at $\mathbf{x}_1^{\mathrm{B}}$—using the corresponding objective function $\mathbf{x}_{i-1}$ is in—multiplied with $\mathbf{s}$ results in $\mathbf{s}\nabla z_1\left(\mathbf{x}_1^{\mathrm{B}}\right) = 2$, which is not smaller than zero. Therefore, $\hat{\mathbf{x}}$ is not within this first section. Border $j = 1$ is not the last section border we consider and thus we increase the section border index by one to $j = 2$. The gradient of the objective of the second section at the first section border multiplied with $\mathbf{s}$ is $\mathbf{s}\nabla z_2\left(\mathbf{x}_1^{\mathrm{B}}\right) = {}^6/_5$, which is still greater

Table 3.16 Iterative solution points of illustrative example with sections

iteration	solution	iteration	solution	iteration	solution
0	(0, 0)	2	(0, 1)	4	(1.0989, 1.0549)
1	(4, 1)	3	(1.1364, 1)	5	(1.0996, 1.0550)

The values are rounded to four digits. Values with less than four post decimal digits indicate an exact value without rounding being necessary.

than zero. This means that $\hat{\mathbf{x}}$ is not directly on the section border, because the objective value still increases along the direction vector. We check the next section border and see that $\mathbf{s}\nabla z_2\left(\mathbf{x}_2^{\mathrm{B}}\right) = -{}^{38}/_{5}$ is smaller than zero. Hence, the wanted point $\hat{\mathbf{x}}$ is situated within this section. The calculation of the step width results in

$$\rho = \rho_{j-1}^{\mathrm{B}} + \frac{\mathbf{s}^{\mathrm{T}}\nabla z\left(\mathbf{x}_{j-1}^{\mathrm{B}}\right)}{\mathbf{s}^{\mathrm{T}}\left(\nabla z\left(\mathbf{x}_{j-1}^{\mathrm{B}}\right) - \nabla z\left(\mathbf{x}_{j}^{\mathrm{B}}\right)\right)}\left(\rho_j^{\mathrm{B}} - \rho_{j-1}^{\mathrm{B}}\right) \tag{3.95}$$

$$= \frac{1}{4} + \frac{(4\ 0)\begin{pmatrix}\frac{3}{10}\\ \frac{3}{10}\end{pmatrix}}{(4\ 0)\left(\begin{pmatrix}\frac{3}{10}\\ \frac{3}{10}\end{pmatrix} - \begin{pmatrix}-\frac{19}{10}\\ -\frac{6}{5}\end{pmatrix}\right)}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{25}{88}\,. \tag{3.96}$$

A further consideration of succeeding sections is not necessary and the algorithm continues as depicted in the flow diagram of Fig. 3.9 or 3.14. Thus, the next iteration point $\hat{\mathbf{x}}$ is $({}^{25}/_{22}, 1)$. It is the solution of the third iteration (see Table 3.16).

Two further iterations are necessary and the optimal solution is found. The complete solution finding path is illustrated in Fig. 3.16. For the sake of completeness it shall be mentioned that for finding the optimal solution of a concave quadratic objective function the solution algorithm by Rosen[35] can also be modified by the elements of the step width determination and the partially defined objective function. But, with many variables and constraints and possibly over-determined feasible solution space, it is numerically difficult to find the optimal solution following the theoretical steps. An according sketch for an algorithm still using LPs first can be found in appendix B.5. With all the considerations above we can now find the optimal solution of our disassembly planning example with standard LP software.

[35] Cf. Rosen (1960): *Gradient projection method*.

3.3.3.4 Mix of quadratic and linear variables

In the above we considered pure quadratic objective functions, i.e., no linear variables exist within the objective function. But many quadratic planning problems contain also linear variables. Therefore, a discussion about this case is necessary in the sequel. The problem with linear variables is that they do not have a finite optimum. The unconstrained maximum and minimum of the function $y = 4\,x$ is plus and minus infinity, respectively. This cannot be used in the presented algorithm. Three possibilities to handle this case are discussed in the following:

1. adding quadratic terms,
2. using the gradient only, and
3. combining the unconstrained optimum and the gradient.

The small example is slightly modified to illustrate this case. The variable x_1 is still quadratic and x_2 becomes linear. The quadratic and linear problem with linear constraints (QLLP) is defined by:

$$\text{(QLLP):}\quad \text{Maximise } z = \left(\begin{cases} -x_1^2 & x_1 \le 1 \\ -\frac{11}{10}\,x_1^2 + \frac{1}{10} & 1 < x_1 \le 2 \\ -\frac{6}{5}\,x_1^2 + \frac{1}{2} & 2 < x_1 \end{cases}\right) + 4\,x_1 + 4\,x_2 + 12 \tag{3.97}$$

$$\text{s.t.}\quad 2\,x_1 + \;\; 5\,x_2 \ge 5 \tag{3.98}$$

$$2\,x_1 - \;\; 3\,x_2 \le 5 \tag{3.99}$$

$$-x_1 + 20\,x_2 \le 20 \tag{3.100}$$

$$x_1 + 20\,x_2 \le 24\,. \tag{3.101}$$

The first option we consider is adding quadratic terms. The idea is that the objective function is extended by a quadratic term of a linear variable with very little influence, e.g., $-10^{-9}\,x_2^2$.[36] Doing so, the unconstrained optimum would be $(20/11, 2\cdot 10^9)$. This equals choosing an arbitrary large number for the linear variables as finite optimum instead of infinity. Note that the coefficient of the additional quadratic term should be as small as possible to avoid a big influence and to assure that the unconstrained optimum is outside the feasible solution space. On the other hand, the coefficient should not be too small to be able to do the calculations with a computer, which has a certain precision. Having added the quadratic terms the presented

[36] The quadratic terms must still assure an overall concave objective function.

solution algorithm (see Fig. 3.14) in its basic form can be applied. A disadvantage—apart from the right choosing of the coefficients—is that the gained "optimal" solution is not really optimal, because of the disturbance by the small quadratic terms.

The second option is the usage of the algorithm depicted in Fig. 3.9. With this no determination of an unconstrained optimum is necessary and the used gradient can be easily calculated. The only drawback might be a possibly worse convergence compared to the third option.

The third option is a combination of both approaches, i.e., a finite optimum for quadratic variables is combined with the gradient for linear variables. A finite optimum does still not exist. That is why we skip the steps marked with an asterisk in Fig. 3.14. But in the succeeding we still use the partly existing finite optimum. The approach is derived from the first option without adding quadratic terms. For all quadratic variables we determine the finite optimal values. For example, the value of x_1 where the maximum value (with respect to x_1) is achieved is $x_1 = {}^{20}\!/_{11}$. For the linear variable x_2 no such optimal value exists. Hence, the feasibility check with the unconstrained optimum is skipped, too. The succeeding steps of the algorithm in Fig. 3.14 are unchanged with the exception of the calculation of $\mathbf{o}$ (marked with a triangle in Fig. 3.14). This vector is calculated in the following way.

For variables with a finite optimum value (e.g., $x_1 = {}^{20}\!/_{11}$) the calculation of $\mathbf{o} = \mathbf{x}_{\text{opt}} - \hat{\mathbf{x}}$ is conducted and for all other variables (e.g., x_2) the gradient is used. For example, if the iterative solution point is $\hat{\mathbf{x}} = (0, 1)^{\mathrm{T}}$ the modified vector pointing to the optimum $\mathbf{o}$ would be $\mathbf{o} = \left((\mathbf{x}_{\text{opt}} - \hat{\mathbf{x}}) \otimes \nabla z(\hat{\mathbf{x}})\right)^{\mathrm{T}} = ({}^{20}\!/_{11} - 0, 4)^{\mathrm{T}} = ({}^{20}\!/_{11}, 4)^{\mathrm{T}}$, because the first order derivative with respect to x_2 is four independent of the value of x_2. Thereby, the operator $\otimes$ denotes the choosing of an element either from its left or right side of the operator depending on a finite value on the left. If $(\mathbf{x}_{\text{opt}} - \hat{\mathbf{x}})$ is finite, this value is taken, otherwise the gradient value. We assume that the convergence is slightly better than an approach only based on the gradient, because—depending on the coefficients of the quadratic variables—the convergence for single variables is better. But for a sound statement further studies are necessary, which are out of the scope of this work.

For the illustrative example the unconstrained optimum is found with either method in five iterations (see Table 3.17). The optimal solution is $({}^{21}\!/_{11}, {}^{241}\!/_{220})^{\mathrm{T}}$ and the numerically found solutions are within a tolerance of $5 \cdot 10^{-10}$. The closest solution is the one found by option three (tolerance of $9 \cdot 10^{-16}$), but this could be by chance. The visualisation of the problem and the solution path are depicted in Fig. 3.22.

Table 3.17 Iterative solution points of illustrative example with mixed variables

iteration	solution	iteration	solution	iteration	solution
0	(0, 0)	2	(0, 1)	4	(1.8182, 1.0909)
1	(4, 1)	3	(1.8182, 1)	5	(1.9091, 1.0955)

The values are rounded to four digits. Values with less than four post decimal digits indicate an exact value without rounding being necessary.

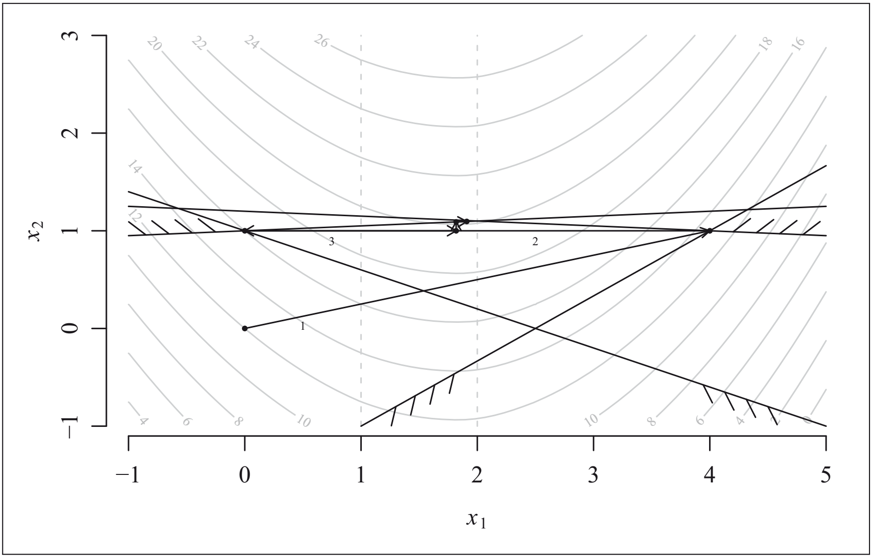

Fig. 3.22 Illustration of (QLLP) example

A special case of variables are the variables with the coefficient zero. These variables do not distribute to the objective function at all. Thus, any value of such a variable is an optimal value, as long as the problem is feasible. These variables need no special treatment, because in the algorithm steps they are automatically handled with when solving a LP or MIQLP. This is also true for problems containing only quadratic variables and variables with zero coefficients.

3.3.3.5 Adaptation to integer problems

So far we have considered the domain of real numbers. When we restrict the problem further to a problem with integrality constraints, we get a so-called pure integer problem, i.e., only integral variables exist, or a mixed integer problem, i.e., integral and real valued variables exist. This also requires an adaptation of the presented solution algorithm. The first adaptation is the determination of the unconstrained integral optimum. This is easily possible for concave quadratic functions without mixed quadratic terms, which is the case for the problems in disassembly planning that we consider. The quadratic functions are symmetric with the axis of symmetry being the maximum (see appendix B.6). Therefore, only the absolute distance to the optimum $\mathbf{x}_{\text{opt}}$ is relevant. Hence, whatever integral value is closer to $\mathbf{x}_{\text{opt}}$ is the integral optimal solution. Thereby, the integral solution is found by rounding. Note that in the case of equidistance both integral values are optimal. For example, if $\mathbf{x}_{\text{opt}} = 3/2$, the values $\mathbf{x}_{\text{opt}}^{\text{int}} = 1$ and $\mathbf{x}_{\text{opt}}^{\text{int}} = 2$ are both optimal, because they both have the highest (identical) function value. Applying this to an n-dimensional problem up to 2^n optimal values can exist where there is only one optimal value in the domain of real numbers.

If one of these optima is feasible the optimal integral solution is found without using a solver software. (Note that for problems with linear variables the unconstrained optimum does not exist, see Sect. 3.3.3.4, which makes the determination of the constrained real optimum the first step.) Otherwise, the constrained optimum of the domain of real numbers is determined. Having this solution we assume that the integral optimum is in the neighbourhood, i.e., within the same section combination. Thus, a MIQLP is formulated which uses an objective function that is only based on the objective functions of the actual section combination. This way we try to keep the necessary solver runs with MIQLP (and MILP) low, because integer solving takes usually more time than LP solving. Solving the MIQLP shows, whether the problem is feasible or not. If it is feasible it also delivers a first integral solution. To find the optimal integral solution the above mentioned algorithms could possibly be modified, but this would include several MILP solvings and, furthermore, the handling of an assisting constraint needs to be modified, because a restriction to the hyperplane as in the QLP solving will most likely omit many relevant integral solutions. Therefore, a different procedure is preferred, that follows the considerations to find the unconstrained optimum with section borders present (see Sect. 3.3.3.3 and here especially Fig. 3.18 and 3.19).

For a better understanding Fig. 3.23 illustrates the following discussion. Depicted are four areas I through IV, which represent the four section com-

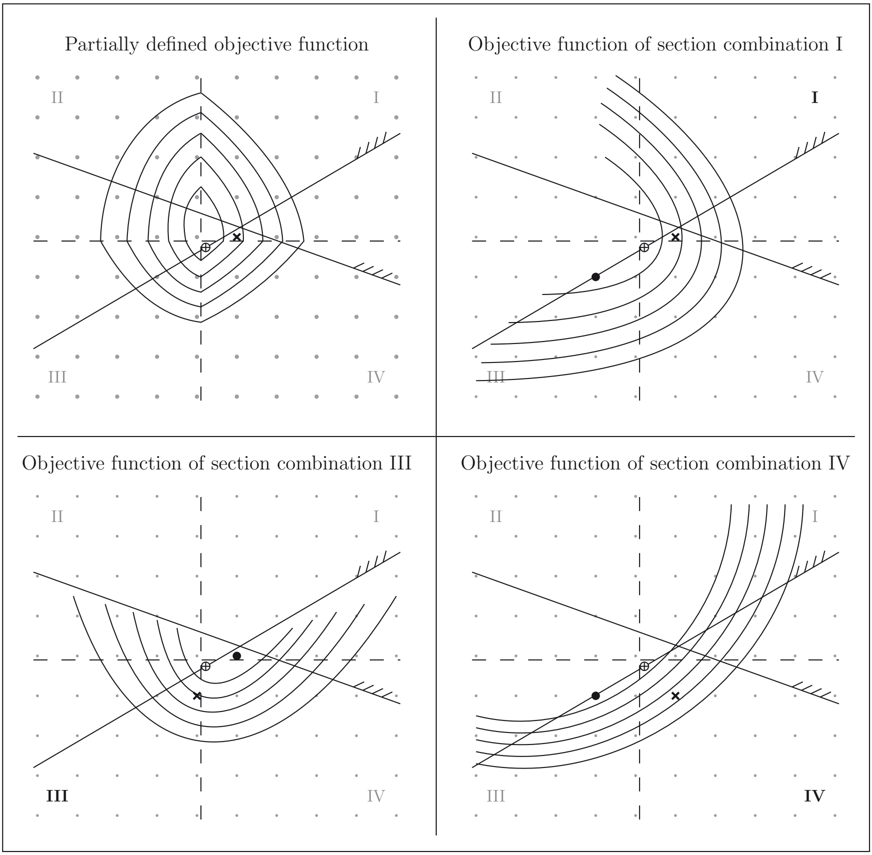

Fig. 3.23 Objective functions and solution space for the section combinations

binations. The section borders are marked by the vertical and horizontal dashed lines. The grid of grey dots indicates the integral solution values. The two constraints (solid black straight lines) limit the feasible solution space to the section combinations I, III, and IV. In the upper left part of the figure we see several levels of the partially defined objective function with the jumping first order derivatives at the section borders. The "$\oplus$" indicates the optimum in the domain of real numbers in all four parts of the figure.

The optimal real valued solution is situated within the section combination IV. Thus, only the objective function of this section combination is used for the first MIQLP solving. The objective function is also set valid for all other sections. This means that no constraints are added to restrict the solution to the focussed section. If the integral solution is situated within the section combination, the optimal integral solution is found. The objective function of the section combination used for the MIQLP is correct for the actual section combination and overestimates the objective function of the other sections (see appendix B.4). So if the optimal integral solution is within the section combination, all other integral solutions especially of neighbouring sections have a lower value. And this lower value is even overestimated by the actual section combination objective function, such that the correct objective value (corresponding to the section) is even lower and can therefore not be better than the optimal one that is found. Note that the variables are independent of each other, i.e., a change of a value of decision variable one does not lead to a change of a value of variable two regarding the objective value.

If the integral solution is not within the section combination we used the objective function of, further steps are necessary. As depicted in the lower right part of Fig. 3.23 the integral solution is outside the section combination. Here, the integral solution "•" is in section combination III instead of IV. Repeating the procedure (i.e., solving a MIQLP) with the objective function of the section combination III leads to the integral solution in section combination I (lower left part). Repeating it with the objective function of section combination I results in the known solution in section combination III (see upper right part in the figure). In order to find the optimal integral solution an objective comparison needs to be applied. Thereby, all section combinations that are part of the oscillating solutions and the ones that are between these are compared.

For each participating section combination the corresponding objective function is used. In addition, this time the section borders are added as additional constraints to limit the integral solution to the corresponding section combination. For each participating section combination a MIQLP is solved and the results are compared afterwards. The optimal integral solution of each section combination (I, III, and IV) is marked with "×" in the corresponding part of the figure. The best of these three solutions is chosen and represented by the "×" in the upper left part of Fig. 3.23. Note that there might be section combinations without an integral solution, like section combination II. When it comes to the comparison the MIQLP solver needs to be run several times which is surely not the fastest way of solving the problem, but we can use standard solvers like GUROBI. In the worst case the solution finding results in a complete enumeration of all section

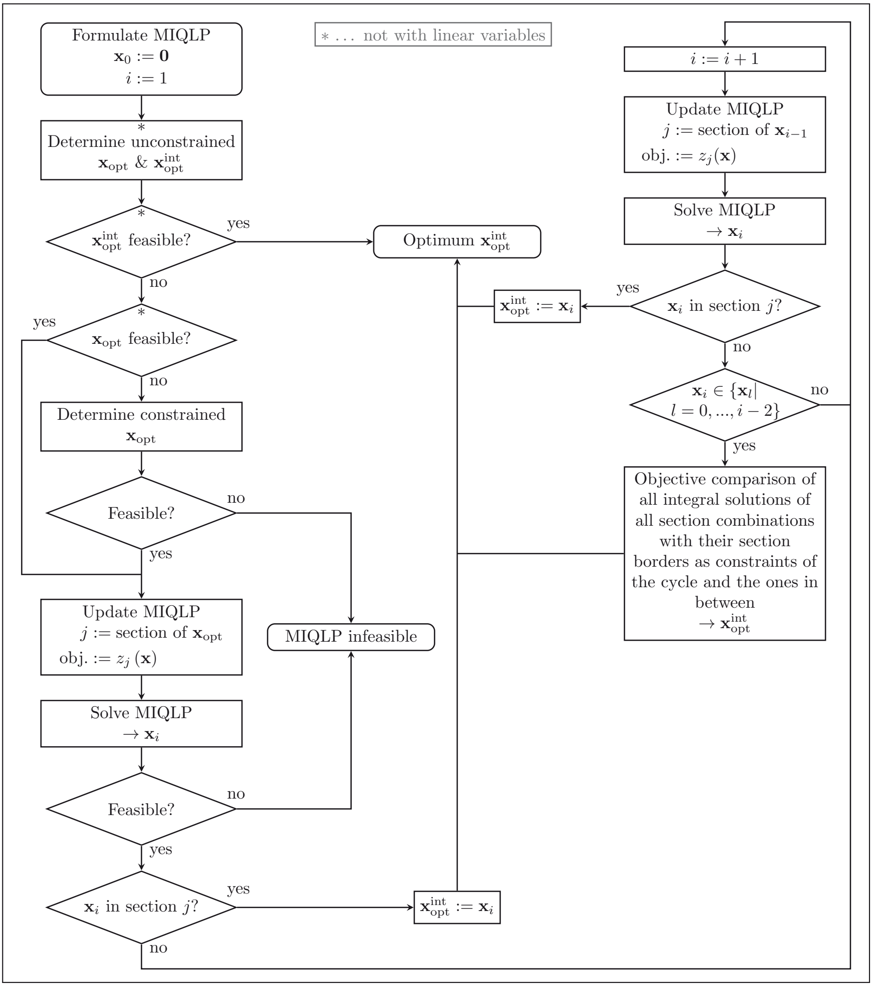

Fig. 3.24 Flowchart of finding the optimal integral solution(s)

combinations.[37] We expect that in most cases the solution is found with just one MIQLP solving. The flowchart is depicted in Fig. 3.24.

A modification of the above used example shall be used to illustrate the algorithm. The modification is necessary, because the unconstrained optimal

[37] Of course, sections where the real valued optimum is lower than an already found integral solution can be excluded from the enumeration.

integral solution is feasible and therefore not the best choice to illustrate the algorithm. For the (MIQLP) the objective function and the section borders are taken from above (see Eq. (3.81)).

$$\text{(MIQLP):}\qquad \text{Maximise } z = \left(\begin{cases} -x_1^2 & x_1 \le 1 \\ -\frac{11}{10}\,x_1^2 + \frac{1}{10} & 1 < x_1 \le 2 \\ -\frac{6}{5}\,x_1^2 + \frac{1}{2} & 2 < x_1 \end{cases}\right) + \left(\begin{cases} -x_2^2 & x_2 \le \frac{1}{2} \\ -\frac{11}{10}\,x_2^2 + \frac{1}{40} & \frac{1}{2} < x_2 \le \frac{3}{2} \\ -\frac{6}{5}\,x_2^2 + \frac{1}{4} & \frac{3}{2} < x_2 \end{cases}\right) - \frac{3}{2}\,x_1\,x_2 + 4\,x_1 + 4\,x_2 + 12 \tag{3.102}$$

The constraints are modified to

$$\text{s.t.}\qquad 2\,x_1 + \ \ 5\,x_2 \ge 5 \tag{3.103}$$
$$2\,x_1 - \ \ 3\,x_2 \le 5 \tag{3.104}$$
$$-2\,x_1 + 10\,x_2 \le 5 \tag{3.105}$$
$$x_1 + 20\,x_2 \le 24 \tag{3.106}$$
$$x_1, x_2 \in \mathbb{Z}\,. \tag{3.107}$$

The problem is depicted in Fig. 3.25. According to the presented algorithm, the optimal unconstrained real valued and integral shall be determined. These are $\mathbf{x}_{\text{opt}} = (40/37, 40/37)^{\mathrm{T}}$ and $\mathbf{x}_{\text{opt}}^{\text{int}} = (1,1)^{\mathrm{T}}$, respectively. Both are infeasible so that the constrained real valued optimum needs to be determined. According to the earlier presented algorithm (see Fig. 3.15) a feasible solution of $\mathbf{x}_{\text{opt}} = (1915/1444, 1105/1444)^{\mathrm{T}}$ is found. This solution is located within the section combination "B II". The corresponding objective function is

$$z_{\mathrm{B\,II}}(\mathbf{x}) = -\frac{11}{10}\,x_1^2 - \frac{11}{10}\,x_2^2 - \frac{3}{2}\,x_1\,x_2 + 4\,x_1 + 4\,x_2 + \frac{97}{8} \tag{3.108}$$

(see Eq. (3.89)). Solving the MIQLP with this objective and the constraints of (MIQLP) results in the integral solution $(3,1)^{\mathrm{T}}$. This solution is not in section combination "B II" anymore. This solution appears the first time. Hence, the objective function is updated with the corresponding section combination "C II".

$$z_{\mathrm{C\,II}}(\mathbf{x}) = -\frac{6}{5}\,x_1^2 - \frac{11}{10}\,x_2^2 - \frac{3}{2}\,x_1\,x_2 + 4\,x_1 + 4\,x_2 + \frac{501}{40} \tag{3.109}$$

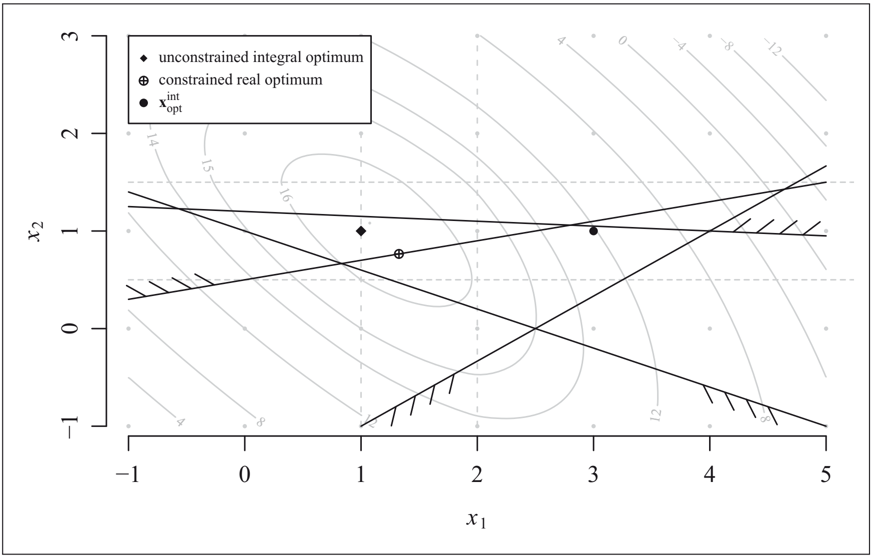

Fig. 3.25 Illustration of (MIQLP) example

Solving this MIQLP results in the solution $(3,1)^{\mathrm{T}}$, which is within the section "C II". Thus, the integral optimum is found with $\mathbf{x}_{\text{opt}}^{\text{int}} = (3,1)^{\mathrm{T}}$.

3.3.4 Numerical example

After developing the finding of the optimal solution of a quadratic optimisation problem with a piecewise defined concave objective function and linear constraints it shall be applied to the disassembly planning problem motivated in Sect. 3.3.1. Other than in Sect. 3.3.2, the section variables (e.g., Q^{I}_{es}) are not used in the sequel, because the developed solution finding is based on a partially defined objective function.[38]

The objective function consists of

$$R = \sum_e r^{\mathrm{I}}_e(Q^{\mathrm{I}}_e)\, Q^{\mathrm{I}}_e + \sum_r r^{\mathrm{R}}_r(Q^{\mathrm{R}}_r)\, Q^{\mathrm{R}}_r \tag{3.110}$$

[38] The section variables are necessary for solvers that do not support partially defined objective functions.

Table 3.18 Data for the piecewise linear price-quantity dependent function

c	$\bar{c}^{\mathrm{A}}_{c}$	$\hat{c}^{\mathrm{A}}_{c,1}$	$\hat{c}^{\mathrm{A}}_{c,2}$	$\check{Q}^{\mathrm{C}}_{c,1}$	e	$\bar{r}^{\mathrm{I}}_{e}$	$\hat{r}^{\mathrm{I}}_{e,1}$	$\hat{r}^{\mathrm{I}}_{e,2}$	$\hat{r}^{\mathrm{I}}_{e,3}$	$\check{Q}^{\mathrm{I}}_{e,1}$	$\check{Q}^{\mathrm{I}}_{e,2}$
1	2,220	2.2	2.4	50	1	31	-0.004	-	-	-	-
2	2,095	2.7	2.9	80	2	300	0	-	-	-	-
3	2,830	2.8	3.1	50	3	2,500	-0.49	-0.55	-0.6	100	200

r	$\bar{r}^{\mathrm{R}}_{r}$	$\hat{r}^{\mathrm{R}}_{r,1}$	$\hat{r}^{\mathrm{R}}_{r,2}$	$\check{Q}^{\mathrm{R}}_{r,1}$	d	$\bar{c}^{\mathrm{D}}_{d}$	$\hat{c}^{\mathrm{D}}_{d,1}$	$\hat{c}^{\mathrm{D}}_{d,2}$	$\check{Q}^{\mathrm{D}}_{d,1}$
1	1.43	0	-0.0000014	50,000	1	0.2	0	0.0000002	50.000
2	1.00	0	-0.0000009	50,000	2	0.4	0.0000004	0.0000006	10.000
3	0.75	0	-0.0000007	50,000					
4	0.45	0	-	-					

and

$$C = \sum_{c} \left(c^{\mathrm{J}}_{c} + c^{\mathrm{A}}_{c}(Q^{\mathrm{C}}_{c})\right) Q^{\mathrm{C}}_{c} + \sum_{d} c^{\mathrm{D}}_{d}(Q^{\mathrm{D}}_{d})\, Q^{\mathrm{D}}_{d} \tag{3.111}$$

with, for instance, $r^{\mathrm{I}}_{e}(Q^{\mathrm{I}}_{e})$ being

$$r^{\mathrm{I}}_{e}(Q^{\mathrm{I}}_{e}) = \begin{cases} \bar{r}^{\mathrm{I}}_{e} + \hat{r}^{\mathrm{I}}_{e,1}\, Q^{\mathrm{I}}_{e} & 0 \le Q^{\mathrm{I}}_{e} \le \check{Q}^{\mathrm{I}}_{e,1} \\ \bar{r}^{\mathrm{I}}_{e} + \sum_{t=2}^{s} \left(\hat{r}^{\mathrm{I}}_{e,t-1} - \hat{r}^{\mathrm{I}}_{et}\right) \check{Q}^{\mathrm{I}}_{e,t-1} + \hat{r}^{\mathrm{I}}_{es}\, \check{Q}^{\mathrm{I}}_{e} & \check{Q}^{\mathrm{I}}_{e,s-1} < Q^{\mathrm{I}}_{e} \le \check{Q}^{\mathrm{I}}_{es} \\ \bar{r}^{\mathrm{I}}_{e} + \sum_{t=2}^{S} \left(\hat{r}^{\mathrm{I}}_{e,t-1} - \hat{r}^{\mathrm{I}}_{et}\right) \check{Q}^{\mathrm{I}}_{e,t-1} + \hat{r}^{\mathrm{I}}_{eS}\, \check{Q}^{\mathrm{I}}_{e} & \check{Q}^{\mathrm{I}}_{e,S-1} < Q^{\mathrm{I}}_{e} \end{cases} . \tag{3.112}$$

According to the data given in Table 3.18 the price and cost functions are:

$$r^{\mathrm{I}}_{1}(Q^{\mathrm{I}}_{1}) = 31 - 0.004\, Q^{\mathrm{I}}_{1} \tag{3.113}$$

$$r^{\mathrm{I}}_{2}(Q^{\mathrm{I}}_{2}) = 300 \tag{3.114}$$

$$r^{\mathrm{I}}_{3}(Q^{\mathrm{I}}_{3}) = \begin{cases} 2500 - 0.49\, Q^{\mathrm{I}}_{3} & 0 \le Q^{\mathrm{I}}_{3} \le 100 \\ 2506 - 0.55\, Q^{\mathrm{I}}_{3} & 100 < Q^{\mathrm{I}}_{3} \le 200 \\ 2516 - 0.6\, Q^{\mathrm{I}}_{3} & 200 < Q^{\mathrm{I}}_{3} \end{cases} \tag{3.115}$$

$$r^{\mathrm{R}}_{1}(Q^{\mathrm{R}}_{1}) = \begin{cases} 1.43 & 0 \le Q^{\mathrm{R}}_{1} \le 50000 \\ 1.5 - 0.0000014\, Q^{\mathrm{R}}_{1} & 50000 < Q^{\mathrm{R}}_{1} \end{cases} \tag{3.116}$$

$$r^{\mathrm{R}}_{2}(Q^{\mathrm{R}}_{2}) = \begin{cases} 1 & 0 \le Q^{\mathrm{R}}_{2} \le 50000 \\ 1.045 - 0.0000009\, Q^{\mathrm{R}}_{2} & 50000 < Q^{\mathrm{R}}_{2} \end{cases} \tag{3.117}$$

$$r^{\mathrm{R}}_{3}(Q^{\mathrm{R}}_{3}) = \begin{cases} 0.75 & 0 \le Q^{\mathrm{R}}_{3} \le 50000 \\ 0.785 - 0.0000007\, Q^{\mathrm{R}}_{3} & 50000 < Q^{\mathrm{R}}_{3} \end{cases} \tag{3.118}$$

$$r^{\mathrm{R}}_{4}(Q^{\mathrm{R}}_{4}) = 0.45 \tag{3.119}$$

$$c_1^{\mathrm{A}}(Q_1^{\mathrm{C}}) = \begin{cases} 2220 + 2.2\,Q_1^{\mathrm{C}} & 0 \le Q_1^{\mathrm{C}} \le 50 \\ 2210 + 2.4\,Q_1^{\mathrm{C}} & 50 < Q_1^{\mathrm{C}} \end{cases} \tag{3.120}$$

$$c_2^{\mathrm{A}}(Q_2^{\mathrm{C}}) = \begin{cases} 2095 + 2.7\,Q_2^{\mathrm{C}} & 0 \le Q_2^{\mathrm{C}} \le 80 \\ 2079 + 2.9\,Q_2^{\mathrm{C}} & 80 < Q_2^{\mathrm{C}} \end{cases} \tag{3.121}$$

$$c_3^{\mathrm{A}}(Q_3^{\mathrm{C}}) = \begin{cases} 2830 + 2.8\,Q_3^{\mathrm{C}} & 0 \le Q_3^{\mathrm{C}} \le 50 \\ 2815 + 3.1\,Q_3^{\mathrm{C}} & 50 < Q_3^{\mathrm{C}} \end{cases} \tag{3.122}$$

$$c_1^{\mathrm{D}}(Q_1^{\mathrm{D}}) = \begin{cases} 0.2 & 0 \le Q_1^{\mathrm{D}} \le 50000 \\ 0.19 + 0.0000002\,Q_1^{\mathrm{D}} & 50000 < Q_1^{\mathrm{D}} \end{cases} \tag{3.123}$$

$$c_2^{\mathrm{D}}(Q_2^{\mathrm{D}}) = \begin{cases} 0.4 + 0.0000004\,Q_2^{\mathrm{D}} & 0 \le Q_2^{\mathrm{D}} \le 10000 \\ 0.398 + 0.0000006\,Q_2^{\mathrm{D}} & 10000 < Q_2^{\mathrm{D}} \end{cases}. \tag{3.124}$$

With these parts the functions for revenues R and cost C can be formulated.

$$\begin{aligned} R = {} & \left(31 - 0.004\,Q_1^{\mathrm{I}}\right) Q_1^{\mathrm{I}} + 300\,Q_2^{\mathrm{I}} \\ & + \left(\begin{cases} 2500 - 0.49\,Q_3^{\mathrm{I}} & 0 \le Q_3^{\mathrm{I}} \le 100 \\ 2506 - 0.55\,Q_3^{\mathrm{I}} & 100 < Q_3^{\mathrm{I}} \le 200 \\ 2516 - 0.6\,Q_3^{\mathrm{I}} & 200 < Q_3^{\mathrm{I}} \end{cases} \right) Q_3^{\mathrm{I}} \\ & + \left(\begin{cases} 1.43 & 0 \le Q_1^{\mathrm{R}} \le 50000 \\ 1.5 - 0.0000014\,Q_1^{\mathrm{R}} & 50000 < Q_1^{\mathrm{R}} \end{cases} \right) Q_1^{\mathrm{R}} \\ & + \left(\begin{cases} 1 & 0 \le Q_2^{\mathrm{R}} \le 50000 \\ 1.045 - 0.0000009\,Q_2^{\mathrm{R}} & 50000 < Q_2^{\mathrm{R}} \end{cases} \right) Q_2^{\mathrm{R}} \\ & + \left(\begin{cases} 0.75 & 0 \le Q_3^{\mathrm{R}} \le 50000 \\ 0.785 - 0.0000007\,Q_3^{\mathrm{R}} & 50000 < Q_3^{\mathrm{R}} \end{cases} \right) Q_3^{\mathrm{R}} + 0.45\,Q_4^{\mathrm{R}} \end{aligned} \tag{3.125}$$

$$\begin{aligned} C = {} & \left(300 + \begin{cases} 2220 + 2.2\,Q_1^{\mathrm{C}} & 0 \le Q_1^{\mathrm{C}} \le 50 \\ 2210 + 2.4\,Q_1^{\mathrm{C}} & 50 < Q_1^{\mathrm{C}} \end{cases} \right) Q_1^{\mathrm{C}} \\ & + \left(280 + \begin{cases} 2095 + 2.7\,Q_2^{\mathrm{C}} & 0 \le Q_2^{\mathrm{C}} \le 80 \\ 2079 + 2.9\,Q_2^{\mathrm{C}} & 80 < Q_2^{\mathrm{C}} \end{cases} \right) Q_2^{\mathrm{C}} \\ & + \left(260 + \begin{cases} 2830 + 2.8\,Q_3^{\mathrm{C}} & 0 \le Q_3^{\mathrm{C}} \le 50 \\ 2815 + 3.1\,Q_3^{\mathrm{C}} & 50 < Q_3^{\mathrm{C}} \end{cases} \right) Q_3^{\mathrm{C}} \end{aligned}$$

$$+\left(\begin{cases}0.2 & 0 \leq Q_1^{\mathrm{D}} \leq 50000\\ 0.19+0.0000002\,Q_1^{\mathrm{D}} & 50000 < Q_1^{\mathrm{D}}\end{cases}\right) Q_1^{\mathrm{D}}$$

$$+\left(\begin{cases}0.4+0.0000004\,Q_2^{\mathrm{D}} & 0 \leq Q_2^{\mathrm{D}} \leq 10000\\ 0.398+0.0000006\,Q_2^{\mathrm{D}} & 10000 < Q_2^{\mathrm{D}}\end{cases}\right) Q_2^{\mathrm{D}} \qquad (3.126)$$

The constraints are unchanged and given in Sect. 3.1.

Generally, the solution finding follows the four steps:

1. determine the unconstrained real and integral optimum,
2. check for feasibility of the unconstrained real and integral optimum,
3. determine the constrained real optimum, and
4. determine the constrained integral optimum.

Since the model not only contains quadratic variables in the objective function, but also linear ones (e.g., the variable Q_2^{I} is linear, whereas Q_1^{I} is quadratic) the first two steps are skipped (see Sect. 3.3.3.4). Hence, we start with the third step. In this step the constrained real valued optimum is determined. For this we can choose between the three given options (see Sect. 3.3.3.4). As we will see, it does not matter which option we choose, because the determination of the constrained optimum stops before an option specific part starts.

According to the algorithm steps in Fig. 3.14, a LP is created that contains the constraints of the quadratic problem and uses the gradient of an arbitrary solution for the objective function. As arbitrary solution we take vector $\mathbf{x} = 0$. That means that all decision variables (i.e., the Q as well as X) are summarised to this vector $\mathbf{x}$ (see Eqs. (3.23) and (3.24)).

$$\mathbf{x} = \left(Q^{\mathrm{I}}_{1..3}, Q^{\mathrm{R}}_{1..4}, Q^{\mathrm{C}}_{1..3}, Q^{\mathrm{D}}_{1..2}, X^{\mathrm{I}}_{1..3,\mathrm{A..H}}, X^{\mathrm{R}}_{1..3,\mathrm{A..H},1..4}, X^{\mathrm{D}}_{1..3,\mathrm{A..H},1..2}\right)^{\mathrm{T}} \qquad (3.127)$$

The resulting gradient of the objective function P at $\mathbf{x} = 0$ is

$$\nabla P(\mathbf{x}) = (31, 300, 2500, 1.43, 1, 0.75, 0.45,\\ -2520, -2375, -3090, -0.2, -0.4, 0\ldots0)^{\mathrm{T}}\,. \qquad (3.128)$$

Solving the LP, results in a feasible solution. The coefficients of the objective function are updated with the gradient at the actual solution and the LP is solved again. This is repeated until after three iterations a cycle is detected. This causes to continue the solution algorithm according to Fig. 3.14 on the right side. After six further iterations the optimal solution is found. The solution is depicted in Table 3.19. The objective values of the iterations in chronological order are 4,170.27, −15,634.66, 12,728.13, 14,051.11,

Table 3.19 Real valued optimal solution of the PLPCF model

variables representing the interfaces							
Q_1^{C}	30	Q_1^{I}	175.37	Q_1^{R}	35,741.47	Q_4^{R}	0
Q_2^{C}	139.86	Q_2^{I}	168.16	Q_2^{R}	58,958.09	Q_1^{D}	1.32
Q_3^{C}	25	Q_3^{I}	163.21	Q_3^{R}	0	Q_2^{D}	6,000

variables																					
	X_{ci}^{I}			X_{cir}^{R}												X_{cid}^{D}					
				$r=1$			$r=2$			$r=3$			$r=4$			$d=1$			$d=2$		
	c			c			c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13.5	62.94	11.25	16.5	76.92	13.75	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
B	13.5	62.94	11.25	16.5	76.92	13.75	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
C	.	.	.	29.96	139.86	24.96	.	.	.	.	.	.	.	.	.	0.05	.	0.04	.	.	.
D	.	.	.	29.96	139.86	24.96	.	.	.	.	.	.	.	.	.	0.05	.	0.04	.	.	.
E	29.7	138.46	.	.	.	.	0.3	1.40	25	.	.	.	.	.	.	.	.	.	.	.	.
F	.	.	.	30	135.83	0.11	.	4.03	24.89	.	.	.	.	.	.	.	.	.	.	.	.
G	.	138.46	24.75	0.71	.	.	29.29	1.40	0.25	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	139.86	25	.	.	.	.	.	.	.	.	.	30	.	.

A dot denotes a value of zero. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

14,060.44, 14,061.77, and lastly the optimal objective of $P = 14{,}061.82$ €. According to the MILP solving (see Fig. 3.24), the objective function of the MIQLP is set to the one of the section the real valued optimum is in. The relevant sections are:

$$0 \le Q_1^{\mathrm{C}} \le 50 \qquad 80 < Q_2^{\mathrm{C}} \qquad 0 \le Q_3^{\mathrm{C}} \le 50 \tag{3.129}$$

$$100 < Q_3^{\mathrm{I}} \le 200 \tag{3.130}$$

$$0 \le Q_1^{\mathrm{R}} \le 50000 \qquad 50000 < Q_2^{\mathrm{R}} \qquad 0 \le Q_3^{\mathrm{R}} \le 50000 \tag{3.131}$$

$$0 \le Q_1^{\mathrm{D}} \le 50000 \qquad 0 \le Q_2^{\mathrm{D}} \le 10000\,. \tag{3.132}$$

The objective function for this section combination is derived from Eqs. (3.125) and (3.126).

$$\begin{aligned} R = {} & \left(31 - 0.004\,Q_1^{\mathrm{I}}\right) Q_1^{\mathrm{I}} + 300\,Q_2^{\mathrm{I}} + \left(2506 - 0.55\,Q_3^{\mathrm{I}}\right) Q_3^{\mathrm{I}} \\ & + 1.43\,Q_1^{\mathrm{R}} + \left(1.045 - 0.0000009\,Q_2^{\mathrm{R}}\right) Q_2^{\mathrm{R}} + 0.75\,Q_3^{\mathrm{R}} + 0.45\,Q_4^{\mathrm{R}} \end{aligned} \tag{3.133}$$

Table 3.20 Integral optimal solution of the PLPCF model

variables representing the interfaces							
Q_1^{C}	30	Q_1^{I}	178	Q_1^{R}	36,360	Q_4^{R}	0
Q_2^{C}	145	Q_2^{I}	172	Q_2^{R}	60,970	Q_1^{D}	32
Q_3^{C}	25	Q_3^{I}	167	Q_3^{R}	460	Q_2^{D}	6,000

integral variables																					
	X_{ci}^{I}			X_{cir}^{R}												X_{cid}^{D}					
				$r=1$			$r=2$			$r=3$			$r=4$			$d=1$			$d=2$		
	c			c			c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13	65	11	1	74	14	.	.	.	16	6	.	.	.	.	.	.	.	.	.	.
B	13	65	11	1	80	14	.	.	.	16	.	.	.	.	.	.	.	.	.	.	.
C	.	.	.	29	145	24	.	.	.	.	.	.	.	.	.	1	.	1	.	.	.
D	.	.	.	29	139	24	.	.	.	.	6	.	.	.	.	1	.	1	.	.	.
E	29	143	.	.	.	.	1	2	25	.	.	.	.	.	.	.	.	.	.	.	.
F	.	.	.	20	127	23	10	18	2	.	.	.	.	.	.	.	.	.	.	.	.
G	.	143	24	.	.	1	30	2	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	145	25	.	.	.	.	.	.	.	.	.	30	.	.

A dot denotes a value of zero.

$$C = \left(2520 + 2.2\,Q_1^{\mathrm{C}}\right) Q_1^{\mathrm{C}} + \left(2359 + 2.9\,Q_2^{\mathrm{C}}\right) Q_2^{\mathrm{C}} + \left(3090 + 2.8\,Q_3^{\mathrm{C}}\right) Q_3^{\mathrm{C}} + 0.2\,Q_1^{\mathrm{D}} + \left(0.4 + 0.0000004\,Q_2^{\mathrm{D}}\right) Q_2^{\mathrm{D}} \tag{3.134}$$

Solving the MIQLP—without forcing the variables to stay within the corresponding sections—results in a feasible solution listed in Table 3.20. This solution is located within the same section combination as the used objective function and thus the solution is the optimal integral one. The objective is $P = 10{,}833.86$ €, which is a significant decrease of 23 % compared to the real valued optimum. In addition, we see that the integral solution cannot be gained by just rounding the real valued optimum.

3.3.5 Summary

In this section we extended the model by the application of piecewise linear quantity-price functions to have a little bit more flexibility in the usage compared to just linear dependencies. We showed that the resulting partially defined quadratic objective function is concave, which is a key property for finding the maximum. In addition, we not only developed a theoretical way of solving the problem (i.e., modified gradient projection method by ROSEN, see appendix B.5), but also an algorithm to solve the problem to optimum with standard solvers. This is a major benefit, because any MIQLP solver can be used—whatever is licensed—and we do not have to implement some individual solving software—facing the problems of numerical solving and over-determined problems—with rather poor performance. Furthermore, if only a real valued optimum is desired, just a linear solver is needed, e.g., the one in the lpSolveAPI package of the R-project.[39]

3.4 Rolling horizon disassembly planning

3.4.1 Motivation

So far the focus was on a static consideration of disassembly planning. Depending on the level of aggregation and the actual situation this is the adequate approach to find the best solution for a planning problem. But sometimes it is necessary to include a further dimension: the time. Doing so, appearing temporary differences in the supply, demand, capacity, etc. can be included in the planning of the optimal disassembly process. When an increasing demand for certain items is expected or known for a future period, it might be necessary to change the ordering and/or the disassembly process in order to make it possible to accommodate the demand or to do it in an optimal way. The temporary change, e.g., of the demand, has therefore influence on preceding and succeeding periods.

Basic planning of the multi-period disassembly can be found in the literature.[40] But these basic approaches need to be extended in order to be usable for the operational and/or tactical planning of the companies. Besides the quantities of cores and items, the profit as objective, contracting aspects, quantity limits, material recycling, disposal, etc. are facets to incorporate. In addition, the models are deterministic, which is a benefit for the acceptance by companies. But the multi-period disassembly planning in the literature requires relevant data for all periods to do the planning. We find that an approach—still deterministic—that incorporates uncertain informa-

[39] LP_SOLVE / KONIS (2011): *lpSolveAPI* together with R DEVELOPMENT CORE TEAM (2011): *R-project*.

[40] Cf. GUPTA / TALEB (1994): *Scheduling disassembly*, TALEB / GUPTA (1997): *Disassembly of multiple products*, and TALEB / GUPTA / BRENNAN (1997): *Disassembly with parts and materials commonality*.

tion with the advantage of increasing the time horizon is more beneficial for the planning. This is presented in the sequel.

The scenario we concentrate on is as follows. The company in focus needs to fix the disassembly plan for the upcoming period. Of course, the past periods are already planned and the results of the planning as well as the realisation are known. The actual period is also already planned, but not finally realised yet. This is a first uncertainty,[41] whether the disassembly is realised as planned or not. In the sequel we will assume the realisation follows the plan, i.e., no backlogging and no exceeding the target occurs. For the upcoming period (the one to be planned) all relevant data is known, which makes a planning possible. For all further periods in the future not all information exists yet and the further in the future the less information exists. This also includes that the further in the future the less accurate information we can get. Still, the company has some means of forecasting including educated guessing. And this information cannot be ignored for planning the upcoming period.[42]

Problems like this bring the dynamic economic lot size model by Wagner / Whitin and its numerous extensions to mind.[43] But a different and more straightforward approach that represents the characteristics of the on-going business and its planning situations is the rolling horizon planning. Wenning indicates the rolling horizon planning as a very responsive method for dynamic problems.[44] As explained above, the upcoming period is planned with some forecast information and as the time goes on the next period is to be planned. This proceeds infinitely because we assume that the business is also aimed to continue forever.[45] But at the moment of planning we do not have the information for all (infinitely many) future periods. Thus, a finite horizon planning is to be applied. And in this case the rolling horizon planning that is approximately optimal for a finite horizon outperforms dynamic lot size planning.[46]

[41] The kind of uncertainty we consider can be categorised as environmental uncertainty (in comparison to system uncertainty), because the uncertainty is beyond the disassembly process. Cf. Mula et al. (2006): *Models for production planning under uncertainty: A review*, p. 271.

[42] Cf. Dawande et al. (2007): *Forecast horizons*, p. 688.

[43] Cf. Wagner / Whitin (1958): *Economic lot size model*.

[44] Cf. Wenning (2010): *Context-based routing*, p. 17.

[45] Of course, one could argument, that on average a business in its current form only continues some years, e.g., 100, which is definitely not infinitely. But for our considerations an infinitely on-going business is assumed.

[46] Cf. Wagner (2004): *Comments on dynamic lot size model*, p. 1776.

When taking a look at the planning problem, a key element of it is the horizon up to which period information is considered. In the literature several horizon types are differentiated. Firstly, the planning problems are categorised into finite and infinite horizon problems.[47] Our planning problem can be seen as infinite horizon problem (see above). On the other hand, the planning for the upcoming period is a finite horizon planning and the end of the periods up to which information is incorporated in the planning of the upcoming period is called study horizon. Thereby, the study horizon (i.e., the number of periods ahead) can be constant or can change from period to period. Either way, the length of the horizon needs to be determined.

In general, the longer the horizon the better for finding the optimal plan. But the horizon does not have to be infinitely long. The so-called forecast horizon denotes the limit of periods to consider. It is defined as minimum number of periods T (used in the planning) where an inclusion of the next period $T+1$ does not change the optimal solution of the planning with only T periods included. For the determination of this horizon several things are to consider. The main trade-off is the cost of forecasting and the uncertainty within the forecast. For example, Sethi / Sorger as well as Dawande et al. discuss finding the optimal forecast horizon length for different planning problems.[48] A literature overview over diverse planning horizons can be found in the work by Chand / Ning Hsu / Sethi.[49] In addition, another trade-off to consider is the solving. In general, it is assumed that the longer the horizon the more complex the solution finding, because more variables need to be determined.

Nevertheless, with time varying data only suboptimal solutions can be generated. To give an example, we assume a planning of the upcoming period with a study horizon of four periods. (The first period of the study horizon is the upcoming period.) The data for the four periods might be 10, 11, 12, and 13. Considering this data the optimal solution of the upcoming period regarding the study horizon is determined. Now, the "rolling" of the horizon takes place, i.e., the next period to be planned is the one after the upcoming period. Because of uncertainty the data changes (the 10 stays fixed) and further data for another period needs to be included. The data relevant for the succeeding planning may be 9, 14, 12, and 15. This means the values 11, 12, and 13 changed to 9, 14, and 12, respectively. Under this

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[47] Cf. here and in the sequel Chand / Ning Hsu / Sethi (2002): *Forecast, solution, and rolling horizons*, pp. 26–35.

[48] Cf. Sethi / Sorger (1991): *Rolling horizon decision making* and Dawande et al. (2007): *Forecast horizons*.

[49] Cf. Chand / Ning Hsu / Sethi (2002): *Forecast, solution, and rolling horizons*.

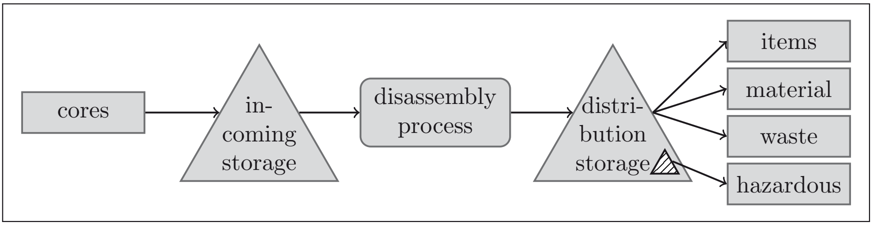

Fig. 3.26 Research focus

condition the solution of the first planning might prevent a better solution of the succeeding planning compared to the case of the planning of the first period with the later known data 10, 9, 14, and 12. Therefore, the planning is comparable to heuristics. Hence, it is assumed to be sufficient to include as many periods in the study horizon as the decision maker thinks necessary for a (very) good solution with respect to the forecasting effort, uncertainty, and solving complexity.[50]

The rolling planning is a deterministic planning that still incorporates uncertainty. Compared to stochastic planning the deterministic planning with linear programming is more accepted by practitioners.[51] Thus, we focus on the deterministic planning. Even though the data is assumed to be deterministic, it changes and might not exist for certain demand or supply values so that a forecast is necessary. This leaves us with the question of how to forecast, achieve some steadiness, and assure feasibility of the future planning from any given state. To apply the dynamic planning[52] we must assure that—independent of the already planned periods—there always exists a feasible solution for the study horizon. But before we discuss the details let us take a look at the planning problem.

3.4.2 Problem description

The general structure of the planning problem is depicted in Fig. 3.26. *Cores* (recovered products) are acquired and stored in the incoming storage. When

[50] Nonetheless, the decision maker can still determine the optimal study horizon, whose length might differ from period to period, but practitioners prefer a constant study horizon. Cf. Chand / Ning Hsu / Sethi (2002): *Forecast, solution, and rolling horizons*, p. 36.

[51] Cf. Dawande et al. (2007): *Forecast horizons*, p. 697.

[52] Note that dynamic planning and dynamic programming are no synonyms.

the disassembly process is scheduled, the cores are taken out of the storage and are disassembled completely. After disassembling the gained items are stored in the distribution storage. A smaller part of the storage can be used for hazardous waste (i.e., hazardous items). This is symbolised by the hatched triangle in the figure. Even though hazardous waste can only be stored in this part all other items and material (i.e., non-hazardous) can also be stored in the hazardous section.

The disassembling always results in items. These can be items in its literal meaning, i.e., the smallest single piece a product consists of, or abstract items for the planning, i.e., modules. If an item—one where the value added is of interest—is distributed or planned for distribution (i.e., stocked), we denote it as *item*. These might be demanded for reuse, refurbish, etc. and are handled piecewise. When the value added is not of interest, only the pure material value is relevant. Thus, these items can be sold for their material value and are distributed to material recycling. These items we call *material*. Items which are not distributable are disposed of. Here we distinguish between (normal or regular) *waste* and the disposal of *hazardous waste*. The latter three categories are handled in weight units.

In the sequel we again consider multiple cores and a multi-period planning. The integration of multiple periods leads to a dynamic planning which requires the incorporation of inventory. The properties from procuring the cores until the distribution of items and material as well as the disposal are explained in the following.

Procuring Cores

The company acquires cores. This includes arranging contracts with suppliers in advance. The contracting minders the uncertainties usually connected with core acquisition.[53] The acquisition generates cost (e.g., transport, labour, and price for cores). When the negotiated number of cores is not recovered by the company a contractual penalty must be paid.[54] In addition, the company guarantees a certain percentage (guarantee level) of cores to be recovered, e.g., 90 % of the contracted quantities. In addition, the guarantee level realises a given minimal quantity of cores to be disassembled by legal enforcement. A guarantee level of 90 % means that contractual penalty must be paid for up to 10 % of the negotiated cores. The price for the cores and other cost might vary from period to period. We assume that

[53] Cf. Mukherjee / Mondal (2009): *Issues relating to remanufacturing*, p. 643.

[54] We do not consider backlogging, but this could also be integrated in our approach.

the guarantee level does not change over time. Nevertheless, the level is specific for every type of core. Thereby, the level of core 1 could be 90 % and that of core 2 85 %.

In the determined period the cores are transported to the facility and stored in the limited storage for incoming cores. This might be an outdoor storage. The storing of the cores induces inventory holding cost. Thereby, a fixed unit cost for each core is assumed.

Disassembly Process

We further assume, the disassembling of the cores is done mostly manually and not interrupted once it is started.[55] We further neglect learning curve effects or the like. Thus, the disassembling unit cost per core is fix. The tools used to disassemble the different cores are similar if not identical so that there is no difference whether two cores of the same type or a different type are disassembled after another, i.e., the disassembly unit cost is not sequence dependent. Additionally, there are no particular set-up actions, which result in extra cost and labour. Thus, a specific inclusion of set-up cost is not necessary. Furthermore, we assume that the disassembly lead time does not change over time (no learning curve effect) for a given core. Therefore, the cost for disassembly can be directly connected with the core, when it is taken out of the storage and does not vary over time. An explicit consideration of the disassembly lead time and a workload limit are not applied.

On the contrary, the core conditions and the possible damaging during the disassembly process are accounted for. We assume that the condition and the damaging are significant for the disassembly process. This means, that no differentiation takes place in the inventory. This again implies, that the condition determines the number of items of a core to be used for the corresponding distribution or disposal. And this determination is relevant in the period the cores are disassembled. Hence, with respect to the core condition the modelling is similar to the basic model.

The cores are completely disassembled. The items are stored on pallets or trays, the material and the disposal in lattice boxes, and the hazardous waste on special trays. All but the hazardous waste goes into the limited distribution storage without a fix slot assignment. As mentioned above, the hazardous waste can only be stored in a special section of the storage. Again, all things in the storage induce inventory holding cost of a fixed rate.

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[55] Cf. Desai / Mital (2003): *Evaluation of disassemblability*, p. 267.

Distribution

The distribution of items and material as well as the disposal is arranged in advance. This means, that contracts are settled up to several periods ahead and the quantities the company is confronted with are not random or uninfluenced by the company. (For example, Kim / Xirouchakis assume a random demand.[56]) Again, a violation of the contracted quantities causes contractual penalties. Other than with cores, no minimum distribution limit is guaranteed. This implies, that even though a contract over 100 items exists, no item has to be distributed. (Of course, the customer is compensated in form of the contractual penalty.) We further assume that the contracted quantities cannot be exceeded, i.e., the company cannot sell more than it has contracted. The revenues for items are based on units and the one for material as well as disposal and hazardous waste are based on the weight. Here, no quantity dependencies exist and all customers are equal in terms of the price. The contractual penalty is a given fraction of the price. The transport of hazardous waste is contracted in advance, too. An example could be that only every second or fourth period a transport is possible. Other than that, the amount of hazardous waste is only limited by the corresponding storage capacity. The only entity not directly contracted is the disposal. Every period disposal can take place to an unlimited amount.[57] Even though the disposal is unlimited, the company is not unaware of the quantities of disposal. The planning gives an estimation of how much shall be disposed of in the next period, so that the quantities are moderately known in advance and thus predictable.

The contracts with material recyclers do not only contain the amount of material. They also include the purity requirements for each material type. For example, a company could require a purity of 95 % in the steel box. This means that up to 5 % impure material can be placed in that box. To fulfil this constraint the material composition of all items relating to the corresponding material type needs to be collected. For example, an item consists of 95 g steel and 5 g plastics. Then the beneficial fraction of the item for the material type steel is 95 % and that for plastics is 5 %. Obviously, this item adheres to the constraint. The same would apply with two items—one with 95 g of pure steel and one of 5 g of pure plastics. Both items together make 100 g—95 % steel and 5 % plastics. For a single period planning this was discussed in Sect. 3.1.2. With the extension of the planning to several periods the question arises whether the additional dimension time

[56] Cf. Kim / Xirouchakis (2010): *Capacitated disassembly scheduling*, p. 7179.

[57] This assumption is caused by modelling as is explained in the sequel.

has influence or not. At least six cases can be distinguished. The purity limit holds

- for each distribution,
- on average over the study horizon,
- on average a fix number of periods into the past,
- on average over the complete history,
- on average a fix number of periods into the past plus the study horizon, and
- on average over the complete history plus the study horizon.

In detail these cases are complicated to model, because one has to consider inventory policies like first in first out, last in first out, lowest in first out, or the like. This would mean another assumption for modelling is required and to keep track of the value and timing of the items put into the storage. To simplify the approach we do not take the time of distribution as reference but the time of disassembling and adding to the storage.[58] When storing the items for material recycling in lattice boxes where the content of a box goes to one recycling company, the decision of what goes to the recycling company is made when adding the items to the box and not when distributing it to the recycling company. Hence, the purity limit holds for adding to inventory

- in every period,
- on average over the study horizon,
- on average a fix number of periods into the past,
- on average over the complete history,
- on average a fix number of periods into the past plus the study horizon, and
- on average over the complete history plus the study horizon.

The first case equals the one of the single period planning, because no aggregation over periods appears. This is the strictest limitation. When adding one or more periods where a purity violation of one period is levelled out (by extra pure material) in another period, the degree of freedom for the solution increases. This applies to case two. Assuming a study horizon of five periods and an overall purity requirement of 95 %, it is possible to plan a 75 % purity in the first period and 100 % in the other four periods of the horizon.[59] When the horizon is rolled on to the next period the 75 % purity are realised, but are forgotten for the next planning. Thus, the same

[58] For the sixth case it does not matter if the distribution or storing is used.

[59] We assume for illustration that in each period the same quantity (i.e., weight) of material is disassembled.

planning might take place and in the end only 75 % purity in each period is achieved. This is not compliant with the requirement of the recycling company and therefore the second case cannot be used in the planning.

In case three the decisions made are not forgotten, because a fixed number of periods (e.g., three) of the past are included in the planning of the upcoming period. When remembering the past decisions an underrun of the given purity limit is not possible. On the contrary, the company might lose possibilities to distribute impure material. An example could be the following. Assume that a company distributed 100 % pure material to the recycling company in the last four periods. When planning the current period (no further inclusion of the study horizon) the company can distribute 80–100 % pure material to achieve an average of 95 % over the past three and current period. Considering the purity of all five periods (i.e., including the one period far into the past) an average purity of 96–100 % is reached, depending on the planning of the current period with 80–100 %. This shows that the difference between the purity realisation (e.g., 100 %) and the purity limit (e.g., 95 %) is "forgotten" if the corresponding period is out of the planning focus (i.e., more than three periods ago). This is to the disadvantage of the disassembly company. To avoid this "forgetting" the number of periods into the past could be extended to the complete history, which equals the case four. But, at least three reasons are against letting this happen.

The first reason is that the company knows about this "forgetting" of pure material in the past and does not want to let it happen. Hence, the company tries to stick as close as possible to the purity limit. Secondly, if we assume disposal cost, less revenue if the impure material is put in another box, and enough material to choose from (e.g., when cores only consist of items with 100 % of steel no impurity can occur) it is profitable to put as many material (especially impure material) into the box with the highest price (in order to get a higher revenue or avoid disposal cost). Hence, the impurity requirement is always limiting this kind of behaviour and so a purity level very close or exactly at the purity requirement is expected, in general.

A third reason comes from the recycling company and can be summarised as: what is the recycling company willing to contract. The recycling process might require a particular purity or for economic reasons the recycling process requires a certain purity level. Otherwise, the process cannot be performed or it is too expensive, respectively. Either way the recycling company wants to assure a minimum purity in each period and would not be satisfied with a scenario where five periods ago a high purity appeared and (to balance the purity) in the current period the purity level is below the minimum. In the light of the above, the recycling company might be willing to accept an average over very view (e.g., two or three) periods, having in mind that the company itself tries to stick with the purity level (as discussed in the first two reasons, but not over the complete past).

Theoretically, there is no difference between the cases one, three, and four assuming the existence of appropriate material, disposal cost, etc. In practice, i.e., depending on the data (e.g., the quantity of material is low, the purity distribution is improper for perfect mixing of material to reach the desired purity level, the demand changes in future periods such that the inventory must be changed, etc.) both companies could agree on a small number of periods to average the purity level.

The cases five and six have the study horizon added to the periods in the past, which increases the freedom of adding impure material in the current period. Several constellations with study horizon length (the period to be planned is included in the study horizon, i.e., the minimum length is one) and periods in the past are listed in Table 3.21. Thereby, the listing of purity levels (given in per cent) demonstrates the worst case, i.e., the one with the maximal underrun of the required purity level of 95 %. We start with 100 % purity in the past. These values are realised and printed in black. The value for the period to be planned is also printed in black, because this value is also going to be realised. On the contrary, the grey numbers denote the values of the study horizon that do not have to be realised. We start with planning period one. The planning might result in the given solution (note, always abstracted to the purity level assuming identical weights in every period) of 60 % purity in period one and 100 % in periods two through five. The past three periods (−2, −1, and 0) are included in the planning for determining the purity average, but the values are fix. The average over the eight values is 95 %, which is exactly the required purity minimum. The focus is then shifted to period two. For this period only one solution exists, because of the value 60 in period one. To balance this seven times 100 % are necessary to achieve 95 % on average. This continues further and the solutions of the first six periods are displayed. The row "1–6" summarises the realised values and lists the average purity over infinitely many periods planned in this manner. The pattern for the first scenario is a repeating group of the four values 60, 100, 100, and 100. The average of those is 90 %.

When the study horizon is reduced by one (3 periods in the past and 4 periods study horizon) the minimal purity in period one increases by 5 %, which equals the allowed impurity of 100 − 95 %, in order to fulfill the purity requirement for the planning. A further decrease of the study horizon leads to a further increase of 5 % per shortened period. If the study horizon is only one period the results are these from case two, where the purity requirement is not underrun. When changing the number of periods

Table 3.21 Illustration of maximal underrun of purity requirement

periods in the past: **3**; study horizon: **5**; purity level: **95** %

planning period	purity level: period −2	−1	0	1	2	3	4	5	6	7	8	9	10	average over past & study
1	100	100	100	60	100	100	100	100						95
2		100	100	60	100	100	100	100	100					95
3			100	60	100	100	100	100	100	100				95
4				60	100	100	100	100	100	100	100			95
5					100	100	100	60	100	100	100	100		95
6						100	100	60	100	100	100	100	100	95
1–6	100	100	100	60	100	100	100	60	100					**90**[a]

periods in the past: **3**; study horizon: **4**; purity level: **95** %

planning period	purity level: period −2	−1	0	1	2	3	4	5	6	7	8	9	10	average over past & study
1	100	100	100	65	100	100	100							95
2		100	100	65	100	100	100	100						95
3			100	65	100	100	100	100	100					95
4				65	100	100	100	100	100	100				95
5					100	100	100	65	100	100	100			95
6						100	100	65	100	100	100	100		95
1–6	100	100	100	65	100	100	100	65	100					**91.25**[a]

periods in the past: **3**; study horizon: **3**; purity level: **95** %

planning period	purity level: period −2	−1	0	1	2	3	4	5	6	7	8	9	10	average over past & study
1	100	100	100	70	100	100								95
2		100	100	70	100	100	100							95
3			100	70	100	100	100	100						95
4				70	100	100	100	100	100					95
5					100	100	100	70	100	100				95
6						100	100	70	100	100	100			95
1–6	100	100	100	70	100	100	100	70	100					**92.5**[a]

periods in the past: **3**; study horizon: **2**; purity level: **95** %

planning period	purity level: period −2	−1	0	1	2	3	4	5	6	7	8	9	10	average over past & study
1	100	100	100	75	100									95
2		100	100	75	100	100								95
3			100	75	100	100	100							95
4				75	100	100	100	100						95
5					100	100	100	75	100					95
6						100	100	75	100	100				95
1–6	100	100	100	75	100	100	100	75	100					**93.75**[a]

periods in the past: **2**; study horizon: **3**; purity level: **95** %														
	purity level													
planning	period													average over
period	−2	−1	0	1	2	3	4	5	6	7	8	9	10	past & study
1		100	100	75	100	100								95
2			100	75	100	100	100							95
3				75	100	100	100	100						95
4					100	100	75	100	100					95
5						100	75	100	100	100				95
6							75	100	100	100	100			95
1–6		100	100	75	100	100	75	100	100					**91 2/3**[a]

periods in the past: **1**; study horizon: **3**; purity level: **95** %														
	purity level													
planning	period													average over
period	−2	−1	0	1	2	3	4	5	6	7	8	9	10	past & study
1			100	80	100	100								95
2				80	100	100	100							95
3					100	80	100	100						95
4						80	100	100	100					95
5							100	80	100	100				95
6								80	100	100	100			95
1–6			100	80	100	80	100	80	100					**90**[a]

[a] This value represents the purity level on average over infinitely many periods.

in the past from three to two the repeating group is shortened by one to three instead of four. In addition this leads to an increase of 5 % to meet the purity requirement per planning (see past periods: 3 and 2 with study horizon: 3). From this we can derive a closed from expression that gives us the maximal underrun ϵ of the required purity level for an infinite runtime. This underrun depends on the study horizon length s, the number of periods of the past p, and the purity level ω.

$$\epsilon = \frac{s-1}{p+1}(1-\omega) \tag{3.135}$$

The $p+1$ denotes the group length and $s-1$ denotes the reduction of the study horizon by the period to be planned.[60] To give an example, the length of the study horizon is four, three periods of the past are considered, and the impurity level is 95 %. The resulting maximal underrun is $\epsilon = \frac{4-1}{3+1}(1 - 0.95) = 0.0375$. This means that for endless repeating of this planning a

[60] For a proof of the formula see appendix B.7.

purity level of $95 - 3.75 = 91.25\,\%$ is realised (see Table 3.21). According to this regularity only three possibilities exist to achieve that the purity requirement is always met, i.e., $\epsilon = 0$. The first one is a study horizon of only one period—this equals the one period planning and the cases where only past periods are considered in the planning. The second one is the inclusion of the complete history ($p \to \infty$ with s being smaller than infinity) and the third is a purity level of $100\,\%$. Following this, the only usefull action to achieve no underrun of the purity is to choose a study horizon of length one, because we explicitly want to consider impurity and the recycling company does not bother with an realisation of infinitely many periods. Note that the term study horizon s here only refers to the purity consideration and not the overall planning.

To conclude this section, in the sequel we only consider the planning of the upcoming period incorporating a limited number of past periods with regard to the purity. This includes the special case of the one-period purity planning, where the purity requirement has to be met every period. This equals the cases one and three (see above). The inclusion of past periods leads only to disadvantages for the disassembly company, which can decide to what extent.

3.4.3 Planning considerations

3.4.3.1 General considerations

The following planning is mainly characterised by

- varying data over time,
- the possibility to appoint numbers of cores and items as well as amounts of material in advance (by contracting), and
- a continuing business, i.e., there exists no finite planning horizon.

Thus, we favour a rolling horizon planning approach that includes a moderate sized study horizon. The length of the study horizon should also influence the steadiness of the results. A short horizon might result in volatile solutions and a long horizon in too steady solutions that prohibit the realisation of better solutions by later varying data in the future. Thereby, varying data does not mean that the values for the same matter in two subsequent periods are different, e.g., core availability is 200 in period one and 300 in period two. This is generally assumed in dynamic planning. It rather means that a value changes from one planning to another. To stick with the core availability example, for the first planning the availabilities are 200 and 300 for the two periods and for the next planning the values might change to 220 and 250 for the same two periods. The longer the study horizon the more likely the data is going to change.

For long study horizons the revenues and costs of future periods do not have the same significance as for the upcoming period. Firstly, the data of future periods might still change and secondly the present value for a future value is usually lower. The latter aspect is assumed to be of marginal influence, because of the relative short time frame of operational and tactical planning. Still, the idea of the net present value for all revenues and cost is a favourable approach for the planning presented here.

In rolling horizon planning infeasible solutions might result, because of various reasons. One of them is the use of "hard" constraints, like the demand.[61] When according constraints exist, the planning and realisation of solutions in earlier periods can lead to infeasibility in future periods. Avoiding infeasibility is achieved by softening the constraints, e.g., setting lower distribution limits for items and material when supply limits exist. (The supply limits are the core availability, which equals the contracted number of cores to be acquired, and the guaranteed level.) In addition, all items can be disposed of, i.e., no upper disposal limit is given. Of course, only as many items as contained in acquired cores over time can be disposed of. Hence, an infinite quantity of disposal is a rather theoretical assumption. Still, limits for disposal, e.g., for the hazardous waste, can be added, but attention must be paid to enable an always feasible solution.

These "technically" motivated constraint relaxations do not influence the relevance of the model or solution. Quite the contrary is the case depending on the point of view. One could argue that the degree of freedom of economic decisions regarding the company can be as extensive as giving up the business. Hence, the entrepreneur has to consider the trade-off between giving up the business and disassembling cores. Whatever is favourable for the entrepreneur should be his or her choice. Of course, the difficulty to measure what is favourable is neglected here. Thus, as long as the decision maker is able to associate a value to the relevant options the removing or relaxing of quantity limits should not decrease but increase the freedom of the decisions.

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[61] Cf. Al-Ameri / Shah / Papageorgiou (2008): *Optimising a rolling horizon framework*, p. 1034.

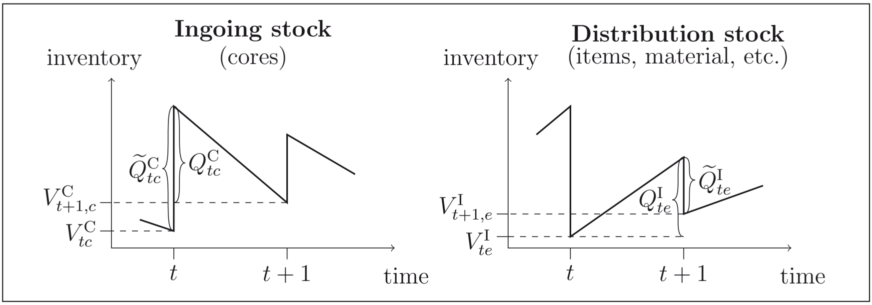

Fig. 3.27 Inventory balance

3.4.3.2 Inventory and steady planning aspects

The inventory is a key component in dynamic planning. Other than just taking the final inventory of a period to calculate the inventory holding cost,[62] we include the inventory development more precisely. Therefore, we approximate the increase and decrease of the inventory during a period by steps and linear functions. An example is given in Fig. 3.27. The inventory of cores is replenished at the beginning of a period. The initial inventory of core c and period t is denoted by V^{C}_{tc} and the replenishing quantity by $\widetilde{Q}^{\mathrm{C}}_{tc}$. The disassembly process linearly decreases the inventory during the period by the quantity Q^{C}_{tc}. Hence, at the end of the period the inventory equals $V^{\mathrm{C}}_{tc} + \widetilde{Q}^{\mathrm{C}}_{tc} - Q^{\mathrm{C}}_{tc}$, which is identical with the initial inventory of the next period $V^{\mathrm{C}}_{t+1,c}$. At the beginning of the next period the procedure repeats. The basis for the calculation of the inventory holding cost for this period is the area under the inventory curve, which equals the average inventory of the period. The average inventory is calculated by

$$V^{\mathrm{C}}_{t+1,c} + \frac{1}{2}\, Q^{\mathrm{C}}_{tc} = V^{\mathrm{C}}_{tc} + \widetilde{Q}^{\mathrm{C}}_{tc} - \frac{1}{2}\, Q^{\mathrm{C}}_{tc}\,. \qquad (3.136)$$

The same applies to the distribution stock with the difference that the inventory is increased linearly and decreased in one step. The initial inventory V^{I}_{te} for a demanded item e is increased by the number of units coming out of the disassembly process Q^{I}_{te} and decreased by the number of units leaving the storage $\widetilde{Q}^{\mathrm{I}}_{te}$. The average inventory equals

[62] Cf. Kim et al. (2009): *A branch and bound algorithm for disassembly scheduling.*

$$V^{\mathrm{I}}_{te} + \frac{1}{2}\,Q^{\mathrm{I}}_{te} = V^{\mathrm{I}}_{t+1,e} + \widetilde{Q}^{\mathrm{I}}_{te} - \frac{1}{2}\,Q^{\mathrm{I}}_{te}\,. \tag{3.137}$$

The inventory development and average calculation for material, waste, and hazardous waste are analogue to the one of items.[63]

The initial inventory for a period is given and according to inflow and outflow updated for the next period. At the end of the study horizon the inventory does not have to be empty, because of an on-going business. Depending on the unit costs the solution of the model could tend to an increased inventory towards the end of the study horizon to prevent, e.g., high disposal cost. This tendency might lead to an always too high inventory and the planning of the next period needs to adjust this. The necessary adjustments might be drastically. In order to avoid this behaviour, we assume that the resulting inventory is stored infinitely. Therefore, we discount the inventory holding like an infinite annuity with the above mentioned net present value method.[64]

The infinite annuity is also applied to revenues and acquisition, disassembly, and disposal cost. This is necessary to have a balance to the infinite inventory holding cost. If only the revenues and cost of the last period of the study horizon are taken as basis for the infinite annuity, the maximisation of the profit will lead to a solution where the revenues are high and cost are low in the last period of the study horizon. To avoid this, we take the average of revenues and cost of the study horizon as basis for the infinite annuity. These average revenues and cost symbolise the expected values for the future periods of the continuing business. Another approach—not considered here—could be the usage of the stock turnover as adequate timeframe instead of the infinite timeframe.

In the rolling planning attention needs to be paid to assure that a solution of an earlier period might cause an infeasible solution of a future period. This is necessary because the study horizon is not infinite, i.e., not all future periods will be integrated in the planning of the current period—either because of computation time for the solving or because not all information is available yet. To avoid such infeasible situations, there always has to exist a feasible solution given an arbitrary state of earlier periods. This follows the practice, because a company can always find a feasible solution from its current state, even if it is by selling the company (in the extreme case). For the scenario considered here, it is sufficient to consider the disposal option at any time and that the demand of items and material does not have to be fulfilled. Of course, disposal and not fulfilling the demand lead to

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[63] Cf. Ullerich / Buscher (2010): *Multi-period product recovery.*

[64] Cf. Eppen / Martin / Schrage (1989): *A scenario approach to capacity planning.*

additional cost. If there still exists a constraint with the possibility of causing an infeasible solution it must be guaranteed by the parameters that such a situation does not occur. To give an example, we assume a storage capacity of 100 units. The transport for emptying the storage takes place only every fourth period. This means that on average 25 units can be processed per period, i.e., put into the storage. If on average 30 cores are contracted in each period and put into the storage no feasible solution can be gained.

3.4.3.3 Optimising the current period

The period to be optimised is denoted by τ. For this period the profit P_τ is to be maximised. The profit is calculated by subtracting the acquisition, disassembly, and disposal cost C_τ, the inventory holding cost C_τ^{V}, and the shortage cost C_τ^{S} from the revenues R_τ.

$$\text{Maximise } P_\tau = R_\tau - C_\tau - C_\tau^{\mathrm{V}} - C_\tau^{\mathrm{S}} \tag{3.138}$$

The revenues can be gained by selling items or material. The number of items sold is denoted by $\widetilde{Q}_{te}^{\mathrm{I}}$ and the price by r_{te}^{I}. The amount of material distributed is denoted by $\widetilde{Q}_{tr}^{\mathrm{R}}$ and the price for a weight unit by r_{tr}^{R}. The revenue of each period is discounted by z $(0 < z < 1)$. Note that z is not tied to an interest rate but rather a value used to achieve the same effect. The study horizon length, the demand position, and the recycling box index are denoted by $\bar{\tau}$, e, and r, respectively. Hence, the revenues are

$$\begin{aligned}
R_\tau = &\sum_{t=\tau}^{\tau+\bar{\tau}-1} \left(\sum_e r_{te}^{\mathrm{I}} \widetilde{Q}_{te}^{\mathrm{I}} + \sum_r r_{tr}^{\mathrm{R}} \widetilde{Q}_{tr}^{\mathrm{R}} \right) z^{t-\tau} \\
&+ \frac{z^{\bar{\tau}}}{\bar{\tau}(1-z)} \sum_{t=\tau}^{\tau+\bar{\tau}-1} \left(\sum_e r_{te}^{\mathrm{I}} \widetilde{Q}_{te}^{\mathrm{I}} + \sum_r r_{tr}^{\mathrm{R}} \widetilde{Q}_{tr}^{\mathrm{R}} \right) \\
= &\sum_{t=\tau}^{\tau+\bar{\tau}-1} \left(\sum_e r_{te}^{\mathrm{I}} \widetilde{Q}_{te}^{\mathrm{I}} + \sum_r r_{tr}^{\mathrm{R}} \widetilde{Q}_{tr}^{\mathrm{R}} \right) \left(z^{t-\tau} + \frac{z^{\bar{\tau}}}{\bar{\tau}(1-z)} \right).
\end{aligned} \tag{3.139}$$

Note that the values R_t are not the revenues of the corresponding periods alone. The revenues of the complete study horizon and the estimation for the infinite horizon are included in the values. The term $\frac{1}{1-z}$ denotes the factor to determine net present value of the infinite annuity of the average revenues. And the average of the revenues is calculated by the division of the sum of revenues over the study horizon divided by the study horizon length $\bar{\tau}$. Furthermore, the infinite annuity starts in the period after the study horizon to avoid double inclusion of the periods of the study horizon. Therefore, the net present value of the infinite annuity is discounted by $z^{\bar{\tau}}$. Hence, put together we get the factor $\frac{z^{\bar{\tau}}}{\bar{\tau}(1-z)}$.

The core acquisition cost depend on the number of cores acquired $\widetilde{Q}^{\mathrm{C}}_{tc}$ and the corresponding unit cost c^{A}_{tc}. The disassembly unit cost c^{J}_{c} multiplied with the cores taken out of the ingoing inventory for processing Q^{C}_{tc} gives the disassembly cost. The cost for disposal form the last term. The amounts of regular waste to dispose of are expressed by $\widetilde{Q}^{\mathrm{D}}_{t1}$ and the ones for hazardous waste by $\widetilde{Q}^{\mathrm{D}}_{t2}$, i.e., they are stored in bin $d = 1$ and $d = 2$, respectively. The corresponding unit cost is c^{D}_{td}. Again, the costs are discounted each period and for a long-term estimation we use the average of the values of the periods to optimise.

$$C_\tau = \sum_{t=\tau}^{\tau+\bar{\tau}-1} \left(\sum_c \left(c^{\mathrm{A}}_{tc} \widetilde{Q}^{\mathrm{C}}_{tc} + c^{\mathrm{J}}_{c} Q^{\mathrm{C}}_{tc} \right) + \sum_d c^{\mathrm{D}}_{td} \widetilde{Q}^{\mathrm{D}}_{td} \right) \left(z^{t-\tau} + \frac{z^{\bar{\tau}}}{\bar{\tau}(1-z)} \right) \tag{3.140}$$

For calculating the inventory holding cost we assume the simplified progression as depicted in Fig. 3.27. The incoming inventory is characterised by an immediate increase of the stock and a linear decrease, whereas the outgoing inventory is characterised by a linear increase and an immediate decrease. The initial inventory of cores for the current period is given by V^{C}_{tc}. The immediate increase results of the incoming number of cores $\widetilde{Q}^{\mathrm{C}}_{tc}$. The linear decrease during the period equals the number of cores taken out of the inventory. The area under the inventory balance curve equals $V^{\mathrm{C}}_{tc} + \widetilde{Q}^{\mathrm{C}}_{tc} - 1/2\, Q^{\mathrm{C}}_{tc}$. The area multiplied with the inventory holding unit cost h^{C}_{c} gives the inventory holding cost for the incoming storage. The outgoing inventory cost is calculated analogously. The initial inventory for items V^{I}_{te}, material V^{R}_{tr}, and waste V^{D}_{td} is linearly increased by the quantities of items Q^{I}_{te}, material Q^{R}_{tr}, and waste Q^{D}_{td} gained from the disassembly process. The average inventory is multiplied with the inventory holding unit cost h^{I}_{e}, h^{R}_{r}, and h^{D}_{d}. The resulting inventory at the end of the study horizon is multiplied with the factor for the infinite annuity.

$$\begin{aligned} C^{\mathrm{V}}_\tau = \sum_{t=\tau}^{\tau+\bar{\tau}-1} \Bigg(& \sum_c h^{\mathrm{C}}_c \left(V^{\mathrm{C}}_{tc} + \widetilde{Q}^{\mathrm{C}}_{tc} - \frac{1}{2} Q^{\mathrm{C}}_{tc} \right) + \sum_e h^{\mathrm{I}}_e \left(V^{\mathrm{I}}_{te} + \frac{1}{2} Q^{\mathrm{I}}_{te} \right) \\ & + \sum_r h^{\mathrm{R}}_r \left(V^{\mathrm{R}}_{tr} + \frac{1}{2} Q^{\mathrm{R}}_{tr} \right) + \sum_d h^{\mathrm{D}}_d \left(V^{\mathrm{D}}_{td} + \frac{1}{2} Q^{\mathrm{D}}_{td} \right) \Bigg) z^{t-\tau} \end{aligned}$$

$$+\left(\sum_c h_c^{\mathrm{C}} V_{\tau+\bar{\tau},c}^{\mathrm{C}} + \sum_e h_e^{\mathrm{I}} V_{\tau+\bar{\tau},e}^{\mathrm{I}} + \sum_r h_r^{\mathrm{R}} V_{\tau+\bar{\tau},r}^{\mathrm{R}} + \sum_d h_d^{\mathrm{D}} V_{\tau+\bar{\tau},d}^{\mathrm{D}}\right) \frac{z^{\bar{\tau}}}{1-z} \tag{3.141}$$

The last part of cost can be summarised as the shortage cost. The σ^{C}, σ^{I}, σ^{R}, and σ^{D} denote fix factors of contractual penalties. For simplicity, we assume that a fixed percentage of the acquisition cost or revenues will be paid to the contracting party when a violation of the contracted quantities occurs. The quantities of shortage are the difference between contracted quantities ($\overline{Q}_{tc}^{\mathrm{C}}$, D_{te}^{I}, D_{tr}^{R}, and $\overline{Q}_{td}^{\mathrm{D}}$) and actually acquired or distributed quantities.

$$\begin{aligned} C_\tau^{\mathrm{S}} = \sum_{t=\tau}^{\tau+\bar{\tau}-1} \Bigg[\Bigg(& \sigma^{\mathrm{C}} \sum_c c_{tc}^{\mathrm{A}} \left(\overline{Q}_{tc}^{\mathrm{C}} - \widetilde{Q}_{tc}^{\mathrm{C}}\right) + \sigma^{\mathrm{I}} \sum_e r_{te}^{\mathrm{I}} \left(D_{te}^{\mathrm{I}} - \widetilde{Q}_{te}^{\mathrm{I}}\right) \\ & + \sigma^{\mathrm{R}} \sum_r r_{tr}^{\mathrm{R}} \left(D_{tr}^{\mathrm{R}} - \widetilde{Q}_{tr}^{\mathrm{R}}\right) + \sigma^{\mathrm{D}} \sum_{d \in \left\{d \middle| \overline{Q}_{td}^{\mathrm{D}} < \infty\right\}} c_{td}^{\mathrm{D}} \left(\overline{Q}_{td}^{\mathrm{D}} - \widetilde{Q}_{td}^{\mathrm{D}}\right) \Bigg) \\ & \cdot \left(z^{t-\tau} + \frac{z^{\bar{\tau}}}{\bar{\tau}(1-z)}\right) \Bigg] \end{aligned} \tag{3.142}$$

Note that the shortage for unlimited disposal does not exist. But just in the case that an upper disposal limit exists, the $\overline{Q}_{td}^{\mathrm{D}}$ is included. The contracted number of cores must not be exceeded. The indices for the relevant periods of the study horizon form the set $\widetilde{T} = \{\tau, \dots, \tau + \bar{\tau} - 1\}$.

$$\widetilde{Q}_{tc}^{\mathrm{C}} \leq \overline{Q}_{tc}^{\mathrm{C}} \quad \forall\, t \in \widetilde{T}, c \tag{3.143}$$

Furthermore, the acquired cores cannot underrun the guaranteed level β_c. The result of the calculation of the lower limit $\beta_c \overline{Q}_{tc}^{\mathrm{C}}$ is rounded up to the next integer, because the cores come only in entire units.

$$\widetilde{Q}_{tc}^{\mathrm{C}} \geq \left\lceil \beta_c \overline{Q}_{tc}^{\mathrm{C}} \right\rceil \quad \forall\, t \in \widetilde{T}, c \tag{3.144}$$

Also, the distributed items, the material, and the waste must not exceed the contracted quantities.

$$\widetilde{Q}_{te}^{\mathrm{I}} \leq D_{te}^{\mathrm{I}} \quad \forall\, t \in \widetilde{T}, e \tag{3.145}$$

$$\widetilde{Q}^{\mathrm{R}}_{tr} \leq D^{\mathrm{R}}_{tr} \quad \forall\, t \in \widetilde{T}, r \tag{3.146}$$

$$\widetilde{Q}^{\mathrm{D}}_{td} \leq \overline{Q}^{\mathrm{D}}_{td} \quad \forall\, t \in \widetilde{T}, d \tag{3.147}$$

If an upper limit for, e.g., disposal does not exist, $\overline{Q}^{\mathrm{D}}_{td}$ could be set to infinity. This is the case for the regular disposal to avoid infeasible solutions, i.e., $\overline{Q}^{\mathrm{D}}_{t,1} = \infty \;\forall\, t$.

Each core is disassembled completely and therefore all containing items are either determined for direct distribution X^{I}_{tci}, for material recycling X^{R}_{tcir}, or for disposal X^{D}_{tcid}. $\bar{I}_c$ denotes the number of items a core consists of.

$$Q^{\mathrm{C}}_{tc} = X^{\mathrm{I}}_{tci} + \sum_r X^{\mathrm{R}}_{tcir} + \sum_d X^{\mathrm{D}}_{tcid} \quad \forall\, t \in \widetilde{T}, c, i \in \{1, \ldots, \bar{I}_c\} \tag{3.148}$$

The number of items determined for material recycling, and waste is multiplied with their weight w_{ci} to obtain the amount of material in weight units in the specific boxes and bins.

$$Q^{\mathrm{R}}_{tr} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X^{\mathrm{R}}_{tcir} \quad \forall\, t \in \widetilde{T}, r \tag{3.149}$$

$$Q^{\mathrm{D}}_{td} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X^{\mathrm{D}}_{tcid} \quad \forall\, t \in \widetilde{T}, d \tag{3.150}$$

Depending on the condition of the core the usage for distribution might be limited. All non-genuine items consisting of the wrong material must be disposed of. The probabilities of core condition and damaging during the disassembly process do not vary over time. Hence, the index t is added compared to the basic model.

$$\sum_d X^{\mathrm{D}}_{tcid} \geq \zeta_{ci} \iota_{ci} Q^{\mathrm{C}}_{tc} \quad \forall\, t \in \widetilde{T}, c, i \in \{1, \ldots, \bar{I}_c\} \tag{3.151}$$

Items that are genuine, functioning, and do not get damaged can be reused.

$$X^{\mathrm{I}}_{tci} \leq (1 - \zeta_{ci})(1 - \eta_{ci})(1 - \theta_{ci}) Q^{\mathrm{C}}_{tc} \quad \forall\, t \in \widetilde{T}, (c, i) \in \bigcup_e \mathcal{P}_e \tag{3.152}$$

Each box has at most one contractual party that requires a certain level of purity within the recycling box. Thereby, the beneficial fraction π_{cir} of all material in the box must exceed the required minimum ω_r. Depending on the contract the purity has to be satisfied not in every period but as discussed above over a certain number of periods in the past. This number of periods is denoted by $\mathcal{I}_r$ and can be individually set for every recycling box r. This might be necessary, because the index r is used to differentiate between material as well as recycling companies.

$$\omega_r \sum_{l=t-\mathcal{I}_r}^{t} Q^{\mathrm{R}}_{lr} \leq \sum_{c} \sum_{i=1}^{\bar{I}_c} w_{ci} \pi_{cir} \sum_{l=t-\mathcal{I}_r}^{t} X^{\mathrm{R}}_{lcir} \quad \forall\, t \in \widetilde{T}, r \tag{3.153}$$

This inclusion of a fixed number of past periods must be treated with caution, because infeasibility might be caused by this inclusion in combination with the limited number of cores to be acquired. Let us assume one past period with high purity material and high quantities. The other past periods between the latter one and the upcoming period to be planned next have low purity material. When rolling on the horizon the period with high purity material is excluded from the purity averaging. Instead of this excluded period the new period to be planned is used to average the purity. As long as enough material is available (i.e., high quantities of cores to be disassembled) this balancing is possible. But, if only a low quantity is available the required amount of (pure) material cannot be gained, because of input limitations. Thus, for the numerical example, which is calculated automatically, no past periods are considered, i.e., $\mathcal{I}_r = 0\ \forall\, r$. This simplifies Eq. (3.153) to the straightforward extension from the basic model.

$$\omega_r Q^{\mathrm{R}}_{tr} \leq \sum_{c} \sum_{i=1}^{\bar{I}_c} w_{ci} \pi_{cir} X^{\mathrm{R}}_{tcir} \quad \forall\, t \in \widetilde{T}, r \tag{3.154}$$

For waste no purity consideration is applied. Because of multiplicity and commonality different items within a core and of several cores satisfy the demand of an item. The added number of items gained by disassembling (X^{I}_{tci}) equals the number of items to satisfy the demand for the position e. The set $\mathcal{P}_e$ contains the core item combination (c, i) for satisfying a specific demand position e.

$$Q^{\mathrm{I}}_{te} = \sum_{(c,i) \in \mathcal{P}_e} X^{\mathrm{I}}_{tci} \quad \forall\, t \in \widetilde{T}, e \tag{3.155}$$

All numbers of items without a demand assignment are zero.

$$X^{\mathrm{I}}_{tci} = 0 \quad \forall\, t, (c, i) \notin \bigcup_{e} \mathcal{P}_e \tag{3.156}$$

A special regard needs to be paid to the hazardous waste. Items that are hazardous have to be treated specifically. They can only be stored in a particular place. This place is used for disposal bin 2 and hazardous items with a demand. Therefore, no hazardous items go into material recycling and regular waste. The set $\mathcal{H}$ contains the core item combinations (c, i) of all hazardous items. The boxes and bins where hazardous items must not be placed, form the basis for setting the corresponding numbers of items gained by disassembly to zero.

$$X^{\mathrm{R}}_{tcir} = 0 \quad \forall\, t, (c, i) \in \mathcal{H}, r \tag{3.157}$$

$$X^{\mathrm{D}}_{tcid} = 0 \quad \forall\, t, (c, i) \in \mathcal{H}, d \in \{1\} \tag{3.158}$$

Note that $d = 1$ denotes regular waste and $d = 2$ hazardous waste. The inventory balance for cores, items, material, and waste is given below. The initial inventory of the next period is calculated by the initial inventory of the actual period plus the inflow and minus the outflow. Thereby, the inventory in period τ, which is the first in $\widetilde{T}$, is given.

$$V^{\mathrm{C}}_{t+1,c} = V^{\mathrm{C}}_{tc} + \widetilde{Q}^{\mathrm{C}}_{tc} - Q^{\mathrm{C}}_{tc} \quad \forall\, t \in \widetilde{T}, c \tag{3.159}$$

$$V^{\mathrm{I}}_{t+1,e} = V^{\mathrm{I}}_{te} + Q^{\mathrm{I}}_{te} - \widetilde{Q}^{\mathrm{I}}_{te} \quad \forall\, t \in \widetilde{T}, e \tag{3.160}$$

$$V^{\mathrm{R}}_{t+1,r} = V^{\mathrm{R}}_{tr} + Q^{\mathrm{R}}_{tr} - \widetilde{Q}^{\mathrm{R}}_{tr} \quad \forall\, t \in \widetilde{T}, r \tag{3.161}$$

$$V^{\mathrm{D}}_{t+1,d} = V^{\mathrm{D}}_{td} + Q^{\mathrm{D}}_{td} - \widetilde{Q}^{\mathrm{D}}_{td} \quad \forall\, t \in \widetilde{T}, d \tag{3.162}$$

The limit for the incoming storage is given by $\overline{V}^1$ and needs to be adhered to each period. In the incoming storage only cores are stored. The individual storage usage is given by the factor ν^{C}_c.

$$\sum_c \nu^{\mathrm{C}}_c \left(V^{\mathrm{C}}_{tc} + \widetilde{Q}^{\mathrm{C}}_{tc} \right) \leq \overline{V}^1 \quad \forall\, t \in \widetilde{T} \tag{3.163}$$

The distribution storage is used for items, material, and waste and is limited by $\overline{V}^2$. Moreover, the place for hazardous waste and items is limited by $\overline{V}^3$. The corresponding storage usage factors are applied here, too.

$$\sum_e \nu^{\mathrm{I}}_e \left(V^{\mathrm{I}}_{te} + Q^{\mathrm{I}}_{te} \right) + \sum_r \nu^{\mathrm{R}}_r \left(V^{\mathrm{R}}_{tr} + Q^{\mathrm{R}}_{tr} \right) + \sum_d \nu^{\mathrm{D}}_d \left(V^{\mathrm{D}}_{td} + Q^{\mathrm{D}}_{td} \right) \leq \overline{V}^2 \quad \forall\, t \in \widetilde{T} \tag{3.164}$$

Table 3.22 Number of decision variables and constraints

variables	$\left(\left\lvert\bigcup_e \mathcal{P}_e\right\rvert + \left(\sum_c \bar{I}_c - \lvert\mathcal{H}\rvert + 1\right)(r+d) + \lvert\mathcal{H}\rvert\, c + e\right)\bar{\tau}$
integer variables	$\left\lvert\bigcup_e \mathcal{P}_e\right\rvert + \left(\sum_c \bar{I}_c - \lvert\mathcal{H}\rvert\right)(r+d) + \lvert\mathcal{H}\rvert$
constraints	$\left(2\sum_c \bar{I}_c + c + \left\lvert\bigcup_e \mathcal{P}_e\right\rvert + 2r + e + d + 3\right)\bar{\tau}$

$$\sum_{e\in\{e\mid\mathcal{P}_e\subseteq\mathcal{H}\}} \nu_e^{\mathrm{I}}\left(V_{te}^{\mathrm{I}} + Q_{te}^{\mathrm{I}}\right) + \nu_2^{\mathrm{D}}\left(V_{t,2}^{\mathrm{D}} + Q_{t,2}^{\mathrm{D}}\right) \le \overline{V}^3 \quad \forall\, t \in \widetilde{T} \tag{3.165}$$

Finally, all decision variables are non-negative and for the upcoming period τ the following variables are integer variables.

$$\widetilde{Q}_{\tau c}^{\mathrm{C}}, X_{\tau ci}^{\mathrm{I}}, X_{\tau cir}^{\mathrm{R}}, X_{\tau cid}^{\mathrm{D}}, \widetilde{Q}_{\tau e}^{\mathrm{I}} \in \mathbb{Z}^* \quad \forall\, c, i \in \{1,\ldots,\bar{I}_c\}, r, d, e \tag{3.166}$$

The resulting model dimension (i.e., the number of constraints and variables) is listed in Table 3.22. The basis is again a compact model version which can be found in the appendix B.8. Here, the variables c and e denote the number of cores and demand positions, respectively. The term $|\cdot|$ represents the cardinality of the corresponding set. As can be seen, the number of constraints and variables increases linearly with any increase of cores, recycling boxes, disposal bins, items a core consists of, demand positions, and the study horizon length. The number of integer variables is independent of the study horizon length. In comparison with the basic model (see Table 3.2) the number of integer variables increases by $\widetilde{Q}_{tc}^{\mathrm{C}}$ and $\widetilde{Q}_{te}^{\mathrm{I}}$, which leads to the adding of c and e in the table. The overall number of decision variables increases of course by the length of the study horizon length $\bar{\tau}$ plus the variables with the tilde, i.e., $\widetilde{Q}_{tc}^{\mathrm{C}}$, $\widetilde{Q}_{te}^{\mathrm{I}}$, $\widetilde{Q}_{tr}^{\mathrm{R}}$, and $\widetilde{Q}_{td}^{\mathrm{D}}$ for every study period. The number of constraints increases, too. Of course, the study horizon length has the major impact for this increase. But since no lower limits for items, material, and disposal are incorporated in the planning a decrease per period can be detected. In addition, the workload limitation is not applied, but instead of it three constraints per period assure that the inventory limits are added.

3.4.3.4 Decision support for contracting

Before we continue with a numerical example a further beneficial aspect for the decision maker is considered. So far we had the quantities of cores, items and material given. These quantities are given by contracts with business partners. In an exemplary company the following might be practise.

The quantities for the cores to be processed are arranged by manager A. The planning of the disassembly process is done by manager B. Manager B uses the rolling horizon planning. On the contrary, manager A uses different means of planning or just a rule of thumb. As a general assumption manager B might assume that more is better. Thus, manager A generates the constraints manager B has to deal with and manager B gets stressed to arrange a good plan of the disassembly. If manager A would have a support for the decisions to be made which is connected with the planning done by manager B, the outcome could lead to a higher profit for the company. Such a possible decision support is presented in the sequel.

Once the current period τ is finally planned, we assume that the result is realised. Hence, the focus is switched to the next period $\tau+1$, which means that the study horizon is rolled one period forward. This includes another period $\tau+\bar{\tau}$ in the planning. This period is the one that was just outside the study horizon when planning the current period. For the planning of the next period the corresponding data of the new period is required. This data is determined in advance, i.e., at least $\bar{\tau}$ periods ahead. But before the next period is finally planned an intermediate planning can be conducted, that does not only focus on the periods of the study horizon, but also of one or more periods further in the future.[65] This intermediate planning we call pre-planning. The difference between the periods of the study horizon and the future period $\tau+\bar{\tau}$—we call it contracting period in the sequel—is that in the contracting period the availability of cores and demand of items and material are initially unlimited. The pre-planning still considers the study horizon as well as the contracting period and tries to find a solution to maximise the profit. The result of the planning could be that not infinitely many cores can be acquired, because of limited resources of the company. With this information of how many cores to acquire in the contracting period manager A could try to arrange contracts to realise these numbers. And these contracts should result in a better profit than just arranging contracts over quantities by rule of thumb.

Of course, already known limitations such as existing long-term contracts or expected seasonal effects can be considered in the pre-planning, too. The pre-planning can be repeated as often as new planning relevant information occurs. Since this pre-planning is guidance to the decision maker it is not necessary to insist on integral solutions. Instead, a relaxed planning problem can be used that does not contain integer variables. This speeds up the solving significantly and should clearly motivate to use this opportunity. The relaxation is without problem, because we do not know whether the

[65] We consider only one period ahead.

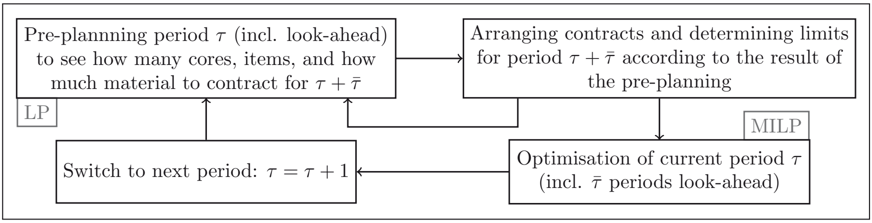

Fig. 3.28 Iterative planning process

resulting quantities can be contracted and, if they are contracted, whether they do not change over time. Note that we assume time varying data in the study horizon.

With this pre-planning it can also be checked whether certain quantities lead to infeasible combinations and what influence the quantities to be contracted might have on the profit. The pre-planning can be conducted iteratively and precedes the planning of the next period. The iterative planning process including the pre-planning is summarised in Fig. 3.28. The current period is planned finally (bottom right in the figure). Then the focus is shifted to the next period (bottom left), because there is nothing else to be done for the realised period. The succeeding pre-planning includes the new study horizon, i.e., based on the old study horizon the old current period is excluded and the contracting period is included. And since the index of the current period τ is shifted to the next period the index of the contracting period is $\tau + \bar{\tau} - 1$ (top left). According to the result of an iteration of the pre-planning the contracts are arranged (top right). If further contracts need to be arranged or other changes occur, the pre-planning is repeated and the changes are adopted. When all aspects are considered or the time runs out, because the next period must be planned finally, the current period is planned finally. Thus, the cycle is closed.

In order to conduct the pre-planning slight changes of the mathematical model have to be made. Nevertheless, most of the equations of the above presented model stay identical. The first change is an increase by one of the period index $\tau = \tau + 1$ (see Fig. 3.28). This represents the rolling of the horizon and does not make any changes in the model necessary. But secondly, in the estimation of the revenues only the average of the periods without the contracting period is included. We believe that with excluding the contracting period for the average calculation the result of the planning is steadier. This makes a change necessary. Thus, Eq. (3.139) is changed to

$$
\begin{aligned}
R_\tau =& \sum_{t=\tau}^{\tau+\bar{\tau}-1} \left(\sum_e r^{\mathrm{I}}_{te} \widetilde{Q}^{\mathrm{I}}_{te} + \sum_r r^{\mathrm{R}}_{tr} \widetilde{Q}^{\mathrm{R}}_{tr} \right) z^{t-\tau} \\
&+ \frac{z^{\bar{\tau}}}{(\bar{\tau}-1)(1-z)} \sum_{t=\tau}^{\tau+\bar{\tau}-2} \left(\sum_e r^{\mathrm{I}}_{te} \widetilde{Q}^{\mathrm{I}}_{te} + \sum_r r^{\mathrm{R}}_{tr} \widetilde{Q}^{\mathrm{R}}_{tr} \right) \\
=& \sum_{t=\tau}^{\tau+\bar{\tau}-2} \left(\sum_e r^{\mathrm{I}}_{te} \widetilde{Q}^{\mathrm{I}}_{te} + \sum_r r^{\mathrm{R}}_{tr} \widetilde{Q}^{\mathrm{R}}_{tr} \right) \left(z^{t-\tau} + \frac{z^{\bar{\tau}}}{(\bar{\tau}-1)(1-z)} \right) \\
&+ \left(\sum_e r^{\mathrm{I}}_{\tau+\bar{\tau}-1,e} \widetilde{Q}^{\mathrm{I}}_{\tau+\bar{\tau}-1,e} + \sum_r r^{\mathrm{R}}_{\tau+\bar{\tau}-1,r} \widetilde{Q}^{\mathrm{R}}_{\tau+\bar{\tau}-1,r} \right) z^{\bar{\tau}-1}. \qquad (3.167)
\end{aligned}
$$

For the same reason the cost calculation in Eq. (3.140) is adapted, too.

$$
\begin{aligned}
C_\tau =& \sum_{t=\tau}^{\tau+\bar{\tau}-2} \left(\sum_c \left(c^{\mathrm{A}}_{tc} \widetilde{Q}^{\mathrm{C}}_{tc} + c^{\mathrm{J}}_{c} Q^{\mathrm{C}}_{tc} \right) + \sum_d c^{\mathrm{D}}_{td} \widetilde{Q}^{\mathrm{D}}_{td} \right) \left(z^{t-\tau} + \frac{z^{\bar{\tau}}}{(\bar{\tau}-1)(1-z)} \right) \\
&+ \left(\sum_c \left(c^{\mathrm{A}}_{\tau+\bar{\tau}-1,c} \widetilde{Q}^{\mathrm{C}}_{\tau+\bar{\tau}-1,c} + c^{\mathrm{J}}_{c} Q^{\mathrm{C}}_{\tau+\bar{\tau}-1,c} \right) \right. \\
&\qquad \left. + \sum_d c^{\mathrm{D}}_{\tau+\bar{\tau}-1,d} \widetilde{Q}^{\mathrm{D}}_{\tau+\bar{\tau}-1,d} \right) z^{\bar{\tau}-1} \qquad (3.168)
\end{aligned}
$$

In the planning of the contractual period no shortages are assumed. Thus, the cost calculation (i.e., Eq. (3.142)) needs to be adapted

$$
\begin{aligned}
C^{\mathrm{S}}_\tau =& \sum_{t=\tau}^{\tau+\bar{\tau}-2} \left[\left(\sigma^{\mathrm{C}} \sum_c c^{\mathrm{A}}_{tc} \left(\overline{Q}^{\mathrm{C}}_{tc} - \widetilde{Q}^{\mathrm{C}}_{tc} \right) + \sigma^{\mathrm{I}} \sum_e r^{\mathrm{I}}_{te} \left(D^{\mathrm{I}}_{te} - \widetilde{Q}^{\mathrm{I}}_{te} \right) \right. \right. \\
&\qquad \left. + \sigma^{\mathrm{R}} \sum_r r^{\mathrm{R}}_{tr} \left(D^{\mathrm{R}}_{tr} - \widetilde{Q}^{\mathrm{R}}_{tr} \right) + \sigma^{\mathrm{D}} \sum_{d \in \left\{ d | \overline{Q}^{\mathrm{D}}_{td} < \infty \right\}} c^{\mathrm{D}}_{td} \left(\overline{Q}^{\mathrm{D}}_{td} - \widetilde{Q}^{\mathrm{D}}_{td} \right) \right) \\
&\qquad \left. \cdot \left(z^{t-\tau} + \frac{z^{\bar{\tau}}}{(\bar{\tau}-1)(1-z)} \right) \right] \qquad (3.169)
\end{aligned}
$$

and the constraints of the contracted quantities (i.e., Eqs. (3.143)–(3.147)), too. These constraints are adapted by substituting the index set from $\widetilde{T}$ to

$\widetilde{T}' = \{\tau, \ldots, \tau + \bar{\tau} - 2\}$ in the equations. If individual limits exist (cores, items, material, and waste), they can be added by constraints like

$$\widetilde{Q}^{\mathrm{C}}_{\tau+\bar{\tau}-1,c} = \overline{Q}^{\mathrm{C}}_{\tau+\bar{\tau}-1,c} \quad \text{for a particular } c, \tag{3.170}$$

where $\overline{Q}^{\mathrm{C}}_{\tau+\bar{\tau}-1,c}$ denotes the fixed value. In the remaining constraints (3.148) through (3.155) and (3.159) through (3.165) the index set $\widetilde{T}$ stays identical, because the item flow during the disassembly process has to be obeyed in the contractual period as well. The integrality constraints (3.166) do not apply for this pre-planning.

3.4.4 Numerical example

3.4.4.1 Data

The planning presented above shall be illustrated by an example of disassembling used forklift trucks. The example is based on the one in Sect. 3.1.3 and the truck is depicted in Fig. 3.4 (on page 46). In total we consider three different trucks (diesel, gas, electricity) consisting of eight items A–H each. The data necessary for this dynamic planning problem is more comprehensive, because of the several periods. We limit the example to 24 periods in order to be able to demonstrate the planning and to have enough iterations to discuss the results.

The time independent data from the former example is the number of items per core $\bar{I}_c$, the weight w_{ci}, and the disassembly cost c^{J}_c (see Table 3.4), the core condition ζ_{ci}, η_{ci}, ι_{ci}, and θ_{ci} (see Table 3.5), the demand positions $\mathcal{P}_e$ (see Table 3.6), the hazardous item $\mathcal{H} = \{(1, \mathrm{H})\}$, as well as the minimum purity requirement ω_r and the beneficial fractions π_{cir} (see Table 3.8). Further time independent data is the length of the study horizon $\bar{\tau} = 5$ and the discounting factor $z = {}^{20}\!/_{21}$, which equals a discounting with a five per cent interest rate. The penalty cost factors σ^{C}, σ^{I}, σ^{R}, and σ^{D} for not meeting the contracting quantities are 1, 0.25, 0.25, and 0.25, respectively. The storage limits $\overline{V}^1$, $\overline{V}^2$, and $\overline{V}^3$ are set to 200, 1,000, and 30, respectively. The remaining time independent data for the inventory, the guarantee level β_c, and the number of past periods for the purity averaging $\underline{\tau}_r$ is summarised in Table 3.23.

The unit costs c^{A}_{tc} and c^{D}_{td} as well as the prices r^{I}_{te} and r^{R}_{tr} for all 24 periods are listed in Table 3.24and the contracted quantities for the first five periods in Table 3.25.[66] The values are chosen randomly. Note that for

Table 3.23 Inventory data, guarantee level, and past periods

cores					items				material					disposal			
c	$V^{\mathrm{C}}_{1,c}$	h^{C}_{c}	ν^{C}_{c}	β_c	e	$V^{\mathrm{I}}_{1,e}$	h^{I}_{e}	ν^{I}_{e}	r	$V^{\mathrm{R}}_{1,r}$	h^{R}_{r}	ν^{R}_{r}	$\mathcal{I}_r$	d	$V^{\mathrm{D}}_{1,d}$	h^{D}_{d}	ν^{D}_{d}
1	10	14.0	1.0	0.9	1	40	2	0.18	1	500	0.004	0.0078	0	1	750	0.002	0.0065
2	7	14.2	1.0	0.9	2	80	10	0.02	2	1700	0.004	0.0077	0	2	660	0.005	0.0070
3	5	14.3	0.9	0.9	3	20	50	5.40	3	53	0.004	0.0020	0				
									4	0	0.004	0.0015	0				

Table 3.24 Unit costs and prices of periods 1–24

	c^{A}_{tc}			r^{I}_{te}			r^{R}_{tr}				c^{D}_{td}	
t	$c=1$	$c=2$	$c=3$	$e=1$	$e=2$	$e=3$	$r=1$	$r=2$	$r=3$	$r=4$	$d=1$	$d=2$
1	2,300	2,600	2,900	30	300	2,400	1.35	0.95	0.75	0.45	0.20	0.40
2	2,345	2,599	2,789	31	315	2,352	1.40	0.93	0.75	0.43	0.21	0.42
3	2,460	2,545	2,881	31	324	2,352	1.37	0.95	0.79	0.44	0.21	0.43
4	2,343	2,530	2,928	31	325	2,357	1.30	0.93	0.79	0.45	0.21	0.42
5	2,316	2,595	2,815	31	329	2,379	1.35	0.96	0.78	0.45	0.21	0.40
6	2,279	2,579	2,689	32	313	2,417	1.41	0.98	0.80	0.45	0.21	0.40
7	2,336	2,473	2,614	33	316	2,410	1.36	1.01	0.80	0.44	0.20	0.41
8	2,236	2,491	2,691	32	329	2,379	1.39	0.94	0.82	0.44	0.19	0.40
9	2,210	2,396	2,732	32	326	2,322	1.32	0.91	0.84	0.43	0.19	0.42
10	2,303	2,333	2,670	31	332	2,356	1.37	0.93	0.88	0.45	0.19	0.43
11	2,212	2,386	2,558	31	342	2,304	1.34	0.97	0.88	0.46	0.20	0.45
12	2,280	2,288	2,582	31	346	2,409	1.33	1.03	0.90	0.45	0.20	0.46
13	2,357	2,349	2,568	30	337	2,398	1.39	1.06	0.86	0.43	0.20	0.46
14	2,305	2,385	2,509	29	343	2,414	1.38	0.99	0.88	0.43	0.20	0.47
15	2,378	2,493	2,510	29	330	2,391	1.32	1.02	0.84	0.42	0.20	0.47
16	2,283	2,476	2,411	30	321	2,481	1.33	0.95	0.88	0.41	0.20	0.48
17	2,295	2,569	2,491	31	327	2,420	1.39	0.98	0.92	0.41	0.20	0.48
18	2,186	2,597	2,448	31	317	2,373	1.39	1.03	0.88	0.40	0.20	0.47
19	2,080	2,568	2,526	33	319	2,372	1.41	0.97	0.88	0.41	0.21	0.46
20	2,071	2,645	2,530	32	311	2,352	1.48	1.01	0.85	0.41	0.21	0.46
21	2,030	2,607	2,616	32	311	2,401	1.50	1.08	0.85	0.40	0.22	0.47
22	2,118	2,725	2,526	32	301	2,371	1.50	1.10	0.90	0.39	0.22	0.48
23	2,091	2,699	2,613	33	288	2,326	1.57	1.01	0.86	0.41	0.23	0.48
24	2,172	2,813	2,600	33	283	2,414	1.62	0.98	0.89	0.42	0.22	0.50

the disposal no specific quantities are contracted and the value infinity is used in the table. This, on the other hand, makes the contractual penalties

[66] Of course, the unit costs and prices are not known this much in advance (see Table 3.24). For the planning only the data of the corresponding study horizon is necessary. Even though we consider time varying data for cost, prices, and quantities, we only vary the quantities to keep the amount of data for the example at a moderate level.

Table 3.25 Contracted quantities of periods 1–5

	$\overline{Q}^{\mathrm{C}}_{tc}$			D^{I}_{te}			D^{R}_{tr}				$\overline{Q}^{\mathrm{D}}_{td}$	
t	$c=1$	$c=2$	$c=3$	$e=1$	$e=2$	$e=3$	$r=1$	$r=2$	$r=3$	$r=4$	$d=1$	$d=2$
1	30	188	25	216	215	210	53,000	61,000	0	0	∞	0
2	34	189	25	191	246	208	50,000	67,000	0	0	∞	0
3	35	185	26	201	261	235	55,000	60,000	0	0	∞	∞
4	33	179	30	212	287	248	48,000	59,000	0	0	∞	0
5	34	158	29	239	269	212	44,000	64,000	0	0	∞	0

σ^{D} irrelevant. However, one restriction does exist. Only every fourth period the hazardous waste is transported to the disposal facility. Hence, the corresponding upper limits $\overline{Q}^{\mathrm{D}}_{t,2}$ are set to zero to avoid a transport in the other three periods. With having the above given data at hand, the planning can begin.

A last word on modelling several suppliers and customers: To model different suppliers with different contracting details for the same core another core needs to be added. This way the index c represents a core from a specific supplier. For example, let us assume another supplier for core $c = 3$. Then a core $c = 4$ is added that has the same data for all material dependent data, but for the guarantee level and/or price different values are used for example. The same applies to the distribution.

3.4.4.2 Rolling planning

Optimising the current period

The planning starts with already contracted quantities for the first five periods as given in Table 3.25. The first period needs to be planned to fix the quantities of cores, items, material, as well as regular and hazardous waste. Thus, we start with the optimisation of the current period (see Fig. 3.28). The period index τ is set to one and the model (see Sect. 3.4.3.3) is solved to optimum. The optimal solution is listed in the Tables 3.26–3.29. The optimal values of X^{I}_{tci}, X^{R}_{tcir}, and X^{D}_{tcid} can be found in the appendix B.9 (see Table B.1).

As can be seen in Tables 3.26 and 3.27, only in the current period the values are strictly integer. This of course leads automatically to integer values for the initial inventories of the second period. As expected the inventory is not empty at the end of the study horizon (e.g., $V^{\mathrm{C}}_{6,1} = 65.02$) such that the

Table 3.26 Optimal solution of first iteration (part one)

	$\widetilde{Q}^{\mathrm{C}}_{tc}$			Q^{C}_{tc}			V^{C}_{tc}		
t	$c=1$	$c=2$	$c=3$	$c=1$	$c=2$	$c=3$	$c=1$	$c=2$	$c=3$
1	27	188	25	24	192	0	10	7	5
2	31	189	25	19.23	191.36	18.74	13	3	30
3	32	185	26	22.91	140.51	62.26	24.77	0.64	36.26
4	30	179	30	0	209.26	30	33.87	45.13	0
5	31	158	29	29.85	172.87	29	63.87	14.87	0
6							65.02	0	0

Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Table 3.27 Optimal solution of first iteration (part two)

	$\widetilde{Q}^{\mathrm{I}}_{te}$			Q^{I}_{te}			V^{I}_{te}		
t	$e=1$	$e=2$	$e=3$	$e=1$	$e=2$	$e=3$	$e=1$	$e=2$	$e=3$
1	212	9	210	172	213	190	40	80	20
2	191	246	208	191	208.48	208	0	284	0
3	201	261	200.74	201	161.79	200.74	0	246.48	0
4	212	287	236.87	212	207.17	236.87	0	147.27	0
5	208.55	268.12	199.85	208.55	200.69	199.85	0	67.43	0
6							0	0	0

Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Table 3.28 Optimal solution of first iteration (part three)

	$\widetilde{Q}^{\mathrm{R}}_{tr}$				Q^{R}_{tr}				V^{R}_{tr}			
	r				r				r			
t	1	2	3	4	1	2	3	4	1	2	3	4
1	46916	53600	0	0	46416	51900	0	0	500	1700	53	0
2	50000	49002.65	0	0	50000	49002.65	0	0	0	0	53	0
3	50948.78	48950.09	0	0	50948.78	48950.09	0	0	0	0	53	0
4	48000	39112.71	0	0	48000	39112.71	0	0	0	0	53	0
5	44000	64000	0	0	44000	64000	0	0	0	0	53	0
6									0	0	53	0

Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Table 3.29 Optimal solution of first iteration (part four)

	$\widetilde{Q}^{\mathrm{D}}_{td}$		Q^{D}_{td}		V^{D}_{td}	
t	$d=1$	$d=2$	$d=1$	$d=2$	$d=1$	$d=2$
1	766	0	16	4800	750	660
2	0.91	0	0.91	3845.44	0	5460
3	2.04	13886.71	2.04	4581.27	0	9305.44
4	0.72	0	0.72	0	0	0
5	0	0	1.41	5969.34	0	0
6					1.41	5969.34

Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

continuing business assumption is realised. Taking a look at Table 3.29 it can be noticed that the only possibility for disposing hazardous waste in period three will probably be used to empty the hazardous inventory completely (i.e., $V^{\mathrm{D}}_{4,2} = 0$). The hazardous limit is $\overline{V}^3 = 200$ and the storage usage factor for the only hazardous item H of core $c = 1$ is $\nu^{\mathrm{D}}_2 = 0.007$ (see Table 3.23). Thus, the limit divided by the factor results in $200/0.007 = 28{,}571.43$ kg as upper limit for $\widetilde{Q}^{\mathrm{D}}_{t,2}$. The value in Table 3.29 (row $t = 3$, column $d = 2$ of $\widetilde{Q}^{\mathrm{D}}_{td}$) of 13,886.71 kg is not at the upper limit.

The revenues R_1 of the optimisation (i.e., study horizon plus the infinite estimation) sum up to 14,450,215.06 €, the acquisition, disassembly, and disposal cost C_1 to 14,205,613.32 €, the inventory holding cost C^{V}_1 to 65,581.15 €, and the contractual penalties C^{S}_1 to 427,607.38 €. This results in an objective of −248,586.79 €. Of course, this is not the profit for the first period only. The values for the first period only can be calculated with the given solution. The revenues are 627,316.60 €, the acquisition, disassembly, and disposal cost 684,513.20 €, the inventory holding cost 10,274.26 €, and the contractual penalties 26,190.85 €. Hence, a profit of −93,661.71 € is gained in the first period.

Pre-planning and contracting

Now, the first period is planned and the results will be realised. Hence, the focus is shifted to the next period. The second period is going to be pre-planned, which is coupled with the negotiating of contracts in period six. What is known in advance is that the hazardous waste is not going to be transported in period six. Thus, the upper limit is set to zero and integrated in the pre-planning model by

Table 3.30 Solution of pre-planning period two

	$\widetilde{Q}^{\mathrm{C}}_{6,c}$			$\widetilde{Q}^{\mathrm{I}}_{6,e}$			$\widetilde{Q}^{\mathrm{R}}_{6,r}$				$\widetilde{Q}^{\mathrm{D}}_{6,d}$	
	c			e			r				d	
	1	2	3	1	2	3	1	2	3	4	1	2
1^{st} run	0	157.23	0	200.03	220.03	155.66	0	124,690.83	5,982.24	0	0	0
2^{nd} run	25	115.03	15	198.05	203	128.73	0	115,332.22	6,003.89	0	0	0
Contracted quantities	25	117	16	215	190	135	0	118,000	6,000	0	∞	0

Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

$$\widetilde{Q}^{\mathrm{D}}_{6,2} = \overline{Q}^{\mathrm{D}}_{6,2} = 0\,. \tag{3.171}$$

Solving the pre-planning model leads to quantities of cores, items, and material as depicted in Table 3.30in the first row. While starting to contract suppliers and customers it becomes evident that for some reason the company is forced to acquire at least 25, 20, and 15 units of core 1, 2, and 3, respectively. Before the contracting is continued these new limitations are also added to the pre-planning model with

$$\widetilde{Q}^{\mathrm{C}}_{6,1} \geq 25, \quad \widetilde{Q}^{\mathrm{C}}_{6,2} \geq 20, \quad \widetilde{Q}^{\mathrm{C}}_{6,3} \geq 15. \tag{3.172}$$

The model is solved another time. The result can be found in the second row of the table. This procedure can repeat many times, but it shall be sufficient for the illustration here. Interestingly, no disposal is considered in period six. The hazardous disposal cannot be transported, i.e., is limited to zero (see Eq. (3.171)). The regular waste $d = 1$ is always unlimited. This indicates that it is beneficial to avoid disposal and that all items can be used for item reuse, material recycling, or are stored. The contracts are now settled for quantities as listed in the third row of Table 3.30.

In order to avoid manually choosing the values for the contracts in the contracting period in our numerical example, the contracted values are determined automatically. This means, a random value of the interval $\pm 10\,\%$ based on the results of the second run are choosen. Afterwards, the values are rounded to integral values for cores and items as well as multiples of 1,000 kg of material and waste. To give an example, 198.05 units of demand position $e = 1$ should be distributed according to the results. Hence, the contracted value is chosen out of the interval $\overline{Q}^{\mathrm{I}}_{6,1} \in [178.25; 217.86]$. One such value is 214.51 and this is rounded to $\overline{Q}^{\mathrm{I}}_{6,1} = 215$. Thereby, the minimal values of 25, 20, and 15 for cores cannot be underrun.

Table 3.31 Contracted quantities for pre-planning period 11

	$\overline{Q}^{\mathrm{C}}_{tc}$			D^{I}_{te}			D^{R}_{tr}				$\overline{Q}^{\mathrm{D}}_{td}$	
	c			e			r				d	
t	1	2	3	1	2	3	1	2	3	4	1	2
11	25	140	16	191	188	155	41,000	49,000	0	0	∞	∞
12	25	159	15	178	165	161	36,000	48,000	5,000	0	∞	0
13	25	143	15	181	195	153	39,000	50,000	5,000	0	∞	0
14	27	137	15	165	162	160	38,000	45,000	6,000	0	∞	0

Table 3.32 Solution of pre-planning period 11

	$\widetilde{Q}^{\mathrm{C}}_{15,c}$			$\widetilde{Q}^{\mathrm{I}}_{15,e}$			$\widetilde{Q}^{\mathrm{R}}_{15,r}$				$\widetilde{Q}^{\mathrm{D}}_{15,d}$	
	c			e			r				d	
	1	2	3	1	2	3	1	2	3	4	1	2
Solution	25	144.08	15	172.72	168.26	164.37	40,545.86	47,614.42	2,235.69	0	0	0
Contracted quantities	25	150	15	157	155	178	44,000	50,000	2,000	0	∞	∞

Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Once all quantities are contracted the values are stored in the parameters $\overline{Q}^{\mathrm{C}}_{6,c}$, $D^{\mathrm{I}}_{6,e}$, $D^{\mathrm{R}}_{6,r}$, and $\overline{Q}^{\mathrm{D}}_{6,d}$ (see Table 3.30 third row). Note that the regular disposal is not limited to maintain a feasible solution. Subsequently, the current period can be optimised as explained above. After eight further iterations period ten is completely planned. During the planning the contracted quantities $\overline{Q}^{\mathrm{C}}_{tc}$, D^{I}_{te}, D^{R}_{tr}, and $\overline{Q}^{\mathrm{D}}_{td}$ developed as listed in Table 3.31. Only the relevant periods are listed for the pre-planning of period 11. With this information, the initial inventory in period 11, the lower limits of 25, 20, and 15 units of cores 1, 2, and 3, respectively, and the possibility to dispose of hazardous waste in period 15 the pre-planning of period 11 takes place. The resulting quantities for period 15 are displayed in Table 3.32. The contracted quantities are derived from this solution and added to the data for the final planning. But in the meantime the already contracted quantities might change. This is an aspect of time varying data. When the changes occur before the pre-planning they can be included. But they can also occur after the pre-planning. This case is illustrated here. The data for the final planning of period 11 is updated and listed in Table 3.33. Thereby, the contracted quantities of period 15 are added and the remaining modified according to the changes taken place. For example, the contracted quantities

Table 3.33 Contracted quantities for planning period 11

	$\overline{Q}^{\mathrm{C}}_{tc}$			D^{I}_{te}			D^{R}_{tr}				$\overline{Q}^{\mathrm{D}}_{td}$	
	c			e			r				d	
t	1	2	3	1	2	3	1	2	3	4	1	2
11	25	140	16	191	190	150	41,000	49,000	0	0	∞	∞
12	25	170	15	178	165	170	36,000	48,000	5,000	0	∞	0
13	25	140	15	190	195	153	39,000	50,000	5,000	0	∞	0
14	27	137	18	160	160	160	38,000	45,000	6,000	0	∞	0
15	25	150	15	157	155	178	44,000	50,000	2,000	0	∞	∞

of core 2 in period 12 changes from 159 to 170—reasons for this can be manifold. With this data the period 11 is finally planned and the pre-planning of the succeeding period comes next.

This iterative procedure repeats as long as the business continues. In the example the iterating stops at period 20. The contracted quantities of all 24 periods are listed in Table 3.34 and the solutions of the periods are depicted in Tables 3.35 and 3.36. The optimal values for the decision variables X^{I}_{tci}, X^{D}_{tcid}, and X^{R}_{tcir} can be found in the appendix B.10 in the Tables B.2, B.3, and B.4, respectively.

The results show some variation in the values from period to period. This indicates that because of the changing data the solution needs to be adjusted from period to period in order to maintain a high profit. The profit (including the causing revenues and cost) for each period is depicted in Table 3.37. The sum of the profit over all 20 periods sums up to 204,376.77 €. In addition, the development of the revenues, cost, and profit are depicted in Fig. 3.29. Starting with the initial data the revenues and cost decrease over the first periods until they level between 500,000 and 600,000 €. On the contrary, the profit increases over the first periods and levels slightly above the zero value. This behaviour is expected, because in later periods the pre-planning and the resulting contracting show the benefits compared to the initial given data. On the other hand, this proves the gain of the pre-planning.

Table 3.34 Contracted quantities of periods 1–24

	$\overline{Q}^{\mathrm{C}}_{tc}$			D^{I}_{te}			D^{R}_{tr}				$\overline{Q}^{\mathrm{D}}_{td}$	
t	$c=1$	$c=2$	$c=3$	$e=1$	$e=2$	$e=3$	$r=1$	$r=2$	$r=3$	$r=4$	$d=1$	$d=2$
1	30	188	25	216	215	210	53,000	61,000	0	0	∞	0
2	34	189	25	191	246	208	50,000	67,000	0	0	∞	0
3	35	185	26	201	261	235	55,000	60,000	0	0	∞	∞
4	33	179	30	212	287	248	48,000	59,000	0	0	∞	0
5	34	158	29	239	269	212	44,000	64,000	0	0	∞	0
6	25	110	15	202	215	129	0	106,000	6,000	0	∞	0
7	25	138	16	179	162	156	36,000	60,000	2,000	0	∞	∞
8	25	162	16	175	164	176	43,000	45,000	5,000	0	∞	0
9	26	157	16	178	182	176	38,000	51,000	6,000	0	∞	0
10	25	169	16	195	173	166	38,000	48,000	6,000	0	∞	0
11	25	140	16	191	190	150	41,000	49,000	0	0	∞	∞
12	25	170	15	178	165	170	36,000	48,000	5,000	0	∞	0
13	25	140	15	190	195	153	39,000	50,000	5,000	0	∞	0
14	27	137	18	160	160	160	38,000	45,000	6,000	0	∞	0
15	25	150	15	157	155	178	44,000	50,000	2,000	0	∞	∞
16	25	176	15	174	194	161	44,000	53,000	5,000	0	∞	0
17	26	138	15	168	187	186	41,000	48,000	0	0	∞	0
18	27	146	16	178	165	173	43,000	46,000	0	0	∞	0
19	27	142	15	187	187	167	39,000	55,000	6,000	0	∞	∞
20	25	147	15	186	191	167	46,000	53,000	0	0	∞	0
21	25	155	15	196	189	157	40,000	47,000	0	0	∞	0
22	27	157	15	180	160	172	40,000	52,000	0	0	∞	0
23	25	156	16	188	196	171	45,000	55,000	0	0	∞	∞
24	25	176	15	190	179	163	44,000	52,000	0	0	∞	0

3.4.4.3 Planning evaluation

Variation of z

In the above presented numerical example the value of z was set to $^{20}/_{21}$. In order to find the appropriate value of z the decision maker has to evaluate the results of the planning for different values of z for a specific problem. Thereby, the remaining parameters should be comparable in each evaluation. Hence, it might be helpful to use historic data as basis and make according changes. Of course, this is difficult especially for the contracted number of quantities, because they are influenced by the planning itself.

Let us assume a set of historic data. Based on this data a planning took place. The result was that 100 cores should be acquired. In the end, only 90 cores were acquired. When the planning is repeated with a different methodology and/or different data, a value other than 100 , e.g., 120, is the result.

Table 3.35 Solution of 20 period planning (part one)

	$\widetilde{Q}_{tc}^{\mathrm{C}}$			Q_{tc}^{C}			V_{tc}^{C}		
t	$c=1$	$c=2$	$c=3$	$c=1$	$c=2$	$c=3$	$c=1$	$c=2$	$c=3$
1	27	188	25	24	192	0	10	7	5
2	31	189	25	19	192	18	13	3	30
3	32	185	26	27	135	63	25	0	37
4	30	179	30	7	200	30	30	50	0
5	33	158	29	29	183	20	53	29	0
6	25	110	15	81	114	24	57	4	9
7	25	138	16	25	138	16	1	0	0
8	24	162	16	25	162	16	1	0	0
9	25	157	16	25	157	16	0	0	0
10	25	169	15	23	160	14	0	0	0
11	23	140	16	14	149	17	2	9	1
12	23	170	15	27	164	0	11	0	0
13	25	140	15	28	146	30	7	6	15
14	27	137	18	25	137	18	4	0	0
15	25	150	15	26	150	0	6	0	0
16	25	176	15	29	176	0	5	0	15
17	26	138	15	27	137	38	1	0	30
18	27	146	16	27	147	23	0	1	7
19	27	142	15	25	142	15	0	0	0
20	25	147	15	27	147	15	2	0	0
	$\widetilde{Q}_{te}^{\mathrm{I}}$			Q_{te}^{I}			V_{te}^{I}		
t	$e=1$	$e=2$	$e=3$	$e=1$	$e=2$	$e=3$	$e=1$	$e=2$	$e=3$
1	212	9	210	172	213	190	40	80	20
2	191	246	207	191	208	207	0	284	0
3	193	261	195	193	159	195	0	246	0
4	212	287	227	212	204	227	0	144	0
5	203	269	200	208	209	200	0	61	0
6	182	193	129	194	192	135	5	1	0
7	177	160	156	160	160	151	17	0	6
8	175	164	176	180	184	175	0	0	1
9	178	182	170	176	179	170	5	20	0
10	179	172	166	176	180	166	3	17	0
11	160	185	150	160	160	158	0	25	0
12	170	165	170	170	188	162	0	0	8
13	180	194	153	180	171	173	0	23	0
14	160	159	160	160	159	152	0	0	20
15	156	155	160	156	173	148	0	0	12
16	174	194	161	184	202	174	0	18	0
17	168	187	185	180	161	172	10	26	13
18	175	165	167	176	171	167	22	0	0
19	183	170	154	160	164	154	23	6	0
20	168	171	159	168	171	159	0	0	0

Table 3.36 Solution of 20 period planning (part two)

	$\widetilde{Q}^{\mathrm{R}}_{tr}$				Q^{R}_{tr}				V^{R}_{tr}			
	r				r				r			
t	1	2	3	4	1	2	3	4	1	2	3	4
1	46,916	53,600	.	.	46,416	51,900	0	0	500	1,700	53	0
2	49,909	49,614	.	.	49,909	49,614	0	0	0	0	53	0
3	50,222	53,000	.	.	50,222	53,000	691	0	0	0	53	0
4	48,000	44,452	.	.	48,000	44,452	0	0	0	0	744	0
5	44,000	61,930	.	.	44,000	61,930	2,500	0	0	0	744	0
6	.	106,000	6,000	.	355	116,616	5,861	0	0	0	3,244	0
7	34,947.49	59,936	2,000	.	35,047	49,320	2,033	0	355	10,616	3,105	0
8	42,785.51	45,000	4,915.30	.	42,331	49,350	3,283	0	454.51	0	3,138	0
9	38,000	51,000	5,968.82	.	40,136	47,800	5,242	0	0	4,350	1,505.70	0
10	38,000	48,000	5,976.88	.	39,604	50,930	5,198	0	2,136	1,150	778.88	0
11	41,000	49,000	.	.	37,260	44,920	252	0	3,740	4,080	0	0
12	36,000	48,000	3,943	.	38,703	49,200	3,691	0	0	0	252	0
13	39,000	50,000	4,107.58	.	40,961	51,220	5,387	0	2,703	1,200	0	0
14	38,000	45,000	5,059.42	.	36,712	45,300	3,780	0	4,664	2,420	1,279.42	0
15	37,253	50,000	2,000	.	33,877	47,590	3,989	0	3,376	2,720	0	0
16	41,640	53,000	5,000	.	41,640	53,020	3,508	0	0	310	1,989	0
17	41,000	48,000	.	.	41,401	49,300	5,237	0	0	330	497	0
18	35,098.03	46,000	.	.	39,374	54,958	0	0	401	1,630	5,734	0
19	35,819.91	55,000	5,831	.	36,257	51,156	97	0	4,676.97	10,588	5,734	0
20	45,552.06	53,000	.	.	40,438	47,500	0	0	5,114.06	6,744	0	0

	$\widetilde{Q}^{\mathrm{D}}_{td}$		Q^{D}_{td}		V^{D}_{td}	
t	$d=1$	$d=2$	$d=1$	$d=2$	$d=1$	$d=2$
1	766	.	16	4,800	750	660
2	32	.	32	3,800	0	5,460
3	32	14,660	32	5,400	0	9,260
4	32	.	32	1,400	0	0
5	12.88	.	32	5,800	0	1,400
6	0	.	22,682	16,200	19.12	7,200
7	0	17,397.57	32	5,000	22,701.12	23,400
8	0	.	32	5,000	22,733.12	11,002.43
9	0	.	32	5,000	22,765.12	16,002.43
10	0	.	32	4,600	22,797.12	21,002.43
11	0	21,487.64	32	2,800	22,829.12	25,602.43
12	1,576.708	.	16	5,400	22,861.12	6,914.79
13	0	.	32	5,600	21,300.43	12,314.79
14	3,631.52	.	982	5,000	21,332.43	17,914.79
15	0	21,543.36	16	5,200	18,682.91	22,914.79
16	0	.	16	5,800	18,698.91	6,571.43
17	0	.	32	5,400	18,714.91	12,371.43
18	0	.	32	5,400	18,746.91	17,771.423
19	0	19,319.19	32	5,000	18,778.91	23,171.43
20	12,798.15	.	3,682	5,400	18,810.91	8,852.24

Dots denote a value of zero, which is known a priori from the contracted quantities (see Table 3.34). Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Table 3.37 Revenues, cost, and profit development

	revenues		cost										profit
						inventory				contractual penalty			
t	items	material	acquisition	disassembly	disposal	cores	items	material	waste	cores	items	material	
1	513,060	114,257	623,400	60,960	153	2,185	7,867	206	17	6,900	15,480	3,811	−93,662
2	570,275	116,014	633,631	64,140	7	2,504	9,246	199	37	7,035	588	4,074	−35,172
3	549,187	119,154	624,451	62,280	6,311	2,728	8,323	208	60	7,380	23,582	3,299	−70,280
4	634,886	103,740	611,000	65,900	7	2,837	8,347	188	4	7,029	12,374	3,382	27,558
5	570,594	118,853	568,073	65,140	3	2,629	6,863	220	22	2,316	7,416	497	36,269
6	378,026	108,680	381,000	62,460	0	1,562	4,549	259	99	0	1,882	0	34,896
7	432,361	109,664	441,498	50,300	7,133	1,283	5,069	229	175	0	174	374	35,789
8	478,260	105,802	500,262	57,020	0	1,440	5,525	204	113	2,236	0	92	17,171
9	459,768	101,584	475,134	55,620	0	1,404	5,531	210	138	2,210	3,483	7	17,615
10	453,749	101,960	491,902	55,340	0	1,567	5,402	208	162	2,670	207	5	−1,754
11	413,830	102,470	425,844	50,340	9,669	1,431	5,160	196	181	4,424	668	0	18,387
12	471,890	100,869	480,130	54,020	315	1,751	5,560	184	94	4,560	62	238	25,844
13	437,672	110,743	426,305	57,080	0	1,503	5,590	211	118	0	159	192	57,256
14	445,417	101,442	434,142	50,540	726	1,360	5,755	205	146	0	86	207	53,692
15	438,234	101,854	471,050	49,800	10,125	1,532	5,321	195	165	0	10,767	2,227	−11,093
16	466,935	110,131	529,016	57,980	0	1,896	5,724	206	85	0	0	785	−18,624
17	514,057	104,030	451,557	56,340	0	1,548	6,215	195	113	0	605	0	101,514
18	454,021	96,166	477,352	55,240	0	1,397	5,250	220	140	0	3,583	2,746	4,260
19	425,557	108,987	458,706	51,160	8,887	1,318	4,936	259	166	0	9,098	1,158	−1,144
20	432,525	120,947	478,540	53,160	2,688	1,340	4,998	223	99	0	6,403	166	5,855
$\sum$	9,540,304	2,157,347	9,982,993	1,134,820	46,024	35,215	121,231	4,224	2,132	46,760	96,616	23,258	204,377

Values are rounded to integer values. The last row lists the sum of the exact values of the periods and therefore differs to the sum of the rounded values of the periods 1 through 20.

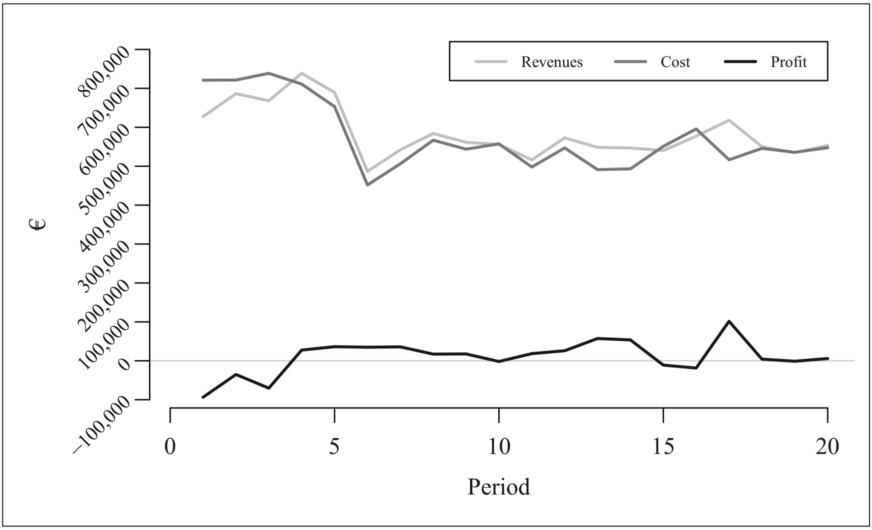

Fig. 3.29 Development of revenues, cost, and profit

In order to keep the planning comparable the 120 units must be changed accordingly to the historic data. This means that an absolute change (from 120 to 100), a proportional change (from 120 to 108), a fix change (from 120 to 90), or even something different is chosen. Which of these options is selected depends on the specific environment and the decision maker has to decide.

For our numerical example we used four different values of z to illustrate such a comparison. Our evaluation is not representative because we performed only one run with each value and the selection of contracting quantities as well as data variation is conducted automatically and independent for each run. The (undiscounted) sums of the profits over all 20 periods for the values $z = {}^{100}/_{104}$, $z = {}^{100}/_{106}$, and $z = {}^{100}/_{107}$ in relation to the reference case of $z = {}^{100}/_{105}$ are 68 %, 94 %, and 87 %, respectively. If the decision maker has to choose between these four values, he or she should select either $z = {}^{100}/_{105}$ or $z = {}^{100}/_{106}$, because they result in the highest sum of profits.

For a detailed analysis of the best value of z the illustrated comparison must be extended to a profound simulation with several runs per value of z and comparable contracting behaviour as discussed above. Moreover, a stochastic approach with the goal to maximise the expected sum of profits and z as decision variable might deliver a good idea for profitable values of z. Of course, this requires that the decision maker knows the density or distribution functions of the stochastic influences.

One period study horizon

To show the benefit of the above presented planning another scenario is illustrated in the sequel. It is motivated by isolated departments for disassembly planning and contracting. In addition, the contracting department still does the contracting in advance. On the contrary, the disassembly planning department does not bother with varying data and therefore performs the planning only for the upcoming period τ, i.e., there is no inclusion of forecast information. Speaking in terms of the rolling horizon planning the study horizon is of length one, whereas the study horizon for the contracting is still longer, e.g., five periods.

This means that the pre-planning model is used as described above with the above given model formulation. The model for the planning of the current period is slightly modified. Firstly, the study horizon length is set to $\bar{\tau} = 1$ for the optimal planning of the period. Secondly, the factor for the discounting and the infinite annuity is removed. Hence, the parts of the objective function are

$$R_\tau = \sum_e r^{\mathrm{I}}_{\tau e} \widetilde{Q}^{\mathrm{I}}_{\tau e} + \sum_r r^{\mathrm{R}}_{\tau r} \widetilde{Q}^{\mathrm{R}}_{\tau r} \tag{3.173}$$

$$C_\tau = \sum_c \left(c^{\mathrm{A}}_{\tau c} \widetilde{Q}^{\mathrm{C}}_{\tau c} + c^{\mathrm{J}}_c Q^{\mathrm{C}}_{\tau c} \right) + \sum_d c^{\mathrm{D}}_{\tau d} \widetilde{Q}^{\mathrm{D}}_{\tau d} \tag{3.174}$$

$$\begin{aligned} C^{\mathrm{V}}_\tau = & \sum_c h^{\mathrm{C}}_c \left(V^{\mathrm{C}}_{\tau c} + \widetilde{Q}^{\mathrm{C}}_{\tau c} - \frac{1}{2} Q^{\mathrm{C}}_{\tau c} \right) + \sum_e h^{\mathrm{I}}_e \left(V^{\mathrm{I}}_{\tau e} + \frac{1}{2} Q^{\mathrm{I}}_{\tau e} \right) \\ & + \sum_r h^{\mathrm{R}}_r \left(V^{\mathrm{R}}_{tr} + \frac{1}{2} Q^{\mathrm{R}}_{tr} \right) + \sum_d h^{\mathrm{D}}_d \left(V^{\mathrm{D}}_{td} + \frac{1}{2} Q^{\mathrm{D}}_{td} \right) \end{aligned} \tag{3.175}$$

$$\begin{aligned} C^{\mathrm{S}}_\tau = & \, \sigma^{\mathrm{C}} \sum_c c^{\mathrm{A}}_{tc} \left(\overline{Q}^{\mathrm{C}}_{\tau c} - \widetilde{Q}^{\mathrm{C}}_{\tau c} \right) + \sigma^{\mathrm{I}} \sum_e r^{\mathrm{I}}_{\tau e} \left(D^{\mathrm{I}}_{\tau e} - \widetilde{Q}^{\mathrm{I}}_{\tau e} \right) \\ & + \sigma^{\mathrm{R}} \sum_r r^{\mathrm{R}}_{\tau r} \left(D^{\mathrm{R}}_{\tau r} - \widetilde{Q}^{\mathrm{R}}_{\tau r} \right) + \sigma^{\mathrm{D}} \sum_{d \in \left\{ d | \overline{Q}^{\mathrm{D}}_{\tau d} < \infty \right\}} c^{\mathrm{D}}_{\tau d} \left(\overline{Q}^{\mathrm{D}}_{\tau d} - \widetilde{Q}^{\mathrm{D}}_{\tau d} \right) . \end{aligned} \tag{3.176}$$

Because of the limitation that only every fourth period hazardous disposal can be transported to the disposal site together with a study horizon of only one period the planner needs to assure that the storage for hazardous disposal is not limiting the disassembly process. Therefore, we apply the rule,

Table 3.38 Sorted absolute changes of acquired cores from period to period

	absolute change from period to period																		
$\bar{\tau}=5$	38	38	30	30	29	28	26	24	21	13	12	8	6	5	5	4	4	3	1
$\bar{\tau}=1$	65	61	51	48	45	37	33	22	21	15	14	12	6	6	6	4	4	1	0

that whenever we have the opportunity to empty the hazardous inventory we use it. This means, that the initial inventory of a period succeeding a period with disposing hazardous waste is always empty. This is the third modification. A fourth change is an extra constraint which ensures that the acquired cores of the succeeding period can be placed into the storage. Otherwise the incoming inventory might be too big to add the guaranteed quantities of cores. The constraint is as follows.

$$\sum_{c} \nu_c^{\mathrm{C}} \left(V_{\tau+1,c}^{\mathrm{C}} + \overline{Q}_{\tau+1,c}^{\mathrm{C}} \right) \leq \overline{V}^1 \tag{3.177}$$

Since the value of $\widetilde{Q}_{\tau+1,c}^{\mathrm{C}}$ is not known before the upcoming period is planned, we approximate the quantities by the corresponding contracted quantities $\overline{Q}_{\tau+1,c}^{\mathrm{C}}$.

Performing the rolling horizon planning as described here, the sum of the undiscounted profits of the 20 periods decreases significantly to −219,769.26 € in a single run. This is a decrease of more than 200 % compared to the rolling planning with a study horizon of five periods. This example should clearly motivate the inclusion of existing information of future periods and not just a single period planning. Furthermore, the resulting revenues, cost, and profits show more volatility than the first presented rolling horizon planning approach (see Fig. 3.30 in comparison with Fig. 3.29). This increased volatility can be caused by the random choosing of contracted quantities. Here the interval is fixed to plus/minus 10 %. Taking a look at the absolute changes of the acquired quantities of core 2 (i.e., $|\widetilde{Q}_{t,2}^{\mathrm{C}} - \widetilde{Q}_{t+1,2}^{\mathrm{C}}|$), we see that for the one period planning ($\bar{\tau}=1$) changes of up to 65 units occur (see Table 3.38). The maximal value of the acquired cores is 189, which means that the 65 significantly exceeds the 10 % of 189. Reasons for this rather large change could be a build-up like that of the bullwhip effect and/or the short study horizon that prevents the planning from smoothing the results.[67]

[67] The reduction of the bullwhip effect (variance amplification of ordering quantities in supply chains) can for example be realised by information enrichment. Cf. Dejonckheere et al. (2004): *The impact of information enrichment on the bullwhip effect*, pp. 729

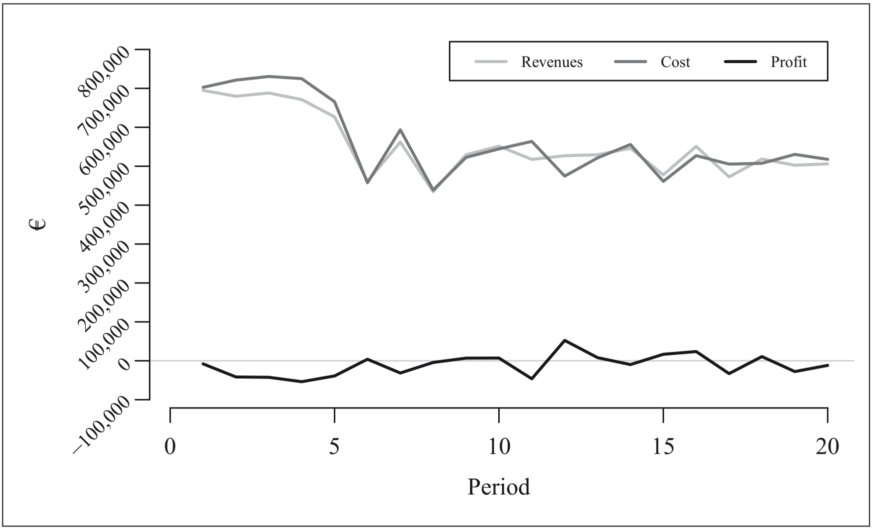

Fig. 3.30 Development of objective values

Ex post (optimal) solution with given contracts and inventory limit

When having planned the twenty periods with the here presented method, we have seen that the results are better than the one period planning. But the question arises: how much potential still exists to increase the profit? The answer is insofar difficult as there exists not really an optimal solution over the planning horizon, because the data becomes available period per period and is not known a priori. Otherwise, the data could have been included in the planning. In addition, even the given data (the contracts) can still change until the current period is planned. Therefore, a comparison can only be conducted ex post.

A somewhat reasonable approach to find a reference for our solution might be the following. Given the realised contracted quantities over the complete planning horizon we try to find the optimal solution when all 20 periods are considered in the planning. In addition, the on-going business is still assumed, which means that the four periods (i.e., 21–24) are included, too. Thereby, the decision variables of the last four periods do not have to be integer values. In order to avoid an increase of inventory at the end of period 24 (to save primarily disposal cost), we limit the final inventory to

and 745 et seq. The extension of the study horizon can be seen as such an information enrichment.

the same as the result of the rolling horizon planning. At the end of period 24 the inventory of cores, items, and material is empty with the exception of core 1 (i.e., $V^{\mathrm{C}}_{25,1} = 14.105$) and the waste (i.e., $V^{\mathrm{D}}_{25,1} = 19{,}513.267$ and $V^{\mathrm{D}}_{25,2} = 28{,}571.429$). The contracted quantities are listed in Table 3.34.

To evaluate the rolling horizon planning with the ex post optimal solution the model needs to be modified. This modification is necessary to extend the integrality constraints to the first 20 periods and the removal of the discounting.

$$\text{Maximise } P = R - C - C^{\mathrm{V}} - C^{\mathrm{S}} \tag{3.178}$$

$$R = \sum_{t=1}^{24} \left(\sum_{e} r^{\mathrm{I}}_{te} \widetilde{Q}^{\mathrm{I}}_{te} + \sum_{r} r^{\mathrm{R}}_{tr} \widetilde{Q}^{\mathrm{R}}_{tr} \right) \tag{3.179}$$

$$C = \sum_{t=1}^{24} \left(\sum_{c} \left(c^{\mathrm{A}}_{tc} \widetilde{Q}^{\mathrm{C}}_{tc} + c^{\mathrm{J}}_{c} Q^{\mathrm{C}}_{tc} \right) + \sum_{d} c^{\mathrm{D}}_{td} \widetilde{Q}^{\mathrm{D}}_{td} \right) \tag{3.180}$$

$$\begin{aligned} C^{\mathrm{V}} = \sum_{t=1}^{24} \Bigg(& \sum_{c} h^{\mathrm{C}}_{c} \left(V^{\mathrm{C}}_{tc} + \widetilde{Q}^{\mathrm{C}}_{tc} - \frac{1}{2} Q^{\mathrm{C}}_{tc} \right) + \sum_{e} h^{\mathrm{I}}_{e} \left(V^{\mathrm{I}}_{te} + \frac{1}{2} Q^{\mathrm{I}}_{te} \right) \\ & + \sum_{r} h^{\mathrm{R}}_{r} \left(V^{\mathrm{R}}_{tr} + \frac{1}{2} Q^{\mathrm{R}}_{tr} \right) + \sum_{d} h^{\mathrm{D}}_{d} \left(V^{\mathrm{D}}_{td} + \frac{1}{2} Q^{\mathrm{D}}_{td} \right) \Bigg) \end{aligned} \tag{3.181}$$

$$\begin{aligned} C^{\mathrm{S}} = \sum_{t=1}^{24} \Bigg(& \sigma^{\mathrm{C}} \sum_{c} c^{\mathrm{A}}_{tc} \left(\overline{Q}^{\mathrm{C}}_{tc} - \widetilde{Q}^{\mathrm{C}}_{tc} \right) + \sigma^{\mathrm{I}} \sum_{e} r^{\mathrm{I}}_{te} \left(D^{\mathrm{I}}_{te} - \widetilde{Q}^{\mathrm{I}}_{te} \right) \\ & + \sigma^{\mathrm{R}} \sum_{r} r^{\mathrm{R}}_{tr} \left(D^{\mathrm{R}}_{tr} - \widetilde{Q}^{\mathrm{R}}_{tr} \right) + \sigma^{\mathrm{D}} \sum_{d \in \left\{ d | \overline{Q}^{\mathrm{D}}_{td} < \infty \right\}} c^{\mathrm{D}}_{td} \left(\overline{Q}^{\mathrm{D}}_{td} - \widetilde{Q}^{\mathrm{D}}_{td} \right) \Bigg) \end{aligned} \tag{3.182}$$

The indices for the relevant periods form the set $\widetilde{T} = \{1, \ldots, 24\}$. Hence, the constraints (3.143) through (3.165) can be kept as they are. In addition, the final inventory limitation is

$$V^{\mathrm{C}}_{25,1} \leq 14.105, \quad V^{\mathrm{C}}_{25,2} = 0, \quad V^{\mathrm{C}}_{25,3} = 0 \tag{3.183}$$

$$V^{\mathrm{I}}_{25,1} = 0, \quad V^{\mathrm{I}}_{25,2} = 0, \quad V^{\mathrm{I}}_{25,3} = 0 \tag{3.184}$$

$$V^{\mathrm{R}}_{25,1} = 0, \quad V^{\mathrm{R}}_{25,2} = 0, \quad V^{\mathrm{R}}_{25,3} = 0, \quad V^{\mathrm{R}}_{25,4} = 0 \tag{3.185}$$

$$V^{\mathrm{D}}_{25,1} \leq 19513.267, \quad V^{\mathrm{D}}_{25,2} \leq 28571.429\,. \tag{3.186}$$

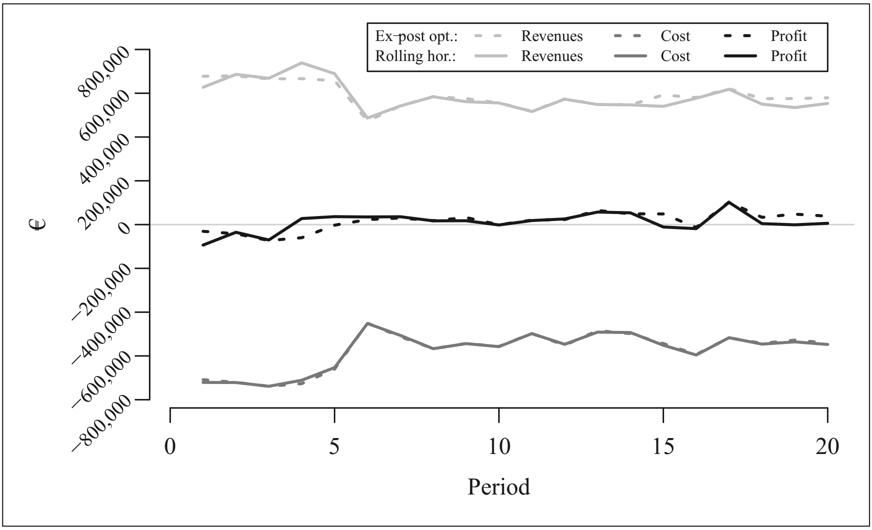

Fig. 3.31 Development of objective values

Furthermore, the integrality constraints need to be extended to

$$\widetilde{Q}^{\mathrm{C}}_{tc}, X^{\mathrm{I}}_{tci}, X^{\mathrm{R}}_{tcir}, X^{\mathrm{D}}_{tcid}, \widetilde{Q}^{\mathrm{I}}_{te} \in \mathbb{Z}^* \quad \forall\, t \in \{1,\ldots,20\}, c, i \in \{1,\ldots,\bar{I}_c\}, r, d, e\,. \tag{3.187}$$

The profit gained in the 20 periods with this model is 308,962.16 €. This profit is significantly greater (about 51 %) than the one of the rolling horizon planning. The development over the periods can be observed in Table 3.39as well as Fig. 3.31. The solid curves are the ones of the rolling horizon planning (see Fig. 3.29) and the dotted lines represent the ex-post optimal solution. Note that the cost is depicted on the negative scale for better viewing. What we see is that there is not much difference in the development on a big scale. It is rather the small differences that cause the difference. What is the main driver for the difference?

We notice that the contractual penalty cost for cores is zero whereas the one of the rolling horizon planning is greater than zero. Let us take a closer look at the quantities of cores, items, and material. Therefore, they are listed in Table 3.40. In comparison to the rolling horizon planning (see Tables 3.35 and 3.36) 19 cores $c = 1$ and one core $c = 3$ are acquired more. Multiplied with the corresponding cost factors we get $44{,}090 + 2{,}670 = 46{,}760$ € more acquisition cost. In addition, the 20 cores need to be disassembled which leads to 5,960 € disassembly cost for the 20 cores. This together is already

Table 3.39 Revenues, cost, and profit development of ex-post solution

	revenues		cost										profit
						inventory				contractual penalty			
t	items	material	acquisition	disassembly	disposal	cores	items	material	waste	cores	items	material	
1	563,580	114,121	630,300	60,960	153	2,227	7,871	206	17	0	2,850	3,845	−30,727
2	560,867	118,050	640,666	63,480	7	2,587	7,282	205	39	0	2,940	3,565	−41,853
3	547,083	118,727	631,831	62,980	6,651	2,868	6,562	209	64	0	24,108	3,406	−72,869
4	552,909	113,704	618,029	65,260	7	3,052	6,285	212	13	0	32,869	891	−60,004
5	541,875	115,514	570,389	64,480	0	2,902	6,179	229	41	0	14,596	1,332	−2,759
6	365,819	107,161	381,000	57,020	0	1,975	4,577	274	85	0	4,933	380	22,736
7	432,432	109,560	441,498	50,660	11,480	1,807	5,590	245	127	0	157	400	30,029
8	478,260	106,170	502,498	55,980	0	2,015	6,016	224	14	0	0	0	17,684
9	473,700	101,610	477,344	57,540	0	1,965	5,991	235	41	0	0	0	32,195
10	454,019	101,980	494,572	54,020	0	2,120	5,564	233	68	0	140	0	−718
11	415,478	102,470	430,268	50,060	9,540	2,080	5,118	219	94	0	256	0	20,314
12	472,138	101,820	484,690	57,980	0	2,351	5,724	225	15	0	0	0	22,973
13	437,529	111,510	426,305	50,060	0	2,197	5,540	235	42	0	195	0	64,465
14	445,074	102,270	434,142	55,820	0	2,081	5,565	217	67	0	172	0	49,280
15	481,301	110,760	471,050	54,020	10,058	1,980	6,000	233	94	0	0	0	48,627
16	466,935	113,270	529,016	55,980	0	2,287	5,772	220	14	0	0	0	−13,084
17	516,477	104,030	451,557	57,200	0	1,959	6,214	192	40	0	0	0	103,345
18	468,352	107,150	477,352	57,400	0	1,738	5,617	220	67	0	0	0	33,108
19	461,948	113,620	458,706	55,100	6,613	1,511	5,637	244	95	0	0	0	47,662
20	457,515	121,610	478,540	54,780	0	1,383	5,443	215	50	0	156	0	38,559
$\sum$	9,593,291	2,195,108	10,029,753	1,140,780	44,508	43,086	118,547	4,490	1,084	0	83,370	13,818	308,962

Values are rounded to integer values. The last row lists the sum of the exact values of the periods and therefore differs to the sum of the rounded values of the periods 1 through 20.

Table 3.40 Ex-post solution of 20 period planning

	$\widetilde{Q}^{\mathrm{C}}_{tc}$			$\widetilde{Q}^{\mathrm{I}}_{te}$			$\widetilde{Q}^{\mathrm{R}}_{tr}$				$\widetilde{Q}^{\mathrm{D}}_{td}$	
t	$c=1$	$c=2$	$c=3$	$e=1$	$e=2$	$e=3$	$r=1$	$r=2$	$r=3$	$r=4$	$d=1$	$d=2$
1	30	188	25	216	177	210	46834	53574	0	0	766	0
2	34	189	25	191	246	203	50000	51667.12	0	0	32	0
3	35	185	26	201	261	194	48384	55200.88	0	0	16	15460
4	33	179	30	210	180	207	48000	55166	0	0	31.35	0
5	34	158	29	204	225	194	43894	58601	0	0	0	0
6	25	110	15	182	154	129	0	106000	4101	0	0	0
7	25	138	16	160	162	156	36000	60000	0	0	0	28000
8	25	162	16	175	164	176	43000	45000	5000	0	0	0
9	26	157	16	178	182	176	38000	51000	6000	0	0	0
10	25	169	16	177	173	166	38000	48000	6000	0	0	0
11	25	140	16	158	190	150	41000	49000	0	0	0	21200
12	25	170	15	178	165	170	36000	48000	5000	0	0	0
13	25	140	15	164	195	153	39000	50000	5000	0	0	0
14	27	137	18	160	158	160	38000	45000	6000	0	0	0
15	25	150	15	157	155	178	44000	50000	2000	0	0	21400
16	25	176	15	174	194	161	44000	53000	5000	0	0	0
17	26	138	15	168	187	186	41000	48000	0	0	0	0
18	27	146	16	178	165	173	43000	46000	0	0	0	0
19	27	142	15	187	187	167	39000	55000	6000	0	0	14375.53
20	25	147	15	186	189	167	46000	53000	0	0	0	0

Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

52,720 €. Several shifts of quantities into other periods and the disposal cost might lead to even higher or lower cost. But even though we have higher cost for more cores in comparison to the rolling planning the ex-post optimal planning results in lower cost with a difference of 13,837.48 €. This means that there exists a major cost saving at some other point. Taking a look at the cost categories acquisition and disassembly, inventory, and contractual penalty we see that the differences are $10{,}029{,}753 + 1{,}140{,}780 + 44{,}508 - (9{,}982{,}993 + 1{,}134{,}820 + 46{,}024) = 51{,}204$ €, $43{,}086 + 118{,}547 + 4{,}490 + 1{,}084 - (35{,}215 + 121{,}231 + 4{,}224 + 2{,}132) = 4{,}405$ €, and $0 + 83{,}370 + 13{,}818 - (46{,}760 + 96{,}616 + 23{,}258) = -69{,}446$ €, respectively. Hence, the major cost saving results from the saving of contractual penalties.

The penalty factors σ^{C}, σ^{I}, and σ^{R} are 1, 0.25, and 0.25, respectively. This means that for each core that is not acquired—even though a contract exists—the penalty cost factor $\sigma^{\mathrm{C}} \cdot c^{\mathrm{A}}_{tc}$ has to be paid to the supplier. The penalty savings for the cores are 44,090 €, 0 €, and 2,670 € for the 19, zero, and one cores more acquired. Because of the increased number of cores to disassemble, the output of items and material increases, too. For the item distribution the penalties reduce by about 62 €, 658 €, and 12,526 €. Furthermore, the penalties for material distribution reduce by 6,815 € and 2,702 € for steel and metal, respectively, increase by 77 € for rubber and stay identical for plastics. We see that together a cost saving of about 69,446 € occurs. Less the increased acquisition and disassembly cost of about 52,720 € an overall cost saving of about 16,726 € results, which explains the major effect of the cost savings.

In addition, for every fulfilment of a contract not only costs are saved, but also revenues are generated. Hence, it is twice beneficial to fulfil contracts to a higher level. With the 20 cores more 40 items of demand position $e = 1$ could be distributed. When we take a look at the solutions $\widetilde{Q}^{\mathrm{I}}_{t,1}$ in Tables 3.35 and 3.40, we notice that eight items are distributed more. Further 30 items are in the inventory at the end of period 20 compared to no item in the rolling horizon planning. This shows that the additional cores are not completely intended for item distribution. For demand position $e = 1$ the eight additional items and the shift of item distribution into other periods leads to an increased revenue of 250 €.[68] For demand position two and three the increased revenue would be 2,631 € and 50,106 €. Furthermore, more material could be distributed also. Here an increase of 27,261 € and 10,808 € for steel and metal, respectively, could be gained. Hence, an overall increase of about 91,056 € of revenues can be expected. Adding the expected cost savings of 16,726 € results in 107,782 € increased profit. The difference to the actual profit increase is caused by additional disposal cost, less revenue for plastics, etc.

From this analysis we see that for this example setting the fulfilment of the contracts of cores would lead to a better solution.[69] Hence, we assume that an increase of the penalty cost factor σ^{C} in the objective function might lead to better solutions. Therefore, we repeat the rolling horizon planning. This leads to a totally different solution, because of the random selection of quantities to contract. The prices and cost factors over the periods are identical. The difference here is that even though σ^{C} equals one we increase the penalty costs for cores in our objective function of the planning of the current period as well as the pre-planning by a factor, e.g., two. The relevant part of the objective function is then

[68] $\sum_{t=1}^{20} r^{\mathrm{I}}_{t,1}(\widetilde{Q}^{\mathrm{I,expost}}_{t,1} - \widetilde{Q}^{\mathrm{I}}_{t,1}) = 30(216 - 212) + 31(191 - 191) + 31(201 - 193) + \cdots + 33(187 - 183) + 32(186 - 168) = 250$. See Tables 3.24, 3.35, and 3.40.

[69] The contracts of items and material are not completely fulfilled.

$$C_\tau^{\mathrm{S}} = \sum_{t=\tau}^{\tau+\bar{\tau}-1} \left[\left(2\sigma^{\mathrm{C}} \sum_c c_{tc}^{\mathrm{A}} \left(\overline{Q}_{tc}^{\mathrm{C}} - \widetilde{Q}_{tc}^{\mathrm{C}} \right) + \sigma^{\mathrm{I}} \sum_e r_{te}^{\mathrm{I}} \left(D_{te}^{\mathrm{I}} - \widetilde{Q}_{te}^{\mathrm{I}} \right) \right.\right.$$
$$\left. + \sigma^{\mathrm{R}} \sum_r r_{tr}^{\mathrm{R}} \left(D_{tr}^{\mathrm{R}} - \widetilde{Q}_{tr}^{\mathrm{R}} \right) + \sigma^{\mathrm{D}} \sum_{d \in \left\{ d \mid \overline{Q}_{td}^{\mathrm{D}} < \infty \right\}} c_{td}^{\mathrm{D}} \left(\overline{Q}_{td}^{\mathrm{D}} - \widetilde{Q}_{td}^{\mathrm{D}} \right) \right)$$
$$\left. \cdot \left(z^{t-\tau} + \frac{z^{\bar{\tau}}}{\bar{\tau}(1-z)} \right) \right]. \tag{3.188}$$

The original one can be found in Eq. (3.142). Surely, the specification for the factor needs more analysis than just guessing, but for the consideration here we just want to illustrate the effect.

The gained new solution of the rolling horizon planning results in a profit of 280,246.93 €. Note that the contracted quantities differ, too. This solution does not have any penalty cost for cores. Hence, the desired effect is achieved. The ex-post solution for this new example leads to a profit of 318,885.34 €.[70] This is still more, but the gap is significantly decreased from 51 % $\left(\frac{308962.16}{204376.77}\right)$ to 14 % $\left(\frac{318885.34}{280246.93}\right)$. Note that this is only one example and must be varified with a greater test set. The remaining gap is hardly to close, because in the ex-post planning the information of all 24 periods is included whereas the rolling horizon planning considers only five periods. Hence, a shift of distribution from period three to 17 or vice versa is not possible in the rolling horizon planning, but entirely possible in the ex-post planning. The development of revenues, cost, and profit in direct comparison can be found in Fig. 3.32. The overall tendency equals that of the example before (see Fig. 3.31). The profit of the rolling horizon planning for the first period is lower and the profit of the periods four and five is higher than of the ex-post planning.[71] But this is balanced with the profit of later periods. This balancing is out of scope for the rolling horizon planning, because of the study horizon of five periods. Hence, there always exists a gap between the two solutions. And this makes it difficult to identify further possible improvements.[72]

[70] The MIP gap is 5.45 %.

[71] Note that the contracted quantities of the first five periods are identical.

[72] In appendix B.11 further aspects are considered.

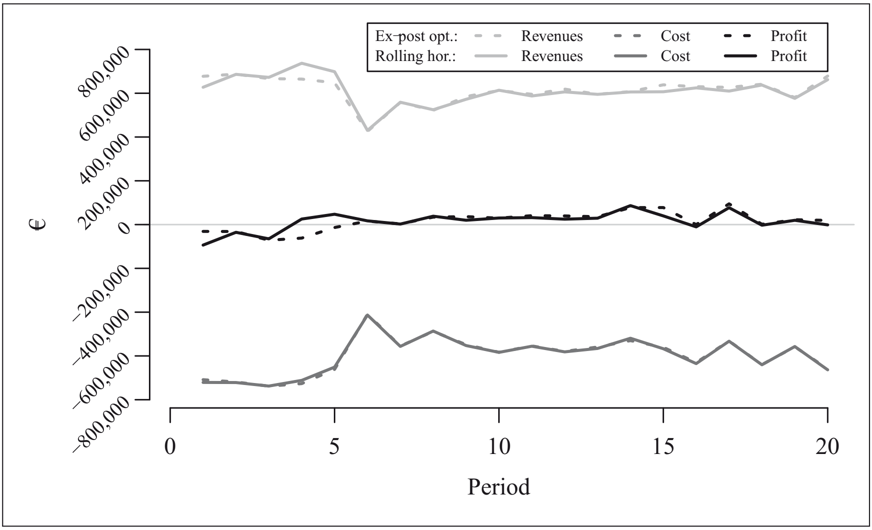

Fig. 3.32 Comparison between rolling horizon planning and ex-post solution for modified penalty factor

3.5 Concluding remarks

In this chapter we presented the complete disassembly planning considering multiple cores, which incorporates aspects of commonality across cores. We further included material purity aspects, core conditions (functionality, genuineness, and wrong material), destruction during the disassembly process, capacity limitations (e.g., workload and storage), distribution limits, and supply limits (e.g., guaranteed number of cores). The considered disassembly process not only incorporates recycling but also item distribution for reuse (either directly or after further processing) and disposal. Thereby, the disposal is further differentiated into regular and hazardous waste. The objective of the disassembly planning is the maximisation of the profit.

Starting with the analysis of the approach by Kongar / Gupta, a basic model is developed that takes the aspects of the disassembly process more detailed into account.[73] This basic model is extended to consider quantity and price or cost dependencies. Hence, a fundamental market influence is considered when determining the optimal disassembly plan. Thereby, the dependencies are either of linear or piecewise linear character to be able

[73] Cf. Kongar / Gupta (2006b): *Disassembly to order*.

to use standard solver software. In this context, a solution approach for problems with a continuous concave objective function with saltus in the first order partial derivatives is developed. The basic model as well as the extended models are static models.

A fourth approach presented in the context of the complete disassembly planning is the dynamic planning in form of the rolling horizon planning. This planning reflects especially the on-going planning with advancing periods and the inclusion of partly available future information. Furthermore, this approach is highly suited to deliver decision support for the contracting, which is usually several periods in advance to the actual period. This so-called pre-planning for the contracting is also suitable for what-if analysis in terms of utilisation and influence on the profit.

The complete disassembly considered in this chapter is not always the most profitable one. It could be that it is more profitable if not only a single disassembly sequence is applied to each unit of a core (e.g., to save disassembly cost). Moreover, this changing of sequences might be necessary if different mutual exclusive modules (i.e., consisting of the same items) are demanded of a core. In this case a more flexible planning is required, which is subject of the following chapter.

Chapter 4
Flexible disassembly planning

4.1 Introduction

This chapter introduces the considerations of disassembly depth into the planning aspects of the previous chapter. This means that not only the availability and condition of cores, the demand of items and material, and the other considered constraints need to be incorporated. With this extension that we will focus on in the following, an explicit demand of different mutual exclusive modules is considered, too. Hence, aspects of multiplicity and commonality across cores not only apply to items, but also to modules.

When integrating disassembly depth several approaches with different degrees of freedom are possible. A first is a successive approach where, in a first step, the disassembly sequencing for each type of core is determined and afterwards the quantities are planned. This approach is used by Kara / Pornprasitpol / Kaebernick and Tseng / Chang / Cheng.[1] The solution to the sequencing problem can be determined by diverse approaches like LP, clustering, graph based, etc.[2] Once the sequence is determined the disassembly planning is conducted. Here, the one optimal sequence leads to items and modules. The modules can be modelled as items, because they are not further disassembled. Hence, the approach presented in the preceding chapter can be used for this optimisation problem. The major disadvantage of this approach is the adjustment of the two objective functions (i.e., for sequencing and planning) and the multitude of constraints.

---

[1] Cf. Kara / Pornprasitpol / Kaebernick (2006): *Selective disassembly sequencing* and Tseng / Chang / Cheng (2010): *Disassembly-oriented assessment methodology*.

[2] Cf. Lambert / Gupta (2005): *Disassembly modeling*, pp. 287–335.

C. Ullerich, *Advanced Disassembly Planning*,
DOI 10.1007/978-3-658-03118-3_4, © Springer Fachmedien Wiesbaden 2014

To avoid the drawback of determining a suboptimal disassembly sequence for the subsequent disassembly planning the sequencing can be integrated in the planning such that they are determined simultaneously. Of course, this increases the complexity of the planning, but the benefit is the optimal sequence for the final objective function (e.g., profit maximisation) and all corresponding constraints. Still, as a result there exists one fix disassembly sequence for the same type of core.

A more flexible approach is when more than one disassembly sequence and depth is allowed per core. This means that when, for example, a quantity of two units of one core is acquired, one of them is disassembled with one sequence and depth and the other one with a different sequence to another depth. Usually, this flexibility is difficult to realise with automatic production systems, but in the disassembling business most is done manually.[3] Hence, the disassembly process is highly flexible and this potential should be used to gain even more profit than with the simultaneous approach.[4] As a matter of course, the planning becomes more complex, but in a business with low profit margins we deem it beneficial.

This flexible disassembly planning approach is presented in the sequel. Thereby, we only focus on a static model. As we will see, the inclusion of the aspects of disassembly depth in conjunction with the core condition leads to a far more complex problem than with complete disassembly. Nevertheless, the static flexible planning can be extended to dynamic planning using the methodology presented in the chapter above.

4.2 Flexible disassembly planning model

4.2.1 Disassembly sequencing

One of the early examples of disassembly sequencing, i.e., determining the best order of operations to separate a core into its constituent items and modules,[5] is that of the so-called Bourjault's ballpoint.[6] It consists of six items and is used to illustrate several methods to model the disassembly process. Usually graphical approaches are suitable to illustrate the structure

[3] Cf. Kopacek / Kopacek (2006): *Intelligent, flexible disassembly*, p. 554.

[4] Cf. Bley et al. (2004): *Human involvement in disassembly*, p. 487.

[5] Cf. Ilgin / Gupta (2010): *ECMPRO: A review*, p. 578.

[6] Cf. Lambert / Gupta (2005): *Disassembly modeling*, p. 171.

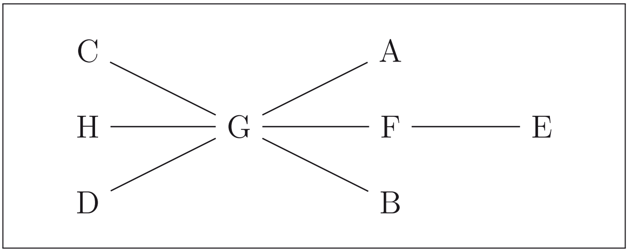

Fig. 4.1 Connection graph of forklift truck

Table 4.1 Connectivity matrix

	item							
item	A	B	C	D	E	F	G	H
A	.	.	.	.	.	.	1	.
B	.	.	.	.	.	.	2	.
C	.	.	.	.	.	.	3	.
D	.	.	.	.	.	.	4	.
E	.	.	.	.	.	5	.	.
F	.	.	.	.	5	.	6	.
G	1	2	3	4	.	6	.	7
H	.	.	.	.	.	.	7	.

A dot denotes a value of zero.

of a core. One such rather simple presentation is the connection graph.[7] It shows which items are directly connected with each other.

For our forklift truck example (see Fig. 3.4 on page 46) the corresponding connection graph (also called liaison graph)[8] is the one depicted in Fig. 4.1. This graph shows all connections between items. This does not mean that all physical connections are listed. For example, if an item is connected to another one with two screws and a tongue and groove joint we see it as one connection between the two items, even though three physical connections exist. For our considerations this level of detail is sufficient.

The front wheels A and B as well as the rear wheels C and D are connected with the chassis G. The lift unit F and engine H are also fixed to the chassis. On the contrary, the fork E is mounted on the lift unit. The graph can also be stored in a matrix with the same information (see Table 4.1). Here the connections have a unique ID one through seven. Thereby, the matrix is

[7] Cf. Lambert / Gupta (2005): *Disassembly modeling*, p. 171 and Kwak / Hong / Cho (2009): *Eco-architecture analysis for end-of-life*, p. 6252.

[8] Cf. Baldwin et al. (1991): *Computer aid for generating assembly sequences*, p. 81.

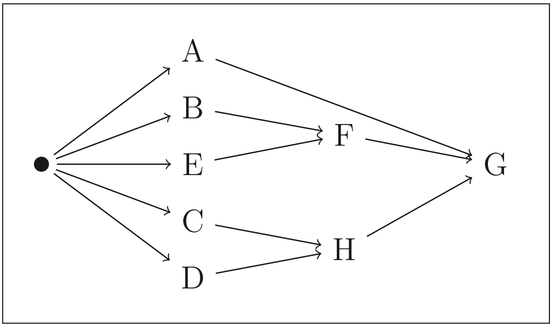

Fig. 4.2 Precedence graph of forklift truck

symmetric and the main diagonal is zero. This matrix and the connection graph are a first good way to gain an overview of the structure of the core. This represents the topological constraints on disassembling the core.[9] But they do not contain all necessary information relevant for the disassembling. For example, it might be necessary to take off the rear wheels to remove the engine. With the connection graph or matrix we know that the engine is fixed to the chassis, but we do not know that we have to take off the rear wheels first. Such precedence information, which is a geometric constraint,[10] is important and therefore needs to be stored somehow. One possibility is the precedence graph or a list of rules.[11] The graph for our example is given in Fig. 4.2. We see that first items B and E have to be removed before F can be taken off. The same applies to C, D, and H as well as A, F, H, and G.

This graph focuses on sequential disassembly. But, we do not consider only cores where the items are taken off item by item. In our example it is possible to separate the module EF from the rest. This information is better given by a list of rules. Each rule of the form "BG not F" can be interpreted as:

> *F cannot be in another module or a single item, when item B and G are together in one module.*

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[9] Cf. Lambert / Gupta (2005): *Disassembly modeling*, p. 140.

[10] Cf. Ibid., p. 154.

[11] Cf. Lambert (2006): *Optimum disassembly sequence with sequence dependent costs*, pp. 542 et seq., Lambert / Gupta (2005): *Disassembly modeling*, pp. 206 et seqq., and Kwak / Hong / Cho (2009): *Eco-architecture analysis for end-of-life*, p. 6252.

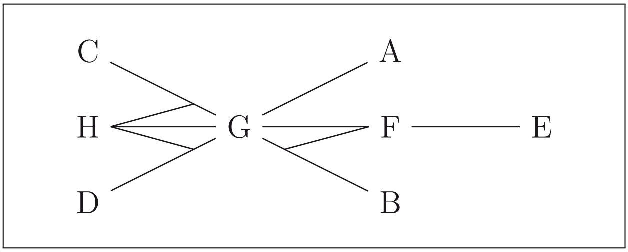

Fig. 4.3 Extended connection graph of forklift truck

The list for our example contains the following entries:

- BG not F,
- CG not H, and
- DG not H.

The reader might miss the entry EG not F as pendant to BG not F in Fig. 4.2. But, the information, that F cannot be separated unless E or G are removed, is already present in the connection graph in Fig. 4.1. Therefore, this entry could be added, but it is redundant.

For straightforward cores, like our example, this information can be integrated in the connection graph. This graph we call extended connection graph and it is given in Fig. 4.3. Compared to the connection graph (see Fig. 4.1) three lines are added. These start on the connection of two items (e.g., between B and G) and end at the item that is blocked by the connection (e.g., F). These need to be interpreted in a way that the connection of F and G can only be disconnected if the connection between B and G is broken up first. The same applies to the other two precedence lines, which represent geometric constraints for the disassembly process. A third category—the technical constraints—can be modelled in the same way as the geometric constraints.[12]

To include the disassembly sequences into the planning we need to know the possible sequences starting with the complete core. Note that we restrict ourselves to complete cores only in the planning. Since our example shows a connection graph of a star structure, the number of sequences is high compared to a linearly structured product with only one possible disassembly sequence. The sequences can be illustrated with a disassembly state graph (see Fig. 4.4).[13] It contains all combinations of items, modules and their dis-

[12] Cf. Lambert / Gupta (2005): *Disassembly modeling*, p. 183.

[13] The disassembly state graph equals the sub-assembly state graph as used in disassembly sequencing. Cf. Ibid.

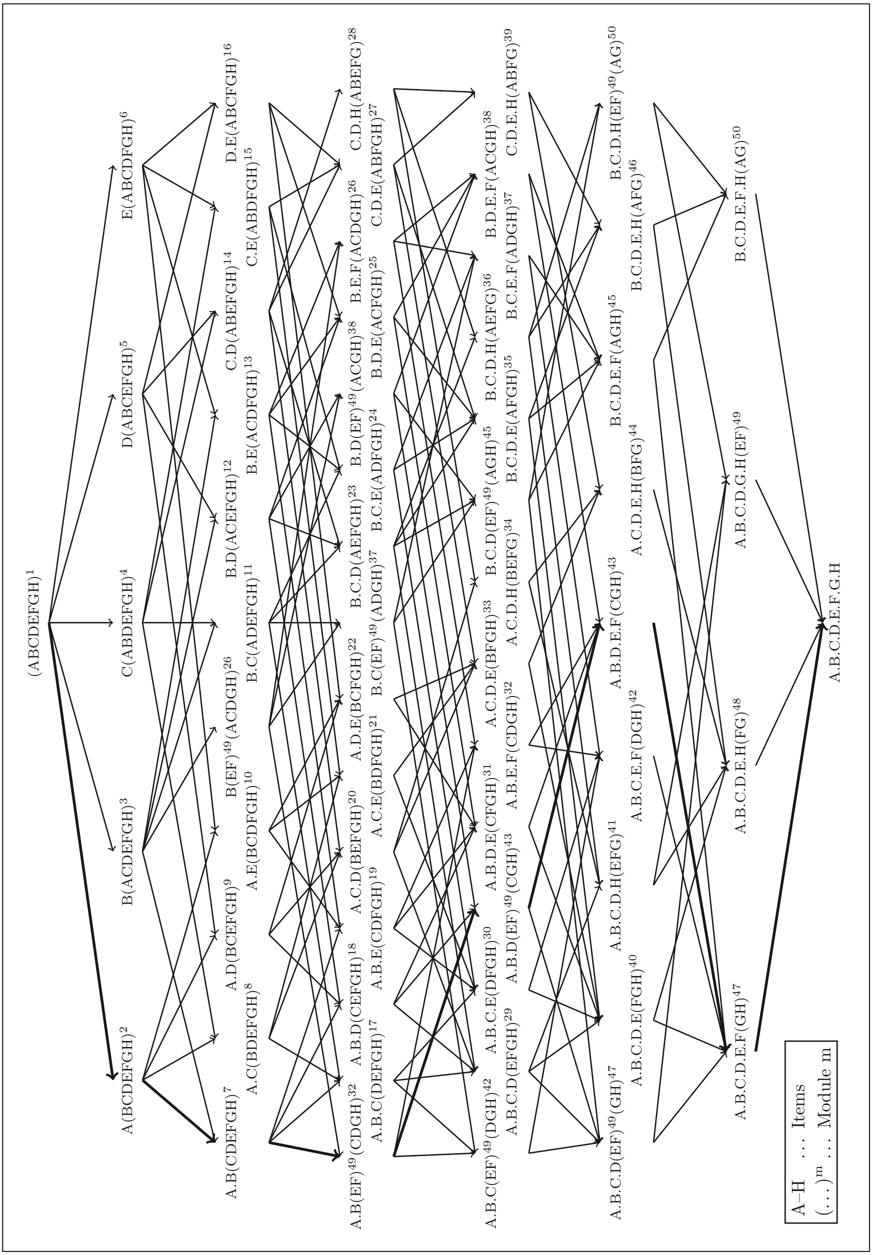

Fig. 4.4 Disassembly state graph of forklift truck

assembly path for one core. The complete core is the starting point, because it is the first module to consider. All paths start from this node and end at the node where all items are taken off, i.e., the core is completely disassembled. For the following considerations the node representing the complete core has the index 1.

In Fig. 4.4 the modules are grouped by round brackets and the already detached items separated by dots. For example a node with the label "A.B(EF)^{49}(CDGH)^{32}" represents a state of the disassembly with items A and B taken off—so that they are two single items—and modules EF and CDGH existing separately. The superscript number denotes the ID of the module, which will be used later in the modelling for the module definition matrix. In order to disassemble the core completely one possibility is the following. (The corresponding path to this sequence is highlighted by slightly thicker arrows in Fig. 4.4.) Firstly, item A is separated by disconnecting the connection 1, i.e., between items A and G (see Table 4.1). Then connection 2 is disjointed, which separates item B from the remaining module BCDEFGH. The next operation could be the cutting of connection 6, which leaves module EF intact but separated from CDGH. After disconnecting the remaining connections 4, 5, 3, and 7 the core is completely disassembled.

But how is this graph generated from the information we have? Starting from module one, all connections are tried to disconnect. Within the first module all seven connections still exist. When separating connection 1 item A is taken off and the module BCDEFGH remains. Checking with the geometric (and technical) constraints BG not F, CG not H, and DG not H we see that none of them applies. Hence the first node in the second level is created. Now the remaining graph can be created breadth or depth first. When we choose breadth first, we try to disjoin the next connection, e.g., 2. This leaves item B separated and the module ACDEFGH. This combination is feasible and the next node is created. This repeats until we reach connection 6. Here a combination of two modules ABCDGH and EF would be generated. Checking the geometric constraints, especially BG not F, we notice that items B and G appear together in module ABCDGH. If this occurs, item F must not appear as single item or in another module, like EF. Therefore, this combination is infeasible and no node is created. The same applies to connection 7. The topological constraints are not directly considered, because we use the connections for generating nodes and not arbitrary item and module combinations. Hence, the topological information coded in the connectivity matrix is directly applied.

Once the second level of the graph is completed, we can continue with generating the third. Therefore, we choose one node of the preceding level, e.g., A(BCDEFGH), check what connections are still intact—all but connection 1—and try to create nodes based on the separation of one connection. At most, six nodes could be created out of that one node with six connections intact. Only four of the possible six nodes are created and we continue with developing the next nodes based on the second node of the preceding level. We notice that we would create nodes that already exist. For example, the separation of connection one and then two leads to the same result as first two and then one. Of course, the different sequences might lead to differences in terms of disassembly time and cost, but since the work is done mostly manually we assume that the sequence is of marginal influence and is therefore neglected. In addition, the condition of the core or connections (like corroded fasteners) leads to a variation of the disassembly time and cost, which might have a greater influence than the actual sequence. Since, the result of the sequence is more of interest than the sequence itself, the two sequences come together in one node, i.e., A.B(CDEFGH). This procedure continues until the complete graph is created, i.e., level eight with a node representing the complete disassembly is reached.

The disassembly state graph contains one more level than connections in a core exist. The upper bound of the number of nodes per level can be calculated based on combinations without repetition. For this bound we assume a core without any geometric or technical constraints. Then, in the second level we could create seven nodes. From these seven nodes we could create $\frac{7 \cdot 6}{2} = 21$ nodes. The successive levels four through eight could be created with 35, 35, 21, seven, and one nodes, respectively. Thereby, the number of nodes for an arbitrary number of connections n per level k is limited by

$$\binom{n}{k-1} = \frac{n!}{(n-k-1)!(k-1)!} \,. \tag{4.1}$$

Thus, the number of nodes of the complete graph (from level one through $n+1$) is limited by:[14]

$$\sum_{k=1}^{n+1} \binom{n}{k-1} = 2^n \,. \tag{4.2}$$

For our example, this means that $2^7 = 128$ is the upper limit of nodes—not considering the geometric constraints. With the geometric constraints the graph results in a size of 60 nodes. Definitively, a better bound could be found, but this is not subject of matter in this work. More important is the number of modules, which are incorporated in the disassembly

[14] The binomial theorem states $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$. Cf. Gellert et al. (1965): *Kleine Enzyklopdie – Mathematik*, p. 45. Substituting $a = b = 1$ we get $2^n = \sum_{k=0}^{n} \binom{n}{k} = \sum_{k=1}^{n+1} \binom{n}{k-1}$.

planning later. The number of different modules in our example is 50 (see Fig. 4.4). Note that a single item is no module. A graph only containing the possible modules is the so-called and/or graph.[15] For our example the and/or graph is depicted in Fig. 4.5.

As in the disassembly state graph the complete core is the top most node. When removing connection 1 item A is separated from the remaining module BCDEFGH. The module is represented by a node in the graph, whereas the single items are not displayed at all. In Fig. 4.5 the path is highlighted that corresponds to the highlighted path in Fig. 4.4. Disjoining connection 2 reduces the module by item B to CDEFGH. Now connection 6 is disconnected and two modules result. One is CDGH and the other is EF. This is an and-relationship, because module CDGH "and" EF result from this operation.[16] Alternatively, which means "or", the module CDEFGH can be reduced to DEFGH, CEFGH, or CDFGH. In each level of the graph the modules with the same length are listed starting from the biggest (with eight items) down to the smallest (with two items). Hence, the and-operation causes a skip of several levels in the graph. We follow the dashed line from node number seven to node number 32 and 49. Node CDGH is stepwise reduced to CGH and GH. In the end both two-item modules GH and EF are separated to single items, which is the bottom of the graph. This graph has as many nodes as modules exist, which is usually less than the number of nodes of the disassembly state graph. In our example this is 50 (module) vs. 60 (state) nodes.

Both graphs are suitable to determine the number and composition of all modules, depending on the items and all relevant constraints. Nonetheless, both graphs are rather big, even for our fairly small example. The good news is, that for the planning the modules with their consisting items are necessary and not the graph as such. This module composition information can be generated automatically (i.e., computer based) based on the connectivity matrix and the constraints. Afterwards, it can also be stored in a different way than as a graph. For controlling purposes the graph is a nice tool. The bad news is, that in a worst case the number of modules still increases exponentially with the number of items. In appendix C.1 a comparison between a linear and a star structured core can be found, which illustrates the lower and upper bound of the resulting graph sizes.

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[15] Cf. Lambert / Gupta (2005): *Disassembly modeling*, pp. 147 et seqq. and 187.

[16] In the example only one connection at each node causes a module split and therefore it is sufficient to indicate the and-relationship with dashed arrows.

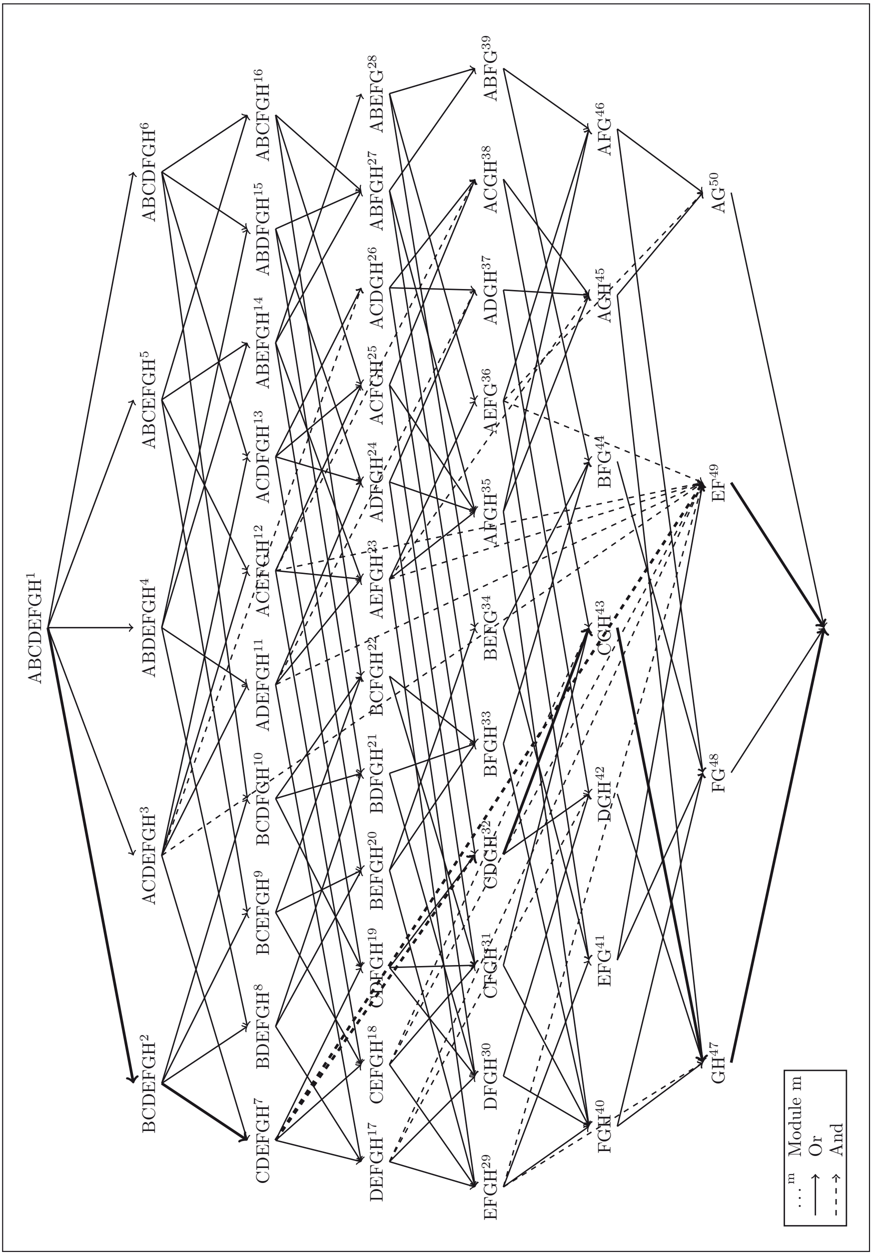

Fig. 4.5 And/Or graph of forklift truck

To find an optimal disassembly sequence several approaches exist. They can be based on petri nets,[17] component fastener graphs,[18] and/or graphs,[19] and even linear programming.[20] The graph based approaches are rather improper to incorporate them with a simultaneous planning with an LP. The linear programming is a good start, but the approaches found in the literature determine only one sequence per type of core. We want to be more flexible with our approach, i.e., we want to be able to determine more than one sequence or depth per core for the planning. One approach realising this requirement is presented in the sequel.

To integrate the module information in our planning approach two matrices (or arrays) are necessary (see Table 4.2). One contains the module definition information, i.e., what items a module consists of. For example, module $m = 37$ consists of the items A, D, G, and H. Thus, the values in the module definition matrix δ_{cmi} equal one if this module contains the item and zero if not. Hence, we observe $\delta_{c,37,\mathrm{A}} = \delta_{c,37,\mathrm{D}} = \delta_{c,37,\mathrm{G}} = \delta_{c,37,\mathrm{H}} = 1$ for an arbitrary core c and $\delta_{c,37,\mathrm{B}} = \delta_{c,37,\mathrm{C}} = \delta_{c,37,\mathrm{E}} = \delta_{c,37,\mathrm{F}} = 0$. The index c is already introduced, because this information is core specific.

The definition of the core is clear. Taking a look at the disassembly state graph (see Fig. 4.4) we observe that module $m = 37$ exists twice in the graph. On the one hand with items B and C as well as module EF and on the other hand with the single items B, C, E, and F. We notice that whenever module $m = 37$ appears in the graph items B and C are always single items along with it. Therefore, they always appear in addition to the module. That is why we call them additional items. Items E and F are not always additional, because they could be kept together as a module EF. This module $m = 49$ itself appears several times in the graph. And the only item it always appears with is item B, which is the only additional item to module EF.

To illustrate the importance of the additional items matrix let us consider a different example. Let a core consist of four items A–D and we assume that the disassembly can only be done in two ways. One way is starting with A, than B, and lastly C and the second way is starting with D, than C, and lastly B. From the first way the modules BCD and CD result. Starting the other way, modules ABC and AB can be gained. In the mathematical programming a decision variable that represents the quantity

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[17] Cf. Moore / Güngör / Gupta (19998): *Petri net approach to disassembly planning*.

[18] Cf. Zhang / Kuo (1997): *Graph-based disassembly sequence planning*.

[19] Cf. Lambert (2002): *Optimum disassembly sequences*.

[20] Cf. Lambert (1999): *Linear programming in disassembly* and Lambert (2006): *Optimum disassembly sequence with sequence dependent costs*.

Table 4.2 Module definition and additional item matrix

module	module definition matrix δ_{cmi}								additional item matrix α_{cmi}							
	item								item							
m	A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H
1	1	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.
2	.	1	1	1	1	1	1	1	1	.	.	.	.	.	.	.
3	1	.	1	1	1	1	1	1	.	1	.	.	.	.	.	.
4	1	1	.	1	1	1	1	1	.	.	1	.	.	.	.	.
5	1	1	1	.	1	1	1	1	.	.	.	1	.	.	.	.
6	1	1	1	1	.	1	1	1	.	.	.	.	1	.	.	.
7	.	.	1	1	1	1	1	1	1	1	.	.	.	.	.	.
8	.	1	.	1	1	1	1	1	1	.	1	.	.	.	.	.
9	.	1	1	.	1	1	1	1	1	.	.	1	.	.	.	.
10	.	1	1	1	.	1	1	1	1	.	.	.	1	.	.	.
11	1	.	.	1	1	1	1	1	.	1	1	.	.	.	.	.
12	1	.	1	.	1	1	1	1	.	1	.	1	.	.	.	.
13	1	.	1	1	.	1	1	1	.	1	.	.	1	.	.	.
14	1	1	.	.	1	1	1	1	.	.	1	1	.	.	.	.
15	1	1	.	1	.	1	1	1	.	.	1	.	1	.	.	.
16	1	1	1	.	.	1	1	1	.	.	.	1	1	.	.	.
17	.	.	.	1	1	1	1	1	1	1	1	.	.	.	.	.
18	.	.	1	.	1	1	1	1	1	1	.	1	.	.	.	.
19	.	.	1	1	.	1	1	1	1	1	.	.	1	.	.	.
20	.	1	.	.	1	1	1	1	1	.	1	1	.	.	.	.
21	.	1	.	1	.	1	1	1	1	.	1	.	1	.	.	.
22	.	1	1	.	.	1	1	1	1	.	.	1	1	.	.	.
23	1	.	.	.	1	1	1	1	.	1	1	1	.	.	.	.
24	1	.	.	1	.	1	1	1	.	1	1	.	1	.	.	.
25	1	.	1	.	.	1	1	1	.	1	.	1	1	.	.	.
26	1	.	1	1	.	.	1	1	.	1	.	.	.	.	.	.
27	1	1	.	.	.	1	1	1	.	.	1	1	1	.	.	.
28	1	1	.	.	1	1	1	.	.	.	1	1	.	.	.	1
29	.	.	.	.	1	1	1	1	1	1	1	1	.	.	.	.
30	.	.	.	1	.	1	1	1	1	1	1	.	1	.	.	.
31	.	.	1	.	.	1	1	1	1	1	.	1	1	.	.	.
32	.	.	1	1	.	.	1	1	1	1	.	.	.	.	.	.
33	.	1	.	.	.	1	1	1	1	.	1	1	1	.	.	.
34	.	1	.	.	1	1	1	.	1	.	1	1	.	.	.	1
35	1	.	.	.	.	1	1	1	.	1	1	1	1	.	.	.
36	1	.	.	.	1	1	1	.	.	1	1	1	.	.	.	1
37	1	.	.	1	.	.	1	1	.	1	1	.	.	.	.	.
38	1	.	1	.	.	.	1	1	.	1	.	1	.	.	.	.
39	1	1	.	.	.	1	1	.	.	.	1	1	1	.	.	1
40	.	.	.	.	.	1	1	1	1	1	1	1	1	.	.	.
41	.	.	.	.	1	1	1	.	1	1	1	1	.	.	.	1
42	.	.	.	1	.	.	1	1	1	1	1	.	.	.	.	.
43	.	.	1	.	.	.	1	1	1	1	.	1	.	.	.	.
44	.	1	.	.	.	1	1	.	1	.	1	1	1	.	.	1
45	1	.	.	.	.	.	1	1	.	1	1	1	.	.	.	.
46	1	.	.	.	.	1	1	.	.	1	1	1	1	.	.	1
47	.	.	.	.	.	.	1	1	1	1	1	1	.	.	.	.
48	.	.	.	.	.	1	1	.	1	1	1	1	1	.	.	1
49	.	.	.	.	1	1	.	.	.	1	.	.	.	.	.	.
50	1	.	.	.	.	.	1	.	.	1	1	1	.	.	.	1

A dot denotes a value of zero.

of the corresponding modules is introduced for every module. Assuming a quantity of four units of a core and considering only the module definition matrix to check whether the items of the modules are disjoint, we could find a solution like: one module ABC, two modules AB, three modules CD, and the rest as single items. But even though the items in the modules AB and CD are disjoint, they cannot be gained at the same time out of a single core. This information is not stored in the module definition matrix. The additional item matrix for such a module AB would contain the information that whenever module AB is gained items C and D appear as single items. When gaining two modules AB automatically two items of C and D are gained or have to be gained. Adding another module ABC with D as single item only one further module CD can result from the fourth unit of a core with the single items A and B. Hence, the solution given above ($1 \times$ ABC, $2 \times$ AB, $3 \times$ CD) is infeasible.

4.2.2 Model extension

4.2.2.1 Objective function and item flow, purity, and limits constraints

The incorporation of the disassembly sequence generation into the existing basic model (see Sect. 3.1) with all its aspects of core condition, demand of material and items, core availabilities, damaging rates, etc. does not require a structural change of the model. It is rather an extension to the complete-disassembly model with changes in the objective function as well as in the constraints. The model structure is displayed in Fig. 4.6. Thereby, in the figure the condition limitation as in Fig. 3.2 is skipped, because they will be developed in the sequel. In comparison with the basic model the extensions, i.e., Y^{M}, Y^{R}, Y^{D}, D^{M}, and Q^{M}, become evident and will be discussed in the following.

When including the distribution of modules in addition to the distribution of items to reuse the resulting revenues of the modules must be added to the already considered revenues of items. The second category influencing the profit directly is the cost. These need to be adapted, too. In comparison to the complete disassembly we can now save cost (and time) to disassemble particular modules further into items. This is possible, if modules are demanded, if all consisting items of a module could be placed into the same recycling box, or if all consisting items of a module would be disposed of. In these cases it is beneficial to save the cost of disassembling.

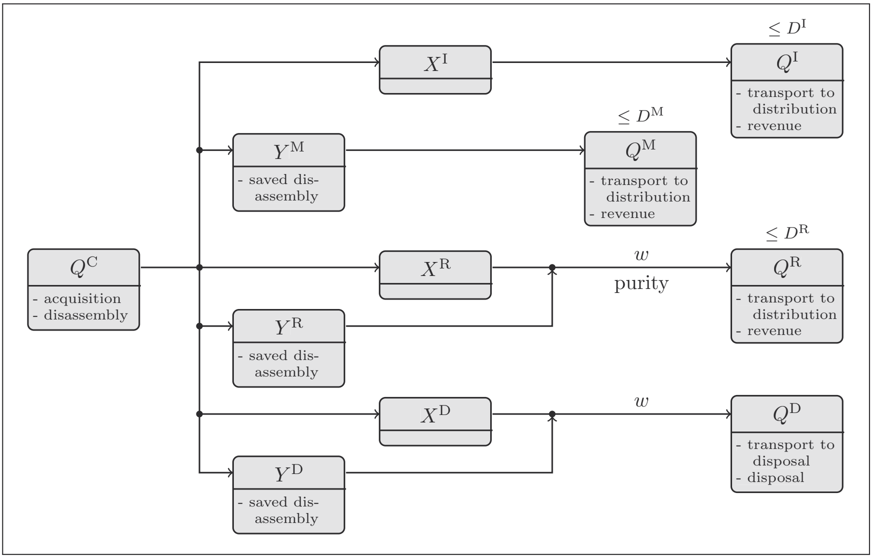

Fig. 4.6 Model structure for flexible disassembly

To determine the cost savings the disassembly cost for each possible module is required. But for the cost determination it is not necessary to disassemble a core into all its possible modules. It is sufficient to evaluate each connection that needs to be separated in order to disassemble the core. Of course, this might not always represent the correct cost, but should give a good estimation. (It is doubtful that precise cost can be given, because of manual labour and variations caused by the condition of the item connections.) For all known special cases, the disassembly cost for the module could be modified afterwards, based on the before calculated values.

Taking a core of four items A through D and assuming that all possible module combinations of a star configuration exist (with D being the centre) then three connections hold the core together (see Fig. C.3). Let us assume that separating the connection AD, BD, and CD costs 1, 2, and 3 €, respectively. The cost for disassembling the complete core is $1 + 2 + 3 = 6$ €, because all three connections have to be separated. The separation of item A causes cost of 1 €. The disassembly of the remaining module BCD would cost $2 + 3 = 5$ € and so on. When we extend this to more than one unit per core, the following example illustrates the determination of the cost.

Three units of a core are acquired. One unit of those three is disassembled to module BCD, one to module AD, and the third to module BD. This means that the connections BD and CD (first unit), AD (second unit), and BD (third unit) are still intact. The disassembly cost of the first unit is 1 € (separating AD), of the second $2 + 3 = 5$ € (separating BD and CD), and of the third $1 + 3 = 4$ € (separating AD and CD). In total the cost is 10 € for the three units. The same can be calculated by subtracting the savings from the complete disassembly cost. The savings of the first unit are $2 + 3 = 5$ €, because the connections BD and CD are still intact in the module BCD. Subtracting the savings from the complete disassembly cost of 6 € the cost of 1 € results. The same applies to the remaining two units. In the mathematical model we use variables representing the quantities of the modules still intact. This means that we would have three units acquired, i.e., $Q^{\mathrm{C}}_{c} = 3$. The complete disassembly cost would be three times 6 €, which is 18 €. On the other hand, one unit of module BCD, one of AD, and one of BD are kept. Subtracting the saved cost (one time 5 €, one time 1 €, and one time 2 €) from the 18 € results in the disassembly cost of 10 €.

Continuing the comment from above about different disassembly cost the illustrative example could be modified as follows. Let us assume that the separation of item C from the core when connections AD and BD still exist is more difficult and thus results in higher disassembly cost. Disassembling the core completely still leads to cost of 6 €, because we can freely decide to start with the connection AD, which is cheaper. Separating connection CD from the complete core causes for example 3.5 € (instead of the 3 €), when connections AD and BD are intact. Since there exist no other (cheaper) way to get module ABD, the cost saving for this core is set to 2.5 € instead of 3 €. Note that the cost to disassemble module ABD completely still is 3 €, but the savings are reduced to 2.5 €. Out of module ABD the modules AD and BD can be generated. But, both these modules can also be gained by other disassembly sequences, which are cheaper. For instance, when first separating item A and then C the module BD is gained. Module AD is also the result of separating first item B and then C. In both cases the expensive separation of CD is avoided. Hence, the saved costs for the modules AD and BD are still 1 € and 2 €, respectively. This already indicates that it is assumed to take the cheapest disassembly sequence to the desired disassembly depth.

Problematic is the case when a connection separation, which leads to more than one separate modules, is more expensive. An example can be found in the disassembly state graph in Fig. 4.4 from node “B(ACDEFGH)” to node “B(EF)(ACDGH)”. The problem is that the modules EF and ACDGH can be found in other nodes as well. Hence, an individual cost saving value cannot be set to the node “B(EF)(ACDGH)”, because there exists only one value for a module independent of its occurrence in any node. To be able to model such cases the model has to be modified, which might be a possibility for future research.

With regard to the flow of items and modules through the disassembly process, the corresponding constraints have to be extended. For one, the complete cores going into to disassembly process are the source of items that can be distributed, recycled, and disposed of. Now they can also be kept as modules. Nonetheless, the quantity of items in modules and as single items—regardless if distributed, recycled, or disposed of—must equal the quantity of items acquired by the cores. The quantity (i.e., the weight) of material in the recycling boxes and disposal bins is determined by the quantity of items and modules together with their weight. A new aspect regarding the item and module flow is the additional items that appear along with certain modules. The necessary information is given by the additional item matrix. Depending on the quantity of modules, at least this many additional single items have to exist, which is indicated by the additional item matrix. Besides the commonality and multiplicity aspects for items, these aspects also apply to modules.

In the recycling boxes only the beneficial weight of all items is of interest, whether it comes from single items or modules does not matter. Hence, a simple extension by the quantity of modules in the corresponding recycling box is necessary. The limitation to hazardous items to be either disposed of or distributed if demanded also applies to the modules. This means that as soon as one hazardous item exists in a module, the complete module is treated hazardous. Furthermore, the lower distribution limits as well as the demand for certain modules is given. And—as with the savings of the disassembly cost—every kept module saves disassembly time, which decreases the workload required for the disassembly. The complete mathematical model is presented later in Sect. 4.2.3.

4.2.2.2 Core condition including superordinate modules only

The extension to consider the condition of the core and its consisting items in the planning with modules requires some explanation. The general assumption for the presented approach is the knowledge about the condition of the cores, i.e., which fractions to expect (see Fig. 3.3). Thereby, we assume independent probabilities of the condition of each individual item. This allows us to plan the optimal quantities given the probabilities. Note that when it comes to disassembling a particular unit of a core, the condition of the consisting items needs to be determined in advance to select the correct disassembly depth such that the desired quantities of output are generated in the end. This is different to the complete disassembly where the items can be checked afterwards. But with sophisticated test routines, experience, and the integration of, for example, RFID into the products the information about the condition of a core increases. Approaches like that of Ondemir / Gupta go into this direction, too.[21]

Now with considering modules, the company needs to set policies on how to handle modules with items and their conditions. For example, a module might contain a non-genuine item which is still fully functioning with the remaining items of the module. The question is: should the module be recycled, disposed of, further disassembled, or can it be distributed as functioning module. Also, when an item of a module is non-genuine and consists of the wrong material, does the complete module has to be disposed of or can it be recycled, etc. In the sequel, we keep the policies rather strictly. This means, that only modules consisting of genuine and functioning items can be distributed for reuse. Furthermore, as soon as a module contains one non-genuine item of the wrong material, the module has to be disposed of. The same applies to hazardous items in modules, i.e., as soon as one hazardous item exists in a module it can either be distributed if demanded or has to be disposed of as hazardous waste.

The fourth aspect with regard to the handling of items is the damaging during the disassembly process. This is still relevant, but items enclosed by surrounding items are most likely not damaged. But, the surrounding items of a module might be damaged during the disassembly process as single separated items might, too. Since the information—of which item is a surrounding one and which not—is not given in our approach, we assume that only the single items that are separated from the modules are damaged during the disassembly process and all items forming a module stay undamaged. Of course, the property that a module is genuine, functioning, or recyclable is independent of this. With regard to the single items nothing changes compared to the basic model with respect to handling based on the condition and damaging. Let us develop the relevant constraints in the following.

The number of modules to distribute is limited to those modules that are genuine and functioning. Hence, all consisting items must be genuine and functioning. Damaging an item in the resulting module during the disassembly process is not considered in this approach. Let us further assume—independent of the otherwise given data—that a demand for module CDGH ($m = 32$) and DGH ($m = 42$) exists. In addition, item C is genuine and functioning with a probability of 80 % and 62.5 %, respectively.

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[21] Cf. Ondemir / Gupta (2011): *Optimal planning for sensor-embedded EoL products.*

This means that item C is genuine and functioning at the same time with a probability of $80\,\% \cdot 62.5\,\% = 50\,\%$. Furthermore, for item D the same values apply and the items G and H are always functioning and genuine (in the sequel when we speak of functioning we mean genuine and functioning at the same time). Given a quantity of 100 units of a core, 50 units of item C and D as well as 100 units of item G and H are functioning, when considered separately. Combining the items into a module reduces the number of modules that are functioning as a whole. For example, a functioning module CDGH is expected only in $50\,\% \cdot 50\,\% \cdot 100\,\% \cdot 100\,\% = 25\,\%$ of the cases, i.e., 25 units out of 100. The module DGH is expected to be functioning in 50 % of the cases, because the 50 % chance of item C being non-functioning or non-genuine is excluded. We see that in total 25 functioning units of module CDGH and 50 units of module DGH can be gained out of the 100 units of the core. Hence, the probability for the single items is to be multiplied to get the required combined probability of the module, when we assume that the individual probabilities are independent of each other.

In fact, when we decide to take the 25 units of functioning modules CDGH, we cannot take another 50 units of module DGH as functioning. Only further 25 units of module DGH can be gained as functioning, because the other 25 units are part of module CDGH where the functioning part DGH is still combined with a functioning item C. Therefore, the quantity of modules where the focussed module is part of must be considered in the determination of the quantity limit for a module. The modules the focussed module is part of we call superordinate. For our small example here, the two restrictions would be that at most

- 25 units of module CDGH (i.e., $Y^{\mathrm{M}}_{c,32} \leq 25$) and
- 50 units of module CDGH together with module DGH (i.e., $Y^{\mathrm{M}}_{c,32} + Y^{\mathrm{M}}_{c,42} \leq 50$)

can be generated. The information that a module is superordinate can be easily detected in the module definition matrix. All entries $\delta_{c\tilde{m}i}$ of a row representing a superordinate module $\tilde{m}$ to module m must be greater or equal than δ_{cmi}, i.e., $\delta_{cmi} \leq \delta_{c\tilde{m}i}\ \forall\, i$. Note that $\tilde{m} \neq m$, i.e., a module is not superordinate to itself. Taking a look at Table 4.2 in row $m = 42$ of the module definition matrix we find the values $\delta_{c,42,i} = (00010011)$. The values of row 32 are (00110011). We notice that the third element of the row of $m = 32$ is greater than that of $m = 42$ and all other elements are equal. Thus, module $m = 32$ is superordinate to module $m = 42$. But not only module $m = 32$ is superordinate. The modules 37, 32, 30, 26, 24, 21, 19, 17, 15, 13, 11, 10, 8, 7, 6, 4, 3, 2, and 1 are superordinate, too. When excluding the decision variables of those modules that are not demanded at all, only the modules of the corresponding core in the sets $\mathcal{R}_f$ are relevant. $\mathcal{R}_f$ is the pendant to $\mathcal{P}_e$ for demanded modules.

$$\sum_{\tilde{m} \in \left\{ \tilde{m} \,\middle|\, \substack{\delta_{cmi} \leq \delta_{c\tilde{m}i} \ \forall\, i, \\ (c,\tilde{m}) \in \bigcup_f \mathcal{R}_f} \right\}} Y^{\mathrm{M}}_{c\tilde{m}} \leq \prod_{i \in \{i | \delta_{cmi}=1\}} (1-\zeta_{ci})(1-\eta_{ci}) Q^{\mathrm{C}}_c \quad \forall\, (c,m) \in \bigcup_f \mathcal{R}_f \tag{4.3}$$

Depending on the choosing of the quantities of the modules to distribute, the quantity of functioning items for distribution is influenced in the same way as explained above. From the small example above, we remember that item C as well as D are functioning in 50 % of the cases and items G and H always. When selecting 25 units of module CDGH and 25 units of module DGH, no more functioning modules are expected. This leaves us with 25 units of item C and 50 units of item D from the 100 possible. We notice that still 25 units of item C are functioning and could therefore be used for item distribution. But, when disassembling items we included a possibility that this item is damaged. The corresponding rate is denoted by θ_{ci}. Let us further assume the damage rate for item C in the process is 40 %. In this case only 60 % of the 25 reamining functioning items are undamaged after the disassembly process. This makes 15 items C to be used for distribution in addition to the selected modules.

To include this in our model formulation we develop the constraint step by step as motivated above. The quantity of items to be distributed (X^{I}) is limited by the totally available genuine $(1-\zeta)$ and functioning $(1-\eta)$ items in the quantity of cores (Q^{C}) less the genuine and functioning items in the superordinate modules to be distributed (Y^{M}).[22] From those remaining items a fraction of θ is damaged in the disassembly process, i.e., $X^{\mathrm{I}} \leq \big((1-\zeta)\,(1-\eta)\,Q^{\mathrm{C}} - Y^{\mathrm{M}}\big)\,(1-\theta)$. Transforming this expression leads to $X^{\mathrm{I}} + (1-\theta)\,Y^{\mathrm{M}} \leq (1-\zeta)\,(1-\eta)\,(1-\theta)\,Q^{\mathrm{C}}$. This consideration is only necessary for items that are demanded (i.e., elements of $\mathcal{P}_e$). The relevant modules are also only the demanded ones (i.e., elements of $\mathcal{R}_f$), because all other quantity variables Y^{M}_{cm} will be set to zero (see Eq. (4.88)). After adding the indices and the necessary confinements the constraint results in

$$X^{\mathrm{I}}_{ci} + (1-\theta_{ci}) \sum_{m \in \left\{ m \,\middle|\, \substack{\delta_{cmi}=1, \\ (c,m) \in \bigcup_f \mathcal{R}_f} \right\}} Y^{\mathrm{M}}_{cm} \leq (1-\zeta_{ci})(1-\eta_{ci})(1-\theta_{ci}) Q^{\mathrm{C}}_c \quad \forall\, (c,i) \in \bigcup_e \mathcal{P}_e\,. \tag{4.4}$$

[22] Superordinate means here that the module contains the item.

In the basic model constraint (3.9) assured that all items, which are non-genuine and of the wrong material, are disposed of. This was modelled by a lower bound of X^{D}_{cid} in relation to Q^{C}_{c}. The same information can be modelled by giving an upper bound for X^{I}_{ci} together with X^{R}_{cir} in relation to Q^{C}_{c}, because if an item is not disposed of, it is either distributed or recycled. This view is adopted in the sequel to limit the quantity of modules and items for distribution and recycling, because of items that are non-genuine and of the wrong material. In general, the constraint is developed like in Eq. (4.3).

Let us assume a core consisting of three items A, B, and C. With a probability of $P(A) = 0.6$ item A is non-genuine and of the wrong material. This value is already the combination (i.e., multiplication) of the probabilities ζ_{ci} and ι_{ci} for some arbitrary core c and item A. The corresponding probabilities for item B and C are $P(B) = 0.2$ and $P(C) = 0.4$. The probability that the complete core is non-genuine and of the wrong material, i.e., all three items are non-genuine and of the wrong material, is $0.6 \cdot 0.2 \cdot 0.4 = 0.048$. On the other hand, the probability that all three items in the core are non-genuine and recyclable or genuine (i.e., the total opposite case) is $(1 - P(A))(1 - P(B))(1 - P(C)) = 0.4 \cdot 0.8 \cdot 0.6 = 0.192$. Many more combinations of item conditions exist where only one or two items are of wrong material. These combinations have in common that at least one item is non-genuine and of wrong material for recycling. According to the selected policy (see section above), as soon as there exists one wrong material item in a module the module has to be disposed of. Hence, with a probability of $1 - 0.192 = 0.808$ the module has to be disposed of. In other words, only with a probability of 0.192 a module of the items A, B, and C can be used for distributing or recycling.

When an item (e.g., C) is separated from the module ABC the module AB remains. With a probability of $(1 - P(A))(1 - P(B)) = 0.4 \cdot 0.8 = 0.32$ this item combination can be used for distributing or recycling, because none of the two items is non-genuine and of the wrong material. This probability calculation can be continued down to the single items. When we acquire 250 units of a core, we expect a fraction of 0.192 of them to be usable for distribution or recycling, because of the above calculated probability. This means we expect $250 \cdot 0.192 = 48$ cores. If we disassemble all 250 cores into modules AB and a single item C, the number of usable modules AB increases to $250 \cdot 0.32 = 80$, because the probability of item C has no effect on this module AB anymore. Of the 250 single items C, $250 \cdot (1 - P(C)) = 250 \cdot 0.6 = 150$ are usable. So we observe the effect, that the quantity of modules that do not have to be disposed of increases with smaller modules, i.e., the more the cores are disassembled.

Table 4.3 Flexible disassembly policy

	module quantity					item quantity					
						item A		item B		item C	
disassmbly depth	max. distribution & recyling (only this module)	max. distribution & recyling (with superord. modules)	selected for distribution & recycling	unused for distribution & recycling	disposal	selected for distribution & recycling	disposal	selected for distribution & recycling	disposal	selected for distribution & recycling	disposal
(ABC)	48	48	45	3	20	—	—	—	—	—	—
A(BC)	120	75	50	25	40	5	85	—	—	—	—
B(AC)	60	15	15	0	10	—	—	5	20	—	—
C(AB)	80	35	30	5	25	—	—	—	—	40	15
A.B.C	—	—	—	—	—	0	15	15	0	0	15
	max. for distribution & recycling					100		200		150	
	selected for distribution & recycling					95		145		150	
	unused for distribution & recycling					5		55		0	

So far we considered the same treatment for all 250 units of a core. But this is not flexible. In order to be a flexible disassembly policy with several disassembly depths—which we consider here—a policy like the following must be feasible (see Table 4.3). The disassembly depth is given in the style of the disassembly state graph, i.e., modules are in braces and several items separated by dots. 65 of the 250 cores are not disassembled at all. 45 of them are intended for recycling (distribution and recycling are subsumed to recycling in the sequel) and 20 for disposal. This leaves three units open until the limit of 48 is reached. The limit is given by the probability that the module ABC is free of any item that is non-genuine and of wrong material. In addition, 90 units of the core are disassembled such that item A is separated and module BC remains. From these 90 units, 50 are intended for recycling, while the limit for recycling is 75 units. If no module ABC is selected for recycling, the 120 units will be the limit. With the 45 units of ABC also 45 units of module BC with recyclable or genuine items are gone, because BC is a subset of ABC.

With this 90 units of module BC, 90 units of item A are generated. Depending on the condition and appearance in other modules these 90 units need to be split into items for recycling and disposal. We select five units for recycling and 85 for disposal. This continues with 15 and 30 units selected

Table 4.4 Flexible policy – solution

$Y^{\mathrm{R}}_{\mathrm{ABC}} = 45$	$Y^{\mathrm{D}}_{\mathrm{ABC}} = 20$	$X^{\mathrm{R}}_{\mathrm{A}} = 5$	$X^{\mathrm{D}}_{\mathrm{A}} = 100$
$Y^{\mathrm{R}}_{\mathrm{BC}} = 50$	$Y^{\mathrm{D}}_{\mathrm{BC}} = 40$	$X^{\mathrm{R}}_{\mathrm{B}} = 20$	$X^{\mathrm{D}}_{\mathrm{B}} = 20$
$Y^{\mathrm{R}}_{\mathrm{AC}} = 15$	$Y^{\mathrm{D}}_{\mathrm{AC}} = 10$	$X^{\mathrm{R}}_{\mathrm{C}} = 40$	$X^{\mathrm{D}}_{\mathrm{C}} = 30$
$Y^{\mathrm{R}}_{\mathrm{AB}} = 30$	$Y^{\mathrm{D}}_{\mathrm{AB}} = 25$		

for recycling of module AC and AB, respectively, as well as ten and 25 units for disposal of module AC and AB, respectively. In total 235 modules with their corresponding items are disassembled, which leaves 15 units of the core to be disassembled completely. The resulting single items need to be allocated to recycling or disposal, which can be seen in the row A.B.C in Table 4.3.

95 of the 100 recyclable or genuine items A (from the 250 units) are included in 45 units of module ABC, 15 units of AC, and 30 units of AB and five further units are selected parallel to module BC. This leaves five units of the 100 items unused for recycling. The same applies to item B and C with 55 and zero units unused for recycling. Transforming the policy into values for decision variables results in the following solution (see Table 4.4). Thereby, $Y^{\mathrm{R}}_{\mathrm{ABC}}$ denotes the quantity of modules ABC chosen for recycling (and distribution) and $Y^{\mathrm{D}}_{\mathrm{ABC}}$ for disposal. The variables $X^{\mathrm{R}}_{\mathrm{A}}$ and $X^{\mathrm{D}}_{\mathrm{A}}$ denote the values for the item A for recycling and disposal, respectively. In addition, the quantity of the core is $Q^{\mathrm{C}} = 250$.

From the discussion we derive that the superordinate modules need to be considered, too. This means, the value of $Y^{\mathrm{R}}_{\mathrm{ABC}}$ has influence on the limit of $Y^{\mathrm{R}}_{\mathrm{BC}}$, $Y^{\mathrm{R}}_{\mathrm{AC}}$, $Y^{\mathrm{R}}_{\mathrm{AB}}$, $X^{\mathrm{R}}_{\mathrm{A}}$, $X^{\mathrm{R}}_{\mathrm{B}}$, and $X^{\mathrm{R}}_{\mathrm{C}}$. In a general formulation this is expressed by

$$\sum_{\tilde{m} \in \left\{ \tilde{m} \,\middle|\, \substack{\delta_{cmi} \le \delta_{c\tilde{m}i} \;\; \forall i, \\ (c,\tilde{m}) \in \{1,\ldots,\overline{M}_c\}} \right\}} \left(Y^{\mathrm{M}}_{c\tilde{m}} + \sum_r Y^{\mathrm{R}}_{c\tilde{m}r} \right) \le \prod_{i \in \{i | \delta_{cmi} = 1\}} (1 - \zeta_{ci}\iota_{ci}) Q^{\mathrm{C}}_c \qquad \forall\; c, m \in \{1, \ldots, \overline{M}_c\} \quad (4.5)$$

for modules, where $\overline{M}_c$ denotes the number of modules of core c, and by

$$\sum_r \left(\sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y^{\mathrm{R}}_{cmr} + X^{\mathrm{R}}_{cir} \right) + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y^{\mathrm{M}}_{cm} + X^{\mathrm{I}}_{ci} \le (1 - \zeta_{ci}\iota_{ci}) Q^{\mathrm{C}}_c \qquad \forall\; c, i \in \{1, \ldots, \bar{I}_c\} \quad (4.6)$$

Table 4.5 Module definition for core ABC

module m	item A	item B	item C
ABC	1	1	1
BC	0	1	1
AC	1	0	1
AB	1	1	0

for the single items.

To illustrate the constraints we continue with the example above. We have four modules and three single items. The probabilities $\zeta \cdot \iota$ are 0.6, 0.2, and 0.4 for the items A, B, and C, respectively. The module definition values are listed in Table 4.5. The four constraints for the modules (see Eq. (4.5)) are:

$$\text{ABC:} \quad Y^{\mathrm{R}}_{\mathrm{ABC}} \leq (1-0.6)(1-0.2)(1-0.4)Q^{\mathrm{C}}, \tag{4.7}$$

$$\text{BC:} \quad Y^{\mathrm{R}}_{\mathrm{ABC}} + Y^{\mathrm{R}}_{\mathrm{BC}} \leq (1-0.2)(1-0.4)Q^{\mathrm{C}}, \tag{4.8}$$

$$\text{AC:} \quad Y^{\mathrm{R}}_{\mathrm{ABC}} + Y^{\mathrm{R}}_{\mathrm{AC}} \leq (1-0.6)(1-0.4)Q^{\mathrm{C}}, \text{ and} \tag{4.9}$$

$$\text{AB:} \quad Y^{\mathrm{R}}_{\mathrm{ABC}} + Y^{\mathrm{R}}_{\mathrm{AB}} \leq (1-0.6)(1-0.2)Q^{\mathrm{C}}. \tag{4.10}$$

For the three single items the constraints (see Eq. (4.6)) result in:

$$\text{A:} \quad Y^{\mathrm{R}}_{\mathrm{ABC}} + Y^{\mathrm{R}}_{\mathrm{AC}} + Y^{\mathrm{R}}_{\mathrm{AB}} + X^{\mathrm{R}}_{\mathrm{A}} \leq (1-0.6)Q^{\mathrm{C}}, \tag{4.11}$$

$$\text{B:} \quad Y^{\mathrm{R}}_{\mathrm{ABC}} + Y^{\mathrm{R}}_{\mathrm{BC}} + Y^{\mathrm{R}}_{\mathrm{AB}} + X^{\mathrm{R}}_{\mathrm{B}} \leq (1-0.2)Q^{\mathrm{C}}, \text{ and} \tag{4.12}$$

$$\text{C:} \quad Y^{\mathrm{R}}_{\mathrm{ABC}} + Y^{\mathrm{R}}_{\mathrm{BC}} + Y^{\mathrm{R}}_{\mathrm{AC}} + X^{\mathrm{R}}_{\mathrm{C}} \leq (1-0.4)Q^{\mathrm{C}}. \tag{4.13}$$

Substituting the variables by their values (see Table 4.4) results in:

$$\text{ABC:} \quad 45 = 45 \leq 48 = (1-0.6)(1-0.2)(1-0.4)250, \tag{4.14}$$

$$\text{BC:} \quad 45+50 = 95 \leq 120 = (1-0.2)(1-0.4)250, \tag{4.15}$$

$$\text{AC:} \quad 45+15 = 60 \leq 60 = (1-0.6)(1-0.4)250, \tag{4.16}$$

$$\text{AB:} \quad 45+30 = 75 \leq 80 = (1-0.6)(1-0.2)250, \tag{4.17}$$

$$\text{A:} \quad 45+15+30+5 = 95 \leq 100 = (1-0.6)250, \tag{4.18}$$

$$\text{B:} \quad 45+50+30+20 = 145 \leq 200 = (1-0.2)250, \text{ and} \tag{4.19}$$

$$\text{C:} \quad 45+50+15+40 = 150 \leq 150 = (1-0.4)250, \tag{4.20}$$

which is definitely feasible. An increase of AC or C is impossible according to these constraints, which can also be seen in Table 4.3 in row B(AC) and column five (unused units) as well as column C and the last row.

But this is not all of the necessary constraints. The following new example shall illustrate a case where an infeasible solution occurs with the given constraints. We use a core ABCD and focus just on the functioning probability to keep the explanation straightforward. All items that are not functioning can only be used for recycling. (The wrong material condition is neglected for the moment.) The probabilities that item A, B, C, and D are functioning are 0.25, 0.4, 0.5, and 0.8, respectively. Let us assume the following solution. Four units of module ABCD, 12 units of module BCD, and eight units of item A are planned for distribution. In addition, 84 units of module ABCD as well as four units of item A are intended for material recycling. Adding the quantities of the items (single and in modules) we find that 100 units of that particular core are required. Given the probabilities (see above) for the items being functioning—and neglecting any wrong material and damaging—the relevant condition constraints regarding the five planned quantities are the following. From Eq. (4.3) we get

$$Y^{\mathrm{M}}_{\mathrm{ABCD}} \leq 0.25 \cdot 0.4 \cdot 0.5 \cdot 0.8\, Q^{\mathrm{C}} \quad \Rightarrow \quad 4 \leq 0.04 \cdot 100 \tag{4.21}$$

$$Y^{\mathrm{M}}_{\mathrm{ABCD}} + Y^{\mathrm{M}}_{\mathrm{BCD}} \leq 0.4 \cdot 0.5 \cdot 0.8\, Q^{\mathrm{C}} \quad \Rightarrow \quad 4 + 12 \leq 0.16 \cdot 100 \tag{4.22}$$

and from Eq. (4.4)

$$X^{\mathrm{M}}_{\mathrm{A}} + Y^{\mathrm{M}}_{\mathrm{ABCD}} + Y^{\mathrm{M}}_{BCD} \leq 0.25\, Q^{\mathrm{C}} \quad \Rightarrow \quad 8 + 4 \leq 0.25 \cdot 100\,. \tag{4.23}$$

As we see they are all feasible. However, taking a closer look at the solution we notice that the solution is indeed infeasible. Of the 100 available units 16 functioning units of module BCD exist ($0.4 \cdot 0.5 \cdot 0.8 = 0.16$). The probability that item A is functioning is one fourth. Hence, considering the functioning 16 units of module BCD four of them come with a functioning item A and 12 without. The four functioning items are used to keep the functioning module ABCD and the other 12 have to be recycled, because they are not functioning. In addition, 84 modules ABCD (i.e., including item A) are recycled, regardless of an item A being functioning or not. Adding the quantities of item A we have already 100 units and no possibility to gain another eight units for distribution, because they are all caught in the recycled units of module ABCD. Even though the given solution is feasible

Table 4.6 Expected item conditions in 100 units of the exemplary core

posi-	item				posi-	item				posi-	item			
tion	A	B	C	D	tion	A	B	C	D	tion	A	B	C	D
1	●	●	●	●	36	○	○	●	●	71	○	○	○	●
2	●	●	●	●	37	○	○	●	●	72	○	○	○	●
3	●	●	●	●	38	○	○	●	●	73	○	○	○	●
4	●	●	●	●	39	○	○	●	●	74	○	○	○	●
5	○	●	●	●	40	○	○	●	●	75	○	○	○	●
6	○	●	●	●	41	●	●	○	●	76	○	○	○	●
7	○	●	●	●	42	●	●	○	●	77	○	○	○	●
8	○	●	●	●	43	●	●	○	●	78	○	○	○	●
9	○	●	●	●	44	●	●	○	●	79	○	○	○	●
10	○	●	●	●	45	○	●	○	●	80	○	○	○	●
11	○	●	●	●	46	○	●	○	●	81	●	●	●	○
12	○	●	●	●	47	○	●	○	●	82	○	●	●	○
13	○	●	●	●	48	○	●	○	●	83	○	●	●	○
14	○	●	●	●	49	○	●	○	●	84	○	●	●	○
15	○	●	●	●	50	○	●	○	●	85	●	○	●	○
16	○	●	●	●	51	○	●	○	●	86	○	○	●	○
17	●	○	●	●	52	○	●	○	●	87	○	○	●	○
18	●	○	●	●	53	○	●	○	●	88	○	○	●	○
19	●	○	●	●	54	○	●	○	●	89	○	○	●	○
20	●	○	●	●	55	○	●	○	●	90	○	○	●	○
21	●	○	●	●	56	○	●	○	●	91	●	●	○	○
22	●	○	●	●	57	●	○	○	●	92	○	●	○	○
23	○	○	●	●	58	●	○	○	●	93	○	●	○	○
24	○	○	●	●	59	●	○	○	●	94	○	●	○	○
25	○	○	●	●	60	●	○	○	●	95	●	○	○	○
26	○	○	●	●	61	●	○	○	●	96	●	○	○	○
27	○	○	●	●	62	●	○	○	●	97	○	○	○	○
28	○	○	●	●	63	○	○	○	●	98	○	○	○	○
29	○	○	●	●	64	○	○	○	●	99	○	○	○	○
30	○	○	●	●	65	○	○	○	●	100	○	○	○	○
31	○	○	●	●	66	○	○	○	●					
32	○	○	●	●	67	○	○	○	●					
33	○	○	●	●	68	○	○	○	●					
34	○	○	●	●	69	○	○	○	●					
35	○	○	●	●	70	○	○	○	●					

The "●" indicates a functioning item and "○" a non-functioning one.

according to the condition constraints developed so far it is not feasible, because there exist dependencies that go beyond the superordinate modules.

4.2.2.3 Core condition using graphs with two usage options

To illustrate the following discussion all expected item combinations for the 100 units of the core ABCD (from above) are displayed in Table 4.6.[23] The value "•" indicates a functioning item and "∘" a non-functioning, which can only be used for recycling. From core position 1–4 the functioning module ABCD is taken. Position 5–16 is used for distributing module BCD. The recycling of 84 units of module ABCD are the remaining positions 17–100. However, there is no position left to gain the eight functioning units of item C for distribution.

In order to find a feasible solution based on the given solution several options exist. These are:

1. reduce $X^{\mathrm{I}}_{\mathrm{A}}$ (and increase $X^{\mathrm{R}}_{\mathrm{A}}$) maximal by eight,
2. reduce $Y^{\mathrm{M}}_{\mathrm{BCD}}$ (and increase $Y^{\mathrm{R}}_{\mathrm{BCD}}$) maximal by eight,
3. reduce $Y^{\mathrm{M}}_{\mathrm{ABCD}}$ (and increase $Y^{\mathrm{R}}_{\mathrm{ABCD}}$ or increase $Y^{\mathrm{M}}_{\mathrm{BCD}}$ and $X^{\mathrm{R}}_{\mathrm{A}}$) maximal by four, and
4. any combination of the above mentioned options.

With option one and two the infeasibility can be cancelled completely. However, option three can only be used four times, because no more than four units of a functioning module ABCD exist. The missing four units need to be compensated with by option one or two. What we see here is that there exist more than one option to find a feasible solution and that once, e.g., ABCD is used to gain BCD the same unit cannot be used for gaining a unit if ACD anymore.

Before we continue we illustrate the Table 4.6 in Fig. 4.7. Each node represents the percentage of units of the core with only these items functioning that are black in the node label. For example, node AB$\underline{\mathrm{C}}\underline{\mathrm{D}}$ indicates that 4 % of the units of the core have functioning items A, B, and D and a non-functioning item $\underline{\mathrm{C}}$. (This equals positions 41–44 in Table 4.6.) Node ABCD means that in 4 % of the units a unit of a core comes in with all items functioning (positions 1–4 in the table) and node $\underline{\mathrm{A}}\underline{\mathrm{B}}\underline{\mathrm{C}}\underline{\mathrm{D}}$ represents the core with no functioning item (positions 97–100 in the table). Adding all percentages, results in 100 %. Note that only the two classes functioning and non-functioning are discussed for now.

The edges of the graph connect the nodes with non-negative flows.[24] In addition, each node inputs the given percentage into the graph for a given quantity of incoming cores. For example, if 100 units of the core are

[23] The expected number of units of functioning item combination A and C is $0.25 \cdot 0.5 = 0.125$, which is 12.5 units of the 100 cores. Therefore an alternative listing with one unit more of functioning A and C combination and one unit less of only functioning C in a core is possible, too. The change only effects position 86 and 96 of the listing.

[24] Even though the following graphs are non-weighted and directed graphs with arrows or directed arcs or edges, they are simply called graphs with nodes and edges. A further distinction is not necessary for the considered problem.

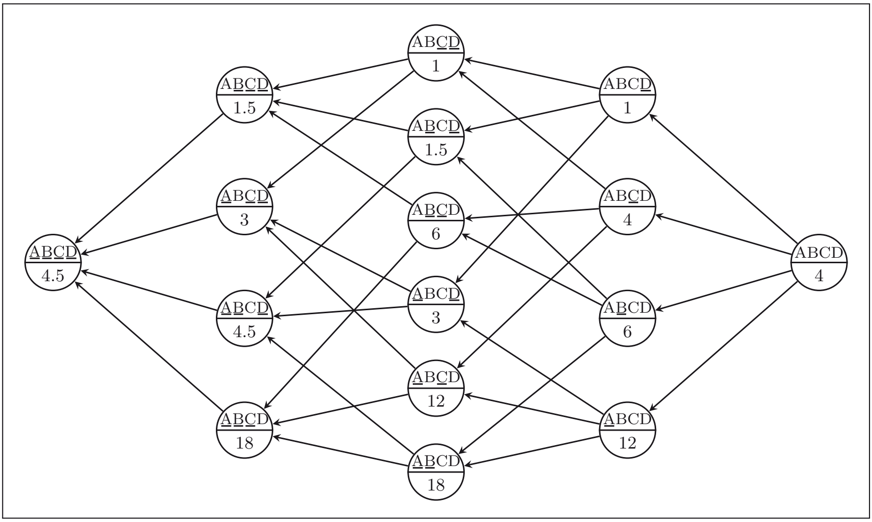

Fig. 4.7 Condition dependencies core graph (2 classes, 4 items)

acquired, six units with functioning items A and D as well as non-functioning items $\underline{\text{B}}$ and $\underline{\text{C}}$ will be expected. This is the input of node A$\underline{\text{BC}}$D, namely $100 \cdot 6\,\% = 6$. Moreover, the output (besides the edges) of each node is the quantity that can be used for item distribution as well as recycling (and disposal). Thereby, node ABCD can be used for everything, i.e., item and module distribution as well as recycling (and disposal), whereas node AB$\underline{\text{CD}}$ can only be used for distributing items A and B, module AB, and recycling any module and item combination of A, B, C, and D. Having one unit of core ABCD and one of AB$\underline{\text{CD}}$ gives the option to use ABCD and AB$\underline{\text{CD}}$ as they are or as two units of AB$\underline{\text{CD}}$. This option needs to be modelled by the edges in the graph. Starting from node AB$\underline{\text{CD}}$ the nodes ABC$\underline{\text{D}}$, AB$\underline{\text{C}}$D, and ABCD are superior with respect to the condition. This means that any quantity of superior cores can be used for the one in focus. But, once a superior unit is used, it cannot be used for another node. For example, if one unit of ABC$\underline{\text{D}}$ is used as AB$\underline{\text{CD}}$, the same unit cannot be used for $\underline{\text{A}}$BC$\underline{\text{D}}$. However, the superior core ABCD to AB$\underline{\text{CD}}$ does not have to be considered directly. It is considered recursively, because ABCD is also superior to ABC$\underline{\text{D}}$. Thus, only the next superior core needs to be considered.[25]

[25] The property of being superior is transitive, because ABC$\underline{\text{D}}$ is superior to AB$\underline{\text{CD}}$ and ABCD is superior to ABC$\underline{\text{D}}$ and therefore ABCD is superior to AB$\underline{\text{CD}}$, too.

Let us denote the flow between the nodes of this core graph with $Z^{\mathrm{C}}_{v\tilde{v}}$ from node v to node $\tilde{v}$. We see that the number of nodes equals the number of all item combinations (here functioning and non-functioning), i.e., $2^{\bar{I}}$ with $\bar{I}$ being four in this small example. From each node an edge per changeable item exists. In node $\mathrm{AB\underline{CD}}$ items A and B can be changed from functioning A and B to non-functioning $\underline{\mathrm{A}}$ and $\underline{\mathrm{B}}$, which leads to the nodes $\mathrm{\underline{A}B\underline{CD}}$ and $\mathrm{A\underline{BCD}}$, respectively. (These are edges to the left in the figure.) In addition, either item C or D was changed to get to the node $\mathrm{AB\underline{CD}}$ from $\mathrm{ABC\underline{D}}$ or $\mathrm{AB\underline{C}D}$. Hence, each node in the graph has four edges (ingoing and outgoing together). Since they are counted double if added, the number of edges in the graph is limited to the number of nodes multiplied by the number of items and divided by two, i.e., $2^{\bar{I}-1}\bar{I}$.

The output of a node is denoted by V^{C}_v. The input of a node depends on the quantity of cores Q^{C} and the condition probabilities. For example, with $Q^{\mathrm{C}} = 100$ units of the core the input of node $\mathrm{AB\underline{CD}}$ is $0.25 \cdot 0.4\,(1 - 0.5)(1 - 0.8)\,Q^{\mathrm{C}} = 1$.[26] For each node v a constraint needs to be formulated to represent the above described behaviour. Everything that goes into a node (e.g., $\mathrm{AB\underline{CD}}$) from superior nodes (e.g., $\mathrm{ABC\underline{D}}$ and $\mathrm{AB\underline{C}D}$) plus the input (e.g., $\rho_{\mathrm{AB\underline{CD}}}Q^{\mathrm{C}}$) must equal the output (e.g., $V^{\mathrm{C}}_{\mathrm{AB\underline{CD}}}$) and the flow to subordinate nodes (e.g., $\mathrm{A\underline{BCD}}$ and $\mathrm{\underline{A}B\underline{CD}}$). Thereby, ρ_v denotes the percentage of units of this core represented by node $\mathrm{AB\underline{CD}}$.

$$
\begin{aligned}
&Z^{\mathrm{C}}_{\mathrm{A\underline{BCD}},\mathrm{AB\underline{CD}}} + Z^{\mathrm{C}}_{\mathrm{\underline{A}B\underline{CD}},\mathrm{AB\underline{CD}}} + V^{\mathrm{C}}_{\mathrm{AB\underline{CD}}} \\
&\qquad\qquad = \rho_{\mathrm{AB\underline{CD}}}\,Q^{\mathrm{C}} + Z^{\mathrm{C}}_{\mathrm{AB\underline{CD}},\mathrm{ABC\underline{D}}} + Z^{\mathrm{C}}_{\mathrm{AB\underline{CD}},\mathrm{AB\underline{C}D}}
\end{aligned}
\tag{4.24}
$$

The edges of the core graph can also be represented by a matrix $E^{\mathrm{C}}_{v\tilde{v}}$. An entry equalling one represents an edge. The resulting matrix is a lower triangle matrix with blocks (see Table 4.7).

This matrix is easily constructed by focusing on the functioning items. Each row has a value of one in those columns where the number of functioning items is one less than in the focussed row and the functioning items of the column are a subset of these in the row. To illustrate this, we have a look at row $\mathrm{ABC\underline{D}}$. In this row the items A, B, and C are functioning. Hence, the number of functioning items is three in this row. An entry of one can only exists in the columns with one less functioning items, i.e., column $\mathrm{AB\underline{CD}}$, $\mathrm{A\underline{B}C\underline{D}}$, $\mathrm{A\underline{BC}D}$, $\mathrm{\underline{A}BC\underline{D}}$, $\mathrm{\underline{A}B\underline{C}D}$, and $\mathrm{\underline{AB}CD}$. Of these six columns only the columns $\mathrm{AB\underline{CD}}$, $\mathrm{A\underline{B}C\underline{D}}$, and $\mathrm{\underline{A}BC\underline{D}}$ have functioning items that

[26] This would equal the term $[(1-\zeta_{\mathrm{A}})(1-\eta_{\mathrm{A}})]\,[(1-\zeta_{\mathrm{B}})(1-\eta_{\mathrm{B}})]\,[1-(1-\zeta_{\mathrm{C}})(1-\eta_{\mathrm{C}})]\,[1-(1-\zeta_{\mathrm{D}})(1-\eta_{\mathrm{D}})]Q^{\mathrm{C}}$, because $(1-\zeta_i)(1-\eta_i)$ is the probability of a functioning item.

Table 4.7 Matrix representation of core graph edges

	core graph edges $E^{\mathrm{C}}_{v\tilde{v}}$															
	node $\tilde{v}$															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
v	$\underline{ABCD}$	$A\underline{BCD}$	$\underline{A}B\underline{CD}$	$\underline{AB}C\underline{D}$	$\underline{ABC}D$	$AB\underline{CD}$	$A\underline{B}C\underline{D}$	$A\underline{BC}D$	$\underline{A}BC\underline{D}$	$\underline{A}B\underline{C}D$	$\underline{AB}CD$	$ABC\underline{D}$	$AB\underline{C}D$	$A\underline{B}CD$	$\underline{A}BCD$	ABCD
$\underline{ABCD}$																
$A\underline{BCD}$	1															
$\underline{A}B\underline{CD}$	1	.														
$\underline{AB}C\underline{D}$	1	.	.													
$\underline{ABC}D$	1	.	.	.												
$AB\underline{CD}$	.	1	1	.	.											
$A\underline{B}C\underline{D}$	.	1	.	1	.	.										
$A\underline{BC}D$	.	1	.	.	1	.	.									
$\underline{A}BC\underline{D}$	.	.	1	1	.	.	.	.								
$\underline{A}B\underline{C}D$	.	.	1	.	1	.	.	.	.							
$\underline{AB}CD$	.	.	.	1	1	.	.	.	.	.						
$ABC\underline{D}$	.	.	.	.	.	1	1	.	1	.	.					
$AB\underline{C}D$	.	.	.	.	.	1	.	1	.	1	.	.				
$A\underline{B}CD$	.	.	.	.	.	.	1	1	.	.	1	.	.			
$\underline{A}BCD$	.	.	.	.	.	.	.	.	1	1	1	.	.	.		
ABCD	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	

Dots denote a value of zero and white spaces are not of interest. Only the framed blocks of cells can have a value other than zero.

are a subset of ABC. Hence, the three entries equalling one in the row are fixed and all other values are zero.

Using this matrix $E^{\mathrm{C}}_{v\tilde{v}}$ the constraints can be formulated by

$$\sum_{\tilde{v}\in\{\tilde{v}\,|\,\tilde{v}<v,\,E^{\mathrm{C}}_{v\tilde{v}}=1\}} Z^{\mathrm{C}}_{v\tilde{v}} + V^{\mathrm{C}}_{v} = \rho_v\, Q^{\mathrm{C}} + \sum_{\tilde{v}\in\{\tilde{v}\,|\,\tilde{v}>v,\,E^{\mathrm{C}}_{\tilde{v}v}=1\}} Z^{\mathrm{C}}_{\tilde{v}v} \quad \forall\, v\,. \tag{4.25}$$

With these constraints the core graph assures that whenever 96 units ABCD are recycled completely (i.e., without disassembly), only four units of the core can be used for anything else.

This core graph alone is not sufficient to model the possibilities of the flexible disassembly considering the conditions. Another graph (or another layer to the existing graph) is necessary. This extra graph is necessary to express the disassembly options that exist. For example, in the core graph the node ABCD represents the core with all four items being functioning. Such an item combination can be used for everything. This means that it can be used for distribution and recycling (this is modelled in the core graph). In addition, it can be used to gain a complete functioning core and any arbitrary module and item combination—down to the four single items

A, B, C, and D just from one unit. The latter aspect is not modelled by the core graph.

The extra graph is called distribution graph and it focusses only on the functioning items. Therefore, the nodes from the core graph are used and labelled only with the functioning items (see Fig. 4.8). The nodes with functioning items are connected by edges. Thereby, the edges already existing in the core graph are the solid ones.[27] A dash-dot edge is an edge that is added to a solid edge. The third type of edge is the dashed one. These edges are completely new. (The different line types have no specific meaning they are only for illustration.) The nodes need to be connected in a way that all possible partitions of a single completely functioning unit of the core are displayed. In the figure each starting edge from a node (e.g., ABCD) splits up into two edges (e.g., AB and CD or ABC and D). This is caused by the partition of the node.

To illustrate this, we consider a completely functioning unit ABCD. The input into the node ABCDis one, i.e., $V^{\mathrm{C}}_{\mathrm{ABCD}} = 1$, which is the output of the core graph. This unit can be separated into ABC and D. Both, module and item exist with one unit parallel. The module ABC can further be separated into A and BC. If we stop here, we have one functioning module BC and two functioning items A and D. (The same can be gained by following a different path through the graph.) If we want only a functioning module ABC and D for recycling, the completely functioning core could be seen as a core with non-functioning item $\underline{\mathrm{D}}$. This is achieved by shifting the one unit from node ABCD in the core graph to node ABC$\underline{\mathrm{D}}$ and set the output $V^{\mathrm{C}}_{\mathrm{ABC\underline{D}}} = 1$ and $V^{\mathrm{C}}_{\mathrm{ABCD}} = 0$. Thus, the input of one unit into the distribution graph appears in node ABC (and not ABCD). And because of this, only the functioning items A, B, and C can be used. Item D is targeted for recycling which will be considered later.

The edges of the graph can also be represented by a lower triangle matrix (see Table 4.8). The edges are denoted by $E^{\mathrm{I}}_{w\tilde{w}}$. The node index is w to illustrate the difference of this graph to the core graph. (Later this is necessary, but for now the indices w and v are identical in principle). In this matrix not only two values exist. The value 0 (or ".") represents the case with no connection between the nodes and the values 1 through 15 represent an edge. The edges are not weighted so that the values greater than 1 have a different meaning. Taking row ABC we find the values 1, 1, 1, 4, 3 and 2. This means that in total six edges or three pairs of edges emerge from node ABC. Each edge pair is identified by an edge with a value of 1 and one with a value greater than 1 (being a node index). To find a pair, an arbitrary

[27] Some of them are combined to one edge.

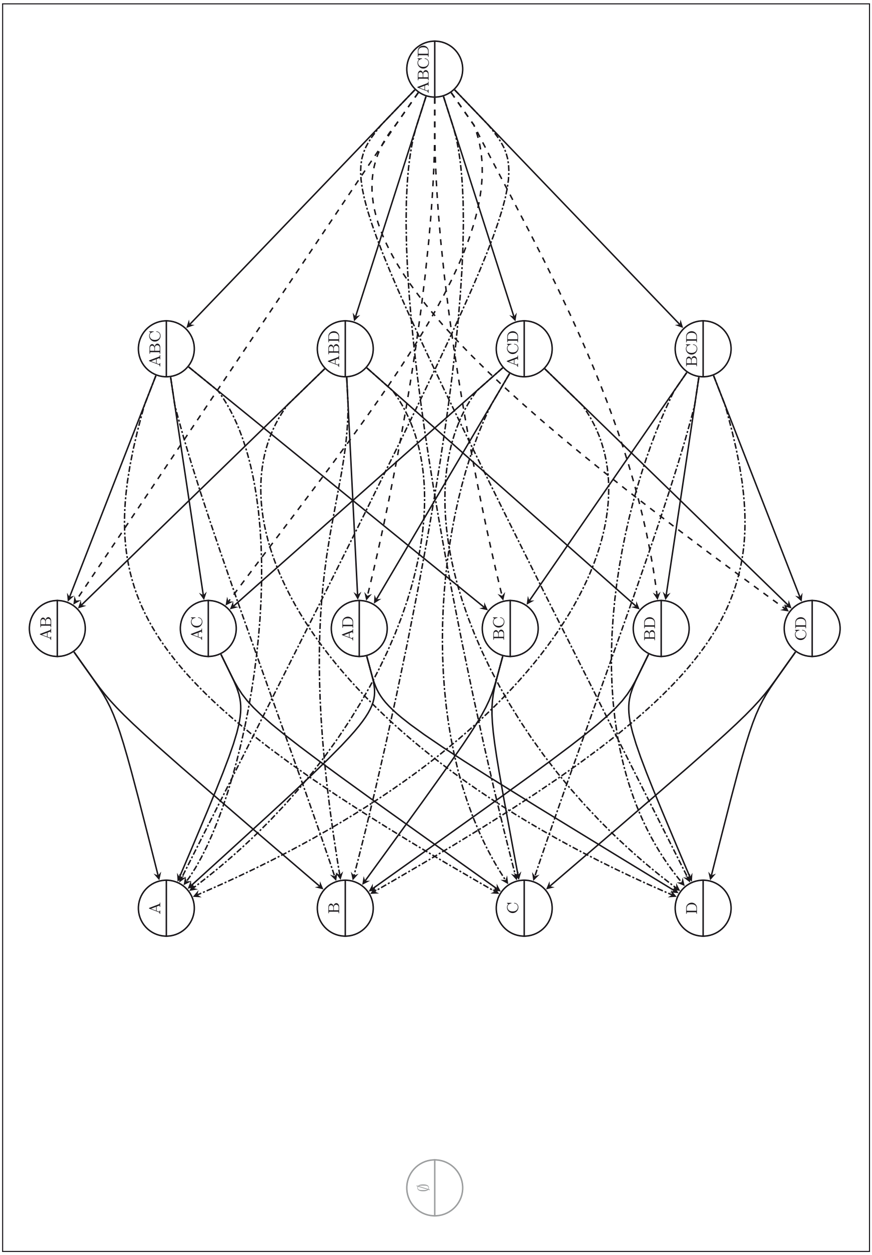

Fig. 4.8 Distribution graph

Table 4.8 Matrix representation of distribution graph edges

		distribution graph edges $E^{\mathrm{I}}_{w\tilde{w}}$															
		node $\tilde{w}$															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	w	∅	A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD
1	∅																
2	A	.															
3	B	.	.														
4	C	.	.	.													
5	D	.	.	.	.												
6	AB	.	1	2	.	.											
7	AC	.	1	.	2	.	.										
8	AD	.	1	.	.	2	.	.									
9	BC	.	.	1	3	.	.	.	.								
10	BD	.	.	1	.	3	.	.	.	.							
11	CD	.	.	.	1	4	.	.	.	.	.						
12	ABC	.	1	1	1	.	4	3	.	2	.	.					
13	ABD	.	1	1	.	1	5	.	3	.	2	.	.				
14	ACD	.	1	.	1	1	.	5	4	.	.	2	.	.			
15	BCD	.	.	1	1	1	.	.	.	5	4	3	.	.	.		
16	ABCD	.	1	1	1	1	1	1	1	8	7	6	5	4	3	2	

Dots denote a value of zero and white spaces are not of interest. Only the framed blocks of cells can have a value other than zero.

value $E^{\mathrm{I}}_{w\tilde{w}}$ greater than one in the focussed row w is selected. Hence, one edge goes from node w to the node with the column index $\tilde{w}$ the selected value $E^{\mathrm{I}}_{w\tilde{w}} > 1$ is in. The second edge of this pair is the one from w to the node $E^{\mathrm{I}}_{w\tilde{w}}$, whose index equals the selected value. This column is marked by the value 1. Coming back to row $w = 12$, i.e., ABC, a first value greater than 1 is $E^{\mathrm{I}}_{w\tilde{w}} = 4$ in column $\tilde{w} = 6$. This means, that one edge of the edge pair goes to node 6 and the other to node 4. Both edges start from the same node $w = 12$. The other two edge pairs got to node 7 and 3 as well as 9 and 2. This concept is equivalent to that of a hypergraph.[28]

The property of the edge pairs is that they have to have the same flow through the edges. This is assured by constraints that set the values of the relevant flow variables equal (see below). The matrix is generated in the following way. The rows are independent of each other so that we pick an arbitrary row, e.g., $w = 12$ (ABC). We select a column $\tilde{w}$ that represents a node with a non-empty strict subset of functioning items of the row (that has not been fixed before). Hence, the columns $\tilde{w} = 1$ ($\emptyset$) and all columns with $\tilde{w} \geq w$ are always skipped. Possible columns are $\tilde{w} \in \{2, 3, 4, 6, 7, 9\}$. We choose column $\tilde{w} = 2$ (A). In this column of the row we set the value to $E^{\mathrm{I}}_{w\tilde{w}} = 1$. The disjoint set of functioning items to this selected column

[28] Cf. Borndörfer et al. (2012): *Vehicle rotation planning*.

in the focussed row we find in column $\hat{w} = 9$ (BC). In this column $\hat{w}$ of the focussed row we store the value of the first column index, i.e., $E^{\mathrm{I}}_{w\hat{w}} = \tilde{w} = 2$. In the focussed row, columns two and nine are fixed. The next column to select is one of $\tilde{w} \in \{3, 4, 6, 7\}$. This procedure continues until all columns in the row are processed and all rows with at least two functioning items are considered.

Before we continue with the formulation of the constraints, the size of the graph is discussed. The number of nodes is limited to one less compared to the core graph, i.e., $2^{\bar{I}} - 1$. The number of edges is based on the number of two-set partitions of the functioning items of a node. A node with k items can be separated into 2^{k-1} partitions. This includes the partition of the empty set and the complete set of k items. Only partitions with non-empty sets are of interest, because there is no edge to the node $w = 1$ ($\emptyset$). In each two-set partition, obviously, two sets exist, which results in $2 \cdot \left(2^{k-1} - 1\right) = 2^k - 2$ sets (excluding the empty set partition). The number of combinations without repetition of k items out of n is $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. Combining the number of outgoing edges per node and the number of nodes leads to

$$\sum_{k=2}^{\bar{I}} \binom{\bar{I}}{k} \left(2^k - 2\right) = \sum_{k=2}^{\bar{I}} \frac{\bar{I}!}{k!\left(\bar{I} - k\right)!} \left(2^k - 2\right) . \tag{4.26}$$

The number of items is given by $\bar{I}$ and the first level to consider is that of two items per node, because there exist no outgoing edges from the nodes 1 through 5. According to the binomial theorem $(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$ and choosing $a = 1$ and $b = 2$ we get[29]

$$\sum_{k=0}^{n} \binom{n}{k} 2^k = 3^n . \tag{4.27}$$

If we subtract two times 2^n on both sides of the equation

$$\sum_{k=0}^{n} \binom{n}{k} 2^k - 2 \cdot 2^n = 3^n - 2 \cdot 2^n \tag{4.28}$$

and replace 2^n by $\sum_{k=0}^{n} \binom{n}{k}$ we get

[29] Cf. Gellert et al. (1965): *Kleine Enzyklopdie – Mathematik*, p. 45.

$$\sum_{k=0}^{n}\binom{n}{k}2^k - 2\sum_{k=0}^{n}\binom{n}{k} = 3^n - 2\cdot 2^n \tag{4.29}$$

$$\sum_{k=0}^{n}\binom{n}{k}\left(2^k - 2\right) = 3^n - 2^{n+1}\,. \tag{4.30}$$

Adding one on both sides leads to

$$\sum_{k=0}^{n}\binom{n}{k}\left(2^k - 2\right) - 1\,(1-2) = 3^n - 2^{n+1} + 1 \tag{4.31}$$

$$\sum_{k=0}^{n}\binom{n}{k}\left(2^k - 2\right) - \binom{n}{0}\left(2^0 - 2\right) = 3^n - 2^{n+1} + 1 \tag{4.32}$$

$$\sum_{k=1}^{n}\binom{n}{k}\left(2^k - 2\right) = 3^n - 2^{n+1} + 1 \tag{4.33}$$

and subtracting zero leads to

$$\sum_{k=1}^{n}\binom{n}{k}\left(2^k - 2\right) - \binom{n}{1}\left(2^1 - 2\right) = 3^n - 2^{n+1} + 1 - 0 \tag{4.34}$$

$$\sum_{k=2}^{n}\binom{n}{k}\left(2^k - 2\right) = 3^n - 2^{n+1} + 1\,. \tag{4.35}$$

Replacing n by $\bar{I}$ results in the number of edges of

$$3^{\bar{I}} - 2^{\bar{I}+1} + 1 \tag{4.36}$$

for the distribution graph.

The distribution graph is the interface between the core graph and planned quantities of functioning items and modules. Therefore, the output of the graph equals the X_i^{I} for the nodes 2 through 5 and Y_m^{M} for the nodes 6 through 16. The input of the nodes of the distribution graph is the output of the core graph. Thus, the flow through the node $w = 12$ (ABC) can be described as

$$\begin{aligned} Z^{\mathrm{I}}_{\mathrm{ABC,AB}} + Z^{\mathrm{I}}_{\mathrm{ABC,AC}} + Z^{\mathrm{I}}_{\mathrm{ABC,BC}} + Z^{\mathrm{I}}_{\mathrm{ABC,A}} + Z^{\mathrm{I}}_{\mathrm{ABC,B}} + Z^{\mathrm{I}}_{\mathrm{ABC,C}} + 2\,Y^{\mathrm{M}}_{\mathrm{ABC}} \\ = 2\,V^{\mathrm{C}}_{\mathrm{ABC\underline{D}}} + 2\,Z^{\mathrm{I}}_{\mathrm{ABCD,ABC}}\,. \end{aligned} \tag{4.37}$$

Thereby, $Z^{\mathrm{I}}_{w\tilde{w}}$ denotes the flow through the edges. The factor two on the input side is necessary, because the outgoing edges appear in pairs and in each edge of the pair the same quantity goes in. For example, one unit from node ABCD $Z^{\mathrm{I}}_{\mathrm{ABCD,ABC}}$ (distribution graph) or one unit from the core graph $V^{\mathrm{C}}_{\mathrm{ABC\underline{D}}}$ result in one unit each to AB and C or AC and B or BC and A or as output $Y^{\mathrm{M}}_{\mathrm{ABC}}$. In order to gain only one unit of output the corresponding variable also needs to be multiplied with two. In addition, the equality of the flows of the edge pairs must also be expressed. The equations for the three pairs are

$$Z^{\mathrm{I}}_{\mathrm{ABC,AB}} = Z^{\mathrm{I}}_{\mathrm{ABC,C}} \tag{4.38}$$

$$Z^{\mathrm{I}}_{\mathrm{ABC,AC}} = Z^{\mathrm{I}}_{\mathrm{ABC,B}} \tag{4.39}$$

$$Z^{\mathrm{I}}_{\mathrm{ABC,BC}} = Z^{\mathrm{I}}_{\mathrm{ABC,A}} \,. \tag{4.40}$$

The exemplary equations need to be formulated in a general way. On the left side of Eq. (4.37) the edges go from the focussed node w to those nodes, where an entry greater than zero exists in the matrix $E^{\mathrm{I}}_{w\tilde{w}}$ in row $\tilde{w}$. Thereby, only the column indices less than the index of the row $\tilde{w}$ need to be considered. Here, both edges of the pairs are included. Furthermore, the output of the node w equals the decision variable Y^{M}_{m}. This relationship can be given by a list where the wth item of the list contains the corresponding index m or i. In this example here the transformation is alternatively possible by using $m = 2^{\bar{I}} + 1 - w$. The node $w = 16$ (ABCD) of the distribution graph is the module $m = 1$, i.e., the whole core. According to the numbering of the modules in the module definition matrix the module without item A, i.e., module BCD, has the index $m = 2$. The module without item B has the index $m = 3$ and so on. The corresponding node indices are $w = 16$, $w = 15$, $w = 14$, etc., respectively.

The right hand side of the equation includes the input into the distribution graph, which is the output of the core graph V^{C}_{v}, and all edges from nodes where the row in column $\tilde{w}$ of matrix $E^{\mathrm{I}}_{w\tilde{w}}$ has a value greater than zero. Thereby, only the rows with an index greater than the column index $\tilde{w}$ need to be considered. Thus, the constraints for the modules related to nodes 6 through 16 are

$$\sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}<w,E^{\mathrm{I}}_{w\tilde{w}}>0\right\}} Z^{\mathrm{I}}_{w\tilde{w}} + 2\,Y^{\mathrm{M}}_{2^{\bar{I}}+1-w} = 2\left(V^{\mathrm{C}}_{w} + \sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}>w,E^{\mathrm{I}}_{\tilde{w}w}>0\right\}} Z^{\mathrm{I}}_{\tilde{w}w}\right)$$

$$\forall\; w \in \{\bar{I}+2,\ldots,2^{\bar{I}}\}. \tag{4.41}$$

For the items the decision variable Y_m^{M} is replaced by the one of items, i.e., X_i^{I}. Thereby, the index relationship between i and w can be expressed by a list or the term $i = w - 1$, because in node $w = 2$ item $i = 1$ is the one in focus. In addition, the constraints are reduced by the outgoing edges, because they do not exist (see Fig. 4.8). The relevant nodes are the ones from node $w = 2$ to $w = \bar{I} + 1 = 5$.

$$X_{w-1}^{\mathrm{I}} = V_w^{\mathrm{C}} + \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} > w, E_{\tilde{w}w}^{\mathrm{I}} > 0\right\}} Z_{\tilde{w}w}^{\mathrm{I}} \quad \forall\, w \in \{2, \ldots, \bar{I} + 1\} \tag{4.42}$$

Finally, the edges of the pairs need to have the same flow. Each pair is identified by the values within a row of the matrix $E_{w\tilde{w}}^{\mathrm{I}}$. One edge of a pair has the value one and the other edge a value greater one, which equals the column index of the first edge. For an arbitrary row $w = 7$, we need the column indices. We select one with an entry in $E_{w\tilde{w}}^{\mathrm{I}}$, which is greater than one, e.g., column $\tilde{w} = 4$. Thereby, for choosing $\tilde{w}$ only columns 2 through $7 - 1 = 6$ need to be considered. The entry in the matrix is $E_{7,4}^{\mathrm{I}} = 2$. This means that the missing edge to this pair is that in column $E_{w\tilde{w}}^{\mathrm{I}}$, i.e., 2. Outgoing edges only exist for “module” nodes, i.e., 6 through 16 in this example.

$$Z_{w\tilde{w}}^{\mathrm{I}} = Z_{w,E_{w\tilde{w}}^{\mathrm{I}}}^{\mathrm{I}} \quad \forall\, w \in \{\bar{I}+2, \ldots, 2^{\bar{I}}\}, \tilde{w} \in \left\{\tilde{w} | \tilde{w} \in \{2, \ldots, w-1\}, E_{w\tilde{w}}^{\mathrm{I}} > 1\right\} \tag{4.43}$$

Instead of constraints (4.41), (4.42), and (4.43), we substitute one of the edges of the pairs by the other so that only the edges with an entry of one in $E_{w\tilde{w}}^{\mathrm{I}}$ remain.[30] This makes the constraint (4.43) dispensable. Thus, the number of constraints and decision variables is significantly reduced, which should speed up the solving. Furthermore, only one of the two edges of a pair has to appear on the output side of the constraint (4.41). This means that not all edges with $E_{w\tilde{w}}^{\mathrm{I}}$ greater than zero are considered, but only the ones with $E_{w\tilde{w}}^{\mathrm{I}}$ equalling one. Since the edges in a pair do not appear twice, the factor two can be removed from the equation. However, the influence on the input side is valid for the module and item constraints. The output variable of the core graph V_v^{C} stays unmodified. But the edges must be changed. Only edges with $E_{w\tilde{w}}^{\mathrm{I}} = 1$ are kept in the model. But still an edge from, e.g., node $w = 7$ to $\tilde{w} = 4$ exists. These edges—more precisely the flow variables of such edges—are substituted by their pair flow, i.e., from

[30] This methodology equals that of an hypergraph, where a (hyper-)edge connects more than just two nodes. In our case it connects one source node with two sink nodes. Cf. Borndörfer et al. (2012): *Vehicle rotation planning*, p. 5.

node $w = 7$ to node $\tilde{w} = 2$. Thus, the sum over the edges in the right hand side of the constraints (4.41) and (4.42) are separated into the edges that have a value of $E^{\mathrm{I}}_{w\tilde{w}} = 1$ and the ones that have a value $E^{\mathrm{I}}_{w\tilde{w}}$ greater than one.

$$\sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}<w,E^{\mathrm{I}}_{w\tilde{w}}=1\right\}} Z^{\mathrm{I}}_{w\tilde{w}} + Y^{\mathrm{M}}_{2^{\bar{I}}+1-w} = V^{\mathrm{C}}_{w} + \sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}>w,E^{\mathrm{I}}_{\tilde{w}w}=1\right\}} Z^{\mathrm{I}}_{\tilde{w}w} + \sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}>w,E^{\mathrm{I}}_{\tilde{w}w}>1\right\}} Z^{\mathrm{I}}_{E^{\mathrm{I}}_{\tilde{w}w},w} \quad \forall\ w \in \{\bar{I}+2,\ldots,2^{\bar{I}}\} \tag{4.44}$$

$$X^{\mathrm{I}}_{w-1} = V^{\mathrm{C}}_{w} + \sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}>w,E^{\mathrm{I}}_{\tilde{w}w}=1\right\}} Z^{\mathrm{I}}_{\tilde{w}w} + \sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}>w,E^{\mathrm{I}}_{\tilde{w}w}>1\right\}} Z^{\mathrm{I}}_{E^{\mathrm{I}}_{\tilde{w}w},w} \quad \forall\ w \in \{2,\ldots,\bar{I}+1\} \tag{4.45}$$

This reduces the number of flow variables for edges by a factor of two.

The same has to be developed for the recycling. The difference is here that the recycling graph is based on the grey items in the nodes of the core graph. These represent non-functioning items. The resulting graph is displayed in Fig. 4.9. The nodes have the same index and the edges go (roughly speaking) in the inverse direction compared to the distribution graph. The resulting edge matrix $E^{\mathrm{R}}_{w\tilde{w}}$ is listed in Table 4.9. It is an upper triangle matrix. This matrix is rotated by 180 degrees or flipped horizontally and vertically compared to $E^{\mathrm{I}}_{w\tilde{w}}$ and all values greater than one are transformed according to $E^{\mathrm{R}}_{w\tilde{w}} = 2^{\bar{I}} + 1 - E^{\mathrm{I}}_{w\tilde{w}}$.

The flow constraints of the network are also similar to the ones of the distribution graph. The output is $Y^{\mathrm{R}}_{m} + Y^{\mathrm{D}}_{m}$ and $X^{\mathrm{R}}_{i} + X^{\mathrm{D}}_{i}$ instead of Y^{M}_{m} and X^{I}_{i}. The input of each node is identical to the other graph, i.e., V^{C}_{v}. Hence, the constraints are

$$\sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}>w,E^{\mathrm{R}}_{w\tilde{w}}>0\right\}} Z^{\mathrm{R}}_{w\tilde{w}} + 2\left(Y^{\mathrm{R}}_{w} + Y^{\mathrm{D}}_{w}\right) = 2\left(V^{\mathrm{C}}_{w} + \sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}<w,E^{\mathrm{R}}_{\tilde{w}w}>0\right\}} Z^{\mathrm{R}}_{\tilde{w}w}\right) \quad \forall\ w \in \left\{1,\ldots,2^{\bar{I}} - \bar{I} - 1\right\} \tag{4.46}$$

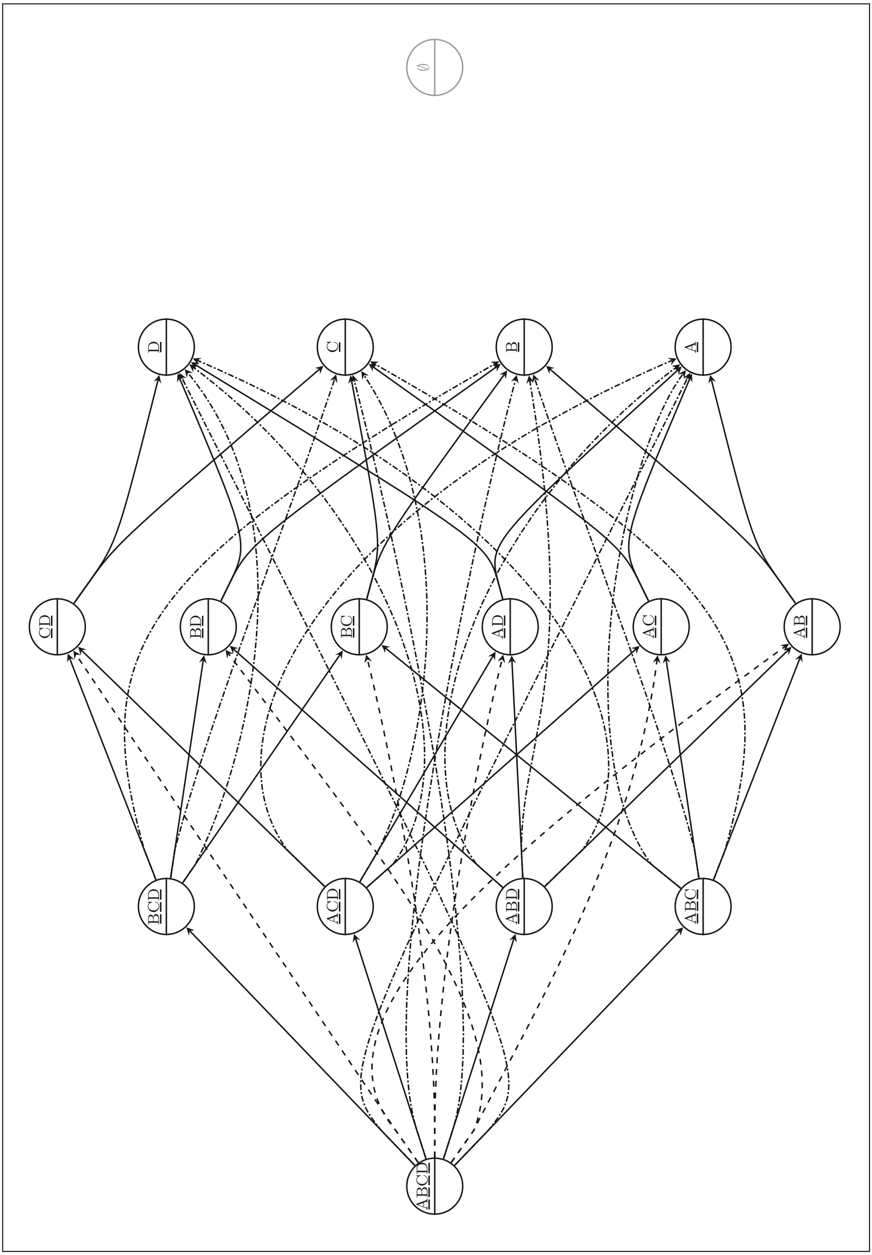

Fig. 4.9 Recycling graph

Table 4.9 Matrix representation of recycling graph edges

		recycling graph edges $E^{\mathrm{R}}_{w\tilde{w}}$															
		node $\tilde{w}$															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	w	ABCD	BCD	ACD	ABD	ABC	CD	BD	BC	AD	AC	AB	D	C	B	A	∅
1	ABCD		15	14	13	12	11	10	9	1	1	1	1	1	1	1	.
2	BCD			.	.	.	14	13	12	.	.	.	1	1	1	.	.
3	ACD				.	.	15	.	.	13	12	.	1	1	.	1	.
4	ABD					.	.	15	.	14	.	12	1	.	1	1	.
5	ABC						.	.	15	.	14	13	.	1	1	1	.
6	CD							.	.	.	.	.	13	1	.	.	.
7	BD								.	.	.	.	14	.	1	.	.
8	BC									.	.	.	.	14	1	.	.
9	AD										.	.	15	.	.	1	.
10	AC											.	.	15	.	1	.
11	AB												.	.	15	1	.
12	D													.	.	.	.
13	C														.	.	.
14	B															.	.
15	A																.
16	∅																

Dots denote a value of zero and white spaces are not of interest. Only the framed blocks of cells can have a value other than zero.

as well as

$$X^{\mathrm{R}}_{2^{\bar{I}}-w} + X^{\mathrm{D}}_{2^{\bar{I}}-w} = V^{\mathrm{C}}_{w} + \sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}<w,E^{\mathrm{R}}_{\tilde{w}w}>0\right\}} Z^{\mathrm{R}}_{\tilde{w}w} \quad \forall\, w \in \{2^{\bar{I}} - \bar{I}, \ldots, 2^{\bar{I}}\} \tag{4.47}$$

and

$$Z^{\mathrm{R}}_{w,\tilde{w}} = Z^{\mathrm{R}}_{w,E^{\mathrm{R}}_{w\tilde{w}}} \quad \forall\, w \in \left\{1, \ldots, 2^{\bar{I}} - \bar{I} - 1\right\}, \tilde{w} \in \left\{\tilde{w}|\tilde{w} > w, E^{\mathrm{R}}_{w\tilde{w}} > 1\right\} \tag{4.48}$$

for the equality of the flows in the edge pairs. The node module conversion is rather easy, because the node index equals the module index. The item index is easily calculated by subtracting the node index from the number of nodes ($2^{\bar{I}}$). Of course, the flow variables have changed to $Z^{\mathrm{R}}_{w\tilde{w}}$, too. Conducting the same transformation as with constraints (4.41), (4.42), and (4.43) to (4.44) and (4.45) we reduce Eqs. (4.46), (4.47), and (4.48) to

$$\sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}>w,E^{\mathrm{R}}_{w\tilde{w}}=1\right\}} Z^{\mathrm{R}}_{w\tilde{w}} + Y^{\mathrm{R}}_{w} + Y^{\mathrm{D}}_{w}$$
$$= V^{\mathrm{C}}_{w} + \sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}<w,E^{\mathrm{R}}_{\tilde{w}w}=1\right\}} Z^{\mathrm{R}}_{\tilde{w}w} + \sum_{\tilde{w}\in\left\{\tilde{w}|\tilde{w}<w,E^{\mathrm{R}}_{\tilde{w}w}>1\right\}} Z^{\mathrm{R}}_{E^{\mathrm{R}}_{\tilde{w}w},w}$$

$$\forall\ w \in \left\{1, \ldots, 2^{\bar{I}} - \bar{I} - 1\right\} \tag{4.49}$$

and

$$X^{\mathrm{R}}_{2^{\bar{I}}-w} + X^{\mathrm{D}}_{2^{\bar{I}}-w} = V^{\mathrm{C}}_w + \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} < w, E^{\mathrm{R}}_{\tilde{w}w} = 1\right\}} Z^{\mathrm{R}}_{\tilde{w}w} + \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} < w, E^{\mathrm{R}}_{\tilde{w}w} > 1\right\}} Z^{\mathrm{R}}_{E^{\mathrm{R}}_{\tilde{w}w}, w}$$
$$\forall\ w \in \{2^{\bar{I}} - \bar{I}, \ldots, 2^{\bar{I}}\}. \tag{4.50}$$

When taking a look at the constraints (4.25), (4.44), (4.45), (4.49), and (4.50), we notice that they all contain the variable V^{C}_v. This can be substituted by transforming Eq. (4.25) to

$$V^{\mathrm{C}}_v = \rho_v\, Q^{\mathrm{C}} + \sum_{\tilde{v} \in \{\tilde{v} | \tilde{v} > v, E^{\mathrm{C}}_{\tilde{v}v} = 1\}} Z^{\mathrm{C}}_{\tilde{v}v} - \sum_{\tilde{v} \in \{\tilde{v} | \tilde{v} < v, E^{\mathrm{C}}_{v\tilde{v}} = 1\}} Z^{\mathrm{C}}_{v\tilde{v}} \quad \forall\ v\,. \tag{4.51}$$

Substituting the variable V^{C}_v in the constraints (4.44), (4.45), (4.49), and (4.50) and v by w leads to

$$\begin{aligned} Y^{\mathrm{M}}_{2^{\bar{I}}+1-w} = \rho_w\, Q^{\mathrm{C}} &+ \sum_{\tilde{w} \in \{\tilde{w} | \tilde{w} > w, E^{\mathrm{C}}_{\tilde{w}w} = 1\}} Z^{\mathrm{C}}_{\tilde{w}w} - \sum_{\tilde{w} \in \{\tilde{w} | \tilde{w} < w, E^{\mathrm{C}}_{w\tilde{w}} = 1\}} Z^{\mathrm{C}}_{w\tilde{w}} \\ &+ \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} > w, E^{\mathrm{I}}_{\tilde{w}w} = 1\right\}} Z^{\mathrm{I}}_{\tilde{w}w} + \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} > w, E^{\mathrm{I}}_{\tilde{w}w} > 1\right\}} Z^{\mathrm{I}}_{E^{\mathrm{I}}_{\tilde{w}w}, w} \\ &- \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} < w, E^{\mathrm{I}}_{w\tilde{w}} = 1\right\}} Z^{\mathrm{I}}_{w\tilde{w}} \qquad \forall\ w \in \{\bar{I} + 2, \ldots, 2^{\bar{I}}\} \end{aligned} \tag{4.52}$$

$$\begin{aligned} X^{\mathrm{I}}_{w-1} = \rho_w\, Q^{\mathrm{C}} &+ \sum_{\tilde{w} \in \{\tilde{w} | \tilde{w} > w, E^{\mathrm{C}}_{\tilde{w}w} = 1\}} Z^{\mathrm{C}}_{\tilde{w}w} - \sum_{\tilde{w} \in \{\tilde{w} | \tilde{w} < w, E^{\mathrm{C}}_{w\tilde{w}} = 1\}} Z^{\mathrm{C}}_{w\tilde{w}} \\ &+ \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} > w, E^{\mathrm{I}}_{\tilde{w}w} = 1\right\}} Z^{\mathrm{I}}_{\tilde{w}w} + \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} > w, E^{\mathrm{I}}_{\tilde{w}w} > 1\right\}} Z^{\mathrm{I}}_{E^{\mathrm{I}}_{\tilde{w}w}, w} \\ &\qquad \forall\ w \in \{2, \ldots, \bar{I} + 1\} \end{aligned} \tag{4.53}$$

$$\begin{aligned} Y^{\mathrm{R}}_w + Y^{\mathrm{D}}_w = \rho_w\, Q^{\mathrm{C}} &+ \sum_{\tilde{w} \in \{\tilde{w} | \tilde{w} > w, E^{\mathrm{C}}_{\tilde{w}w} = 1\}} Z^{\mathrm{C}}_{\tilde{w}w} - \sum_{\tilde{w} \in \{\tilde{w} | \tilde{w} < w, E^{\mathrm{C}}_{w\tilde{w}} = 1\}} Z^{\mathrm{C}}_{w\tilde{w}} \\ &+ \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} < w, E^{\mathrm{R}}_{\tilde{w}w} = 1\right\}} Z^{\mathrm{R}}_{\tilde{w}w} + \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} < w, E^{\mathrm{R}}_{\tilde{w}w} > 1\right\}} Z^{\mathrm{R}}_{E^{\mathrm{R}}_{\tilde{w}w}, w} \end{aligned}$$

$$- \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} > w, E^{\mathrm{R}}_{w\tilde{w}} = 0\right\}} Z^{\mathrm{R}}_{w\tilde{w}} \qquad \forall\, w \in \left\{1, \ldots, 2^{\bar{I}} - \bar{I} - 1\right\} \tag{4.54}$$

$$\begin{aligned} X^{\mathrm{R}}_{2^{\bar{I}}-w} + X^{\mathrm{D}}_{2^{\bar{I}}-w} = \rho_w\, Q^{\mathrm{C}} &+ \sum_{\tilde{w} \in \{\tilde{w} | \tilde{w} > w, E^{\mathrm{C}}_{\tilde{w}w} = 1\}} Z^{\mathrm{C}}_{\tilde{w}w} - \sum_{\tilde{w} \in \{\tilde{w} | \tilde{w} < w, E^{\mathrm{C}}_{w\tilde{w}} = 1\}} Z^{\mathrm{C}}_{w\tilde{w}} \\ &+ \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} < w, E^{\mathrm{R}}_{\tilde{w}w} = 1\right\}} Z^{\mathrm{R}}_{\tilde{w}w} + \sum_{\tilde{w} \in \left\{\tilde{w} | \tilde{w} < w, E^{\mathrm{R}}_{\tilde{w}w} > 0\right\}} Z^{\mathrm{R}}_{E^{\mathrm{R}}_{\tilde{w}w}, w} \\ &\forall\, w \in \{2^{\bar{I}} - \bar{I}, \ldots, 2^{\bar{I}}\}. \end{aligned} \tag{4.55}$$

So far we considered the two usage options of distribution and recycling (including disposal). With just these two options the variables v and w could be used synonymously, as done above. But if items consist of the wrong material, recycling is not an option anymore, because we assume that the wrong material is not suitable to meet the demand of material to recycle. Therefore, these items have to be disposed of. Extending the core graph with this third specification of items increases the number of nodes from 2^n to 3^n and makes the differentiation of v and w necessary.

4.2.2.4 Core condition using graphs with three usage options

The number of edges of the core graph depends on the number of items $\bar{I}$ and number of usage categories u. The graph consists of $(u-1)\bar{I}u^{\bar{I}-1}$ edges (see appendix C.2). For our example the core graph has $3^{4-1}4(3-1) = 216$ edges, because we consider the three categories (i.e., $u = 3$): distribution, recycling, and disposal.[31] The number of nodes and the edges for the distribution and recycling graph are identical, only that a third graph—the disposal graph—is added with the same size as the other two. Obviously, this size cannot be handled manually. Therefore, the extension to integrate the disposal option is described in the sequel with a main focus on the automatic generation.

First, the node indexing of the core graph is developed. Given the number of items $\bar{I}$, the number of nodes is calculated by $3^{\bar{I}}$. The indexing can be in any way as long as the nodes have a unique index. The node representing the case where all items have to be disposed of shall be number 1. From

[31] For the start of the consideration we differentiated only two categories (=classes), which results in $2^{4-1}4(2-1) = 32$ edges for the core graph.

this node the remaining indices of the nodes can be developed according to a three-base numbering system.[32] Each item of a core is represented by a digit. Thereby, only three values for each digit are allowed. The values could be 0, 1, and 2 or $\underline{\underline{\mathrm{A}}}$, $\underline{\mathrm{A}}$, and A, respectively. Thereby, the value zero is related to an item that has to be disposed of (e.g., $\underline{\underline{\mathrm{A}}}$). The value one represents an item that can be recycled or disposed of (e.g., $\underline{\mathrm{A}}$) and the value two is used to identify an item that can be used for everything, even distribution (e.g., A). Hence, a number with four digits is, for example, $\underline{\underline{\mathrm{A}}}\underline{\mathrm{B}}\mathrm{C}\underline{\underline{\mathrm{D}}}$. In addition, the order of the digits is reversed. This is only applied to achieve that the counting starts with item A to get node number 1, 2, 3, etc. no matter how many items exist. Since the counting starts with zero and ends with $3^n - 1$, a value of one is added afterwards to get a node indexing of 1 through 3^n. According to this definition, the number of a node representing the combination $\underline{\underline{\mathrm{A}}}\underline{\mathrm{B}}\mathrm{C}\underline{\mathrm{D}}$ is equivalent to $1210_3 \rightarrow 48_{10} + 1 = 49_{10}$.[33]

This calculation is reversible. In order to get the item coding from a node number the following steps are necessary. Node number 18 shall be converted into the item representation. First, we subtract one from the node number and convert it to the base three system, i.e., $18 - 1 = 17_{10} = 0122_3$. In a second step this order of digits is reversed to 2210. Afterwards, the single digits are replaced with a letter format. Thereby, the first digit is always item A and the last item $\bar{I}$, i.e., $2210 \equiv \mathrm{AB}\underline{\mathrm{C}}\underline{\underline{\mathrm{D}}}$. Note that the letters are only used for a better understanding. For the automatic generation of the core graph only the node number and the reversed three-base number are of interest.

Now that we have the node number and the corresponding item combination, the probability that exactly this item combination appears can be calculated. The probability is denoted by ρ_v for every node v. It is calculated according to the condition of an item. Fig. 3.3 on page 36 illustrates the classification. The damaging is irrelevant for the core graph, because only the core condition matters. The damaging happens within the disassembly process and (per definition) only to single items that have been separated from modules. This means that with a probability of $(1 - \zeta_i)$ times $(1 - \eta_i)$ it is genuine and functioning, i.e., it can be used for distribution, recycling, and disposal. Such an item is represented by the value 2 in the three-base system (i.e., A, B, C, and D). With a probability of ζ_i times ι_i an item i is non-genuine and of the wrong material. In this case the item has to be

[32] The binary, decimal, and hexadecimal numbering systems are widely used.

[33] The transformation in long: $\underline{\underline{\mathrm{A}}}\underline{\mathrm{B}}\mathrm{C}\underline{\mathrm{D}}$ equals 0121 and in reverse order 1210. This number belongs to a three-base numbering system, i.e., 1210_3. The equivalent number of the decimal system is 48_{10}, because $1 \cdot 3^3 + 2 \cdot 3^2 + 1 \cdot 3^1 + 0 \cdot 3^0 = 48$. Adding 1 results in the node index of 49 for the node labelled with $\underline{\underline{\mathrm{A}}}\underline{\mathrm{B}}\mathrm{C}\underline{\mathrm{D}}$.

Table 4.10 Condition probabilities

	item i			
	A	B	C	D
ζ_i	$3/4$	$3/5$	$1/2$	$1/5$
η_i	0	0	0	0
ι_i	$4/15$	$5/12$	$2/5$	$1/2$

disposed of, i.e., it can neither be used for recycling nor for distribution. Such an item is represented by the value 0 (i.e., $\underline{\underline{\mathrm{A}}}$, $\underline{\underline{\mathrm{B}}}$, $\underline{\underline{\mathrm{C}}}$, and $\underline{\underline{\mathrm{D}}}$). With the remaining probability $1-(1-\zeta_i)(1-\eta_i)-\zeta_i\,\iota_i$ the item can be used for recycling and disposal, which is represented by the value 1 (i.e., $\underline{\mathrm{A}}$, $\underline{\mathrm{B}}$, $\underline{\mathrm{C}}$, and $\underline{\mathrm{D}}$). To finally calculate the probability of the item combination of a particular node the single probabilities have to be multiplied. For node $v = 18$ we have the item classification 2210. Hence, item A and B can be used for everything, item $\underline{\mathrm{C}}$ for recycling and disposal, and item $\underline{\underline{\mathrm{D}}}$ only for disposal. The probabilities for items A, B, $\underline{\mathrm{C}}$, and $\underline{\underline{\mathrm{D}}}$ are $(1-\zeta_{\mathrm{A}})(1-\eta_{\mathrm{A}})$, $(1-\zeta_{\mathrm{B}})(1-\eta_{\mathrm{B}})$, $1-(1-\zeta_{\mathrm{C}})(1-\eta_{\mathrm{C}})-\zeta_{\mathrm{C}}\,\iota_{\mathrm{C}}$, and $\zeta_{\mathrm{D}}\,\iota_{\mathrm{D}}$, respectively. Using the probabilities given in Table 4.10 leads to a (node) probability of such an item combination in a unit of a core of

$$
\begin{aligned}
\rho_{18} &= [(1-\zeta_{\mathrm{A}})(1-\eta_{\mathrm{A}})]\,[(1-\zeta_{\mathrm{B}})\,(1-\eta_{\mathrm{B}})]\,[1-(1-\zeta_{\mathrm{C}})\,(1-\eta_{\mathrm{C}})-\zeta_{\mathrm{C}}\,\iota_{\mathrm{C}}]\,[\zeta_{\mathrm{D}}\,\iota_{\mathrm{D}}] \\
&= \left[\left(1-\frac{3}{4}\right)(1-0)\right]\left[\left(1-\frac{3}{5}\right)(1-0)\right]\left[1-\left(1-\frac{1}{2}\right)(1-0)-\frac{1}{2}\cdot\frac{2}{5}\right]\left[\frac{1}{5}\cdot\frac{1}{2}\right] \\
&= 0.003\,.
\end{aligned} \tag{4.56}
$$

The sum over all node probabilities equals one, i.e., $\sum_v \rho_v = 1$.

The next step is the creation of the edge matrix of the core graph $E^{\mathrm{C}}_{v\tilde{v}}$. This is an iterative process through all the rows and columns of the matrix, but each element can be determined independent of all the others. Only in the lower triangle (excluding the diagonal) exist values unequal zero. Thereby, the last column (3^n) and the first row have only zero values, because there exist no edge to node 3^n and no edge from node 1. All other nodes have edges according to the following rule. An edge goes from one node to another if and only if for one item of all $\bar{I}$ there is a reduction by exactly one classification value. From node 2210 exist edges to the nodes 1210, 2110, and 2200. The differences between the starting node and the ending nodes are 1000, 0100, and 0010, respectively. This means the Hamming distance is one, i.e., only one digit is changed, and the change of the one digit is a reduction by one classification value from starting to ending node of the edge. Translated into the node numbers it means: node $v = 18$ is the starting node and the nodes 17, 15, and 9 are the ending ones. Thus, in row $v = 18$ the columns $\tilde{v} \in \{9, 15, 17\}$ have a value of 1 and all other elements in row 18 are 0.

Using the edge matrix of the core graph $E^{\mathrm{C}}_{v\tilde{v}}$, the flow variables $Z^{\mathrm{C}}_{v\tilde{v}}$, the node probability ρ_v, the core acquisition quantity Q^{C}, as well as the output of each node of the core graph V^{C}_v to the distribution, recycling, and disposal graph, the constraints for the nodes are

$$\sum_{\tilde{v} \in \{\tilde{v} | \tilde{v} < v, E^{\mathrm{C}}_{v\tilde{v}} = 1\}} Z^{\mathrm{C}}_{v\tilde{v}} + V^{\mathrm{C}}_v = \rho_v \, Q^{\mathrm{C}} + \sum_{\tilde{v} \in \{\tilde{v} | \tilde{v} > v, E^{\mathrm{C}}_{\tilde{v}v} = 1\}} Z^{\mathrm{C}}_{\tilde{v}v} \quad \forall \, v \qquad (4.57)$$

(see Eq. (4.25)). For non-existing edges (i.e., $E^{\mathrm{C}}_{v\tilde{v}} = 0$) the value of the flow variable $Z^{\mathrm{C}}_{v\tilde{v}}$ can be set to zero.

$$Z^{\mathrm{C}}_{v\tilde{v}} = 0 \quad \forall \, v, \tilde{v} \in \left\{ \tilde{v} \,\middle|\, E^{\mathrm{C}}_{v\tilde{v}} = 0 \right\} \qquad (4.58)$$

The resulting core graph for four items is depicted in Fig. 4.10. The different line styles of the edges are only for a better visibility.

The next steps are the distribution, recycling and disposal graphs. An approach with using the node numbering of the core graph might be cumbersome, because in the three graphs the number of nodes is $2^{\bar{I}}$ whereas the number of nodes in the core graph is $3^{\bar{I}}$. Thus, a one-to-one relationship does not exist. As mentioned above, it is also possible to use a list (or matrix) to store the node relationships. Also with this approach, the three graphs have the same representation, which makes only one edge matrix necessary. As basis we use the distribution graph (see Fig. 4.8). This graph contains $2^{\bar{I}}$ nodes, where node number $2^{\bar{I}}$ is not connected to the remaining nodes and could be excluded. Node number 1 shall be the one representing the complete core.

Node number 2 of the distribution graph is for example node BCD. The input of this node is the output of the nodes of the core graph, which contain the item combination BCD as distributable (i.e., class two) together with remaining items of lower classes (i.e., item $\underline{\mathrm{A}}$ or $\underline{\underline{\mathrm{A}}}$). Hence, this node of the distribution graph is connected with two nodes of the core graph. When the modules get smaller (i.e., less number of items within a module), the number of connections with the core graph increases by the factor two per reduced item. Thus, the item nodes have $2^{\bar{I}-1}$ connections with the core graph. To find the right connections we take an arbitrary node of the distribution graph w, e.g., BC. This node has four connections. These are with the nodes $\underline{\mathrm{A}}\mathrm{BC}\underline{\mathrm{D}}$, $\underline{\mathrm{A}}\mathrm{BC}\underline{\underline{\mathrm{D}}}$, $\underline{\underline{\mathrm{A}}}\mathrm{BC}\underline{\mathrm{D}}$, and $\underline{\underline{\mathrm{A}}}\mathrm{BC}\underline{\underline{\mathrm{D}}}$. Thus, the connection

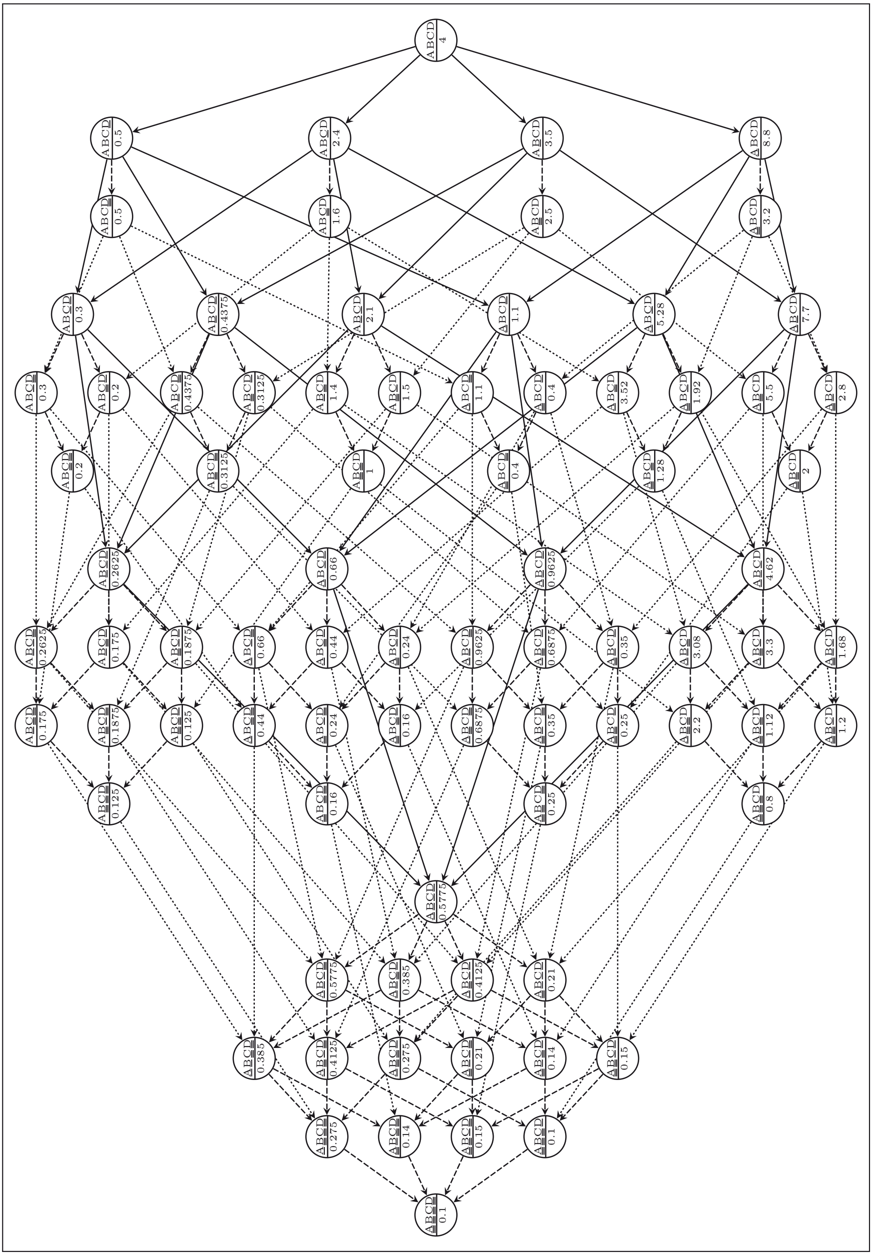

Fig. 4.10 Condition dependencies core graph (3 classes, 4 items)

exists between the node BC of the distribution graph and all nodes of the core graph with the value 2 for the item positions B and C and values unequal 2 for all remaining item positions, i.e., A and D. Let us suppose the set of connections between the distribution and core graph is denoted by L_w^{I}. Then, this set would contain the entries: $L_{\mathrm{BC}}^{\mathrm{I}} = \{53, 26, 52, 25\}$. The set for node 1 is $L_{\mathrm{ABCD}}^{\mathrm{I}} = \{81\}$.

Applying this procedure to node $\underline{\mathrm{B}}\underline{\mathrm{C}}$ of the recycling graph, it leads to connections to the nodes $\mathrm{A}\underline{\mathrm{B}}\underline{\mathrm{C}}\mathrm{D}$, $\mathrm{A}\underline{\mathrm{B}}\underline{\mathrm{C}}\underline{\underline{\mathrm{D}}}$, $\underline{\underline{\mathrm{A}}}\underline{\mathrm{B}}\underline{\mathrm{C}}\mathrm{D}$, and $\underline{\underline{\mathrm{A}}}\underline{\mathrm{B}}\underline{\mathrm{C}}\underline{\underline{\mathrm{D}}}$. The list would have the elements $L_{\mathrm{BC}}^{\mathrm{R}} = \{69, 15, 67, 13\}$. Lastly, the connections between node $\underline{\underline{\mathrm{B}}}\underline{\underline{\mathrm{C}}}$ of the disposal graph to the core graph exist to the nodes $\mathrm{A}\underline{\underline{\mathrm{B}}}\underline{\underline{\mathrm{C}}}\mathrm{D}$, $\mathrm{A}\underline{\underline{\mathrm{B}}}\underline{\underline{\mathrm{C}}}\underline{\mathrm{D}}$, $\underline{\mathrm{A}}\underline{\underline{\mathrm{B}}}\underline{\underline{\mathrm{C}}}\mathrm{D}$, and $\underline{\mathrm{A}}\underline{\underline{\mathrm{B}}}\underline{\underline{\mathrm{C}}}\underline{\mathrm{D}}$, i.e., $L_{\mathrm{BC}}^{\mathrm{D}} = \{57, 30, 56, 29\}$. The edge matrix $E_{w\tilde{w}}^{\mathrm{D}}$ of the three graphs is equal and an upper triangle matrix of dimension $2^{\bar{I}} \times 2^{\bar{I}}$. To set the values in the matrix, an arbitrary element can be chosen. If the column label is not a strict subset of the row label (e.g., row: AB, column: AC), the element is 0. Otherwise, it is set to 1 (e.g., row: ABC, column: AC) and the column with the disjoint label (i.e., B) is set to the node number of the first chosen column (i.e., AC). The resulting edge matrix is listed in Table 4.11.

Before we come to the equations of the constraints another mapping is necessary. It is the one between the module index m and the node index w of the three graphs. The reason why this mapping is required is the fact that one or more modules of the $2^{\bar{I}} - \bar{I} - 1$ theoretically possible modules do not exist, because of geographical, technical, or topological constraints. But these nodes representing such a module cannot be simply removed. Let us assume a module with six items: ABCDEF. Disconnecting one joint might lead to three modules AB, CD, and EF. This is a three-partition. Hence, the modules ABCD, ABEF, and CDEF do not exist. If the nodes representing these three non-existing modules would be removed from the graph it would not be possible to get the modules AB, CD, and EF out of one module ABCDEF. This would exclude a feasible solution from the planning. Therefore, an allocation between the module index m and the node index w is required.

For every node w an entry exists in L_w^{A} that equals the module index m or item index i if they exist. In the case that a module does not exist, the mapping value L_w^{A} equals zero. Using this information two sets of constraints are developed for the modules, i.e., one where a module exists and one where no module exists. With the node connection lists L_w^{I}, L_w^{R}, and L_w^{D}, the edge matrix $E_{w\tilde{w}}^{\mathrm{D}}$, as well as the knowledge that the nodes from $w = 1$ through $w = 2^{\bar{I}} - \bar{I} - 1$ represent modules, the constraints for the three graphs can be formulated. We start with the distribution graph. The output of a node, i.e., edges to other nodes and the planned quantity of modules and items

Table 4.11 Matrix representation of distribution, recycling, and disposal graph edges

		distribution, recycling, disposal graph edges $E^{\mathrm{D}}_{w\tilde{w}}$															
		node $\tilde{w}$															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	w	ABCD	BCD	ACD	ABD	ABC	CD	BD	AD	BC	AC	AB	A	B	C	D	∅
1	ABCD		1	1	1	1	1	1	1	8	7	6	5	4	3	2	.
2	BCD			.	.	.	1	1	.	1	.	.	.	6	7	9	.
3	ACD				.	.	1	.	1	.	1	.	6	.	8	10	.
4	ABD					.	.	1	1	.	.	1	7	8	.	11	.
5	ABC						.	.	.	1	1	1	9	10	11	.	.
6	CD							.	.	.	.	.	.	.	1	14	.
7	BD								.	.	.	.	.	1	.	13	.
8	AD									.	.	.	1	.	.	12	.
9	BC										.	.	.	1	13	.	.
10	AC											.	1	.	12	.	.
11	AB												1	12	.	.	.
12	A													.	.	.	.
13	B														.	.	.
14	C															.	.
15	D																.
16	∅																

Dots denote a value of zero and white spaces are not of interest. Only the framed blocks of cells can have a value other than zero.

to distribute, is on the left hand side. This output must equal the input, which is the connection from the core graph as well as the ingoing edges from other nodes. Of the outgoing edges only one per pair is used. We use the one with the value 1 in the edge matrix. For modules the variable Y^{M}_{m} denotes the planned quantity for distribution. The index m is stored in L^{A}_{w}. For the input side the output of the core graph nodes is added according to the node list L^{I}_{w}. The resulting constraints for the nodes representing existing modules are

$$\sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{w\tilde{w}}=1\right\}} Z^{\mathrm{I}}_{w\tilde{w}} + Y^{\mathrm{M}}_{L^{\mathrm{A}}_{w}}$$

$$= \sum_{\tilde{w}\in L^{\mathrm{I}}_{w}} Y^{\mathrm{C}}_{\tilde{w}} + \sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{\tilde{w}w}=1\right\}} Z^{\mathrm{I}}_{\tilde{w}w} + \sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{\tilde{w}w}>1\right\}} Z^{\mathrm{I}}_{\tilde{w},E^{\mathrm{D}}_{\tilde{w}w}}$$

$$\forall\ w \in \left\{w\,\middle|\,w \in \left\{1,\ldots,2^{\bar{I}}-\bar{I}-1\right\}, L^{\mathrm{A}}_{w} > 0\right\} \quad (4.59)$$

and representing non-existing modules are

$$\sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{w\tilde{w}}=1 \right\}} Z^{\mathrm{I}}_{w\tilde{w}} = \sum_{\tilde{w} \in L^{\mathrm{I}}_{w}} Y^{\mathrm{C}}_{\tilde{w}} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{\tilde{w}w}=1 \right\}} Z^{\mathrm{I}}_{\tilde{w}w} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{\tilde{w}w}>1 \right\}} Z^{\mathrm{I}}_{\tilde{w},E^{\mathrm{D}}_{\tilde{w}w}}$$
$$\forall\ w \in \left\{ w \middle| w \in \left\{ 1,\dots,2^{\bar{I}} - \bar{I} - 1 \right\}, L^{\mathrm{A}}_{w} = 0 \right\}. \quad (4.60)$$

In the latter, the planning variable Y^{M}_{m} is missing.

For the recycling and disposal graph the constraints are slightly modified so that they contain the appropriate variables.

$$\sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{w\tilde{w}}=1 \right\}} Z^{\mathrm{R}}_{w\tilde{w}} + Y^{\mathrm{R}}_{L^{\mathrm{A}}_{w}}$$
$$= \sum_{\tilde{w} \in L^{\mathrm{R}}_{w}} Y^{\mathrm{C}}_{\tilde{w}} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{\tilde{w}w}=1 \right\}} Z^{\mathrm{R}}_{\tilde{w}w} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{\tilde{w}w}>1 \right\}} Z^{\mathrm{R}}_{\tilde{w},E^{\mathrm{D}}_{\tilde{w}w}}$$
$$\forall\ w \in \left\{ w \middle| w \in \left\{ 1,\dots,2^{\bar{I}} - \bar{I} - 1 \right\}, L^{\mathrm{A}}_{w} > 0 \right\} \quad (4.61)$$

$$\sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{w\tilde{w}}=1 \right\}} Z^{\mathrm{R}}_{w\tilde{w}} = \sum_{\tilde{w} \in L^{\mathrm{R}}_{w}} Y^{\mathrm{C}}_{\tilde{w}} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{\tilde{w}w}=1 \right\}} Z^{\mathrm{R}}_{\tilde{w}w} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{\tilde{w}w}>1 \right\}} Z^{\mathrm{R}}_{\tilde{w},E^{\mathrm{D}}_{\tilde{w}w}}$$
$$\forall\ w \in \left\{ w \middle| w \in \left\{ 1,\dots,2^{\bar{I}} - \bar{I} - 1 \right\}, L^{\mathrm{A}}_{w} = 0 \right\} \quad (4.62)$$

$$\sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{w\tilde{w}}=1 \right\}} Z^{\mathrm{D}}_{w\tilde{w}} + Y^{\mathrm{D}}_{L^{\mathrm{A}}_{w}}$$
$$= \sum_{\tilde{w} \in L^{\mathrm{D}}_{w}} Y^{\mathrm{C}}_{\tilde{w}} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{\tilde{w}w}=1 \right\}} Z^{\mathrm{D}}_{\tilde{w}w} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{\tilde{w}w}>1 \right\}} Z^{\mathrm{D}}_{\tilde{w},E^{\mathrm{D}}_{\tilde{w}w}}$$
$$\forall\ w \in \left\{ w \middle| w \in \left\{ 1,\dots,2^{\bar{I}} - \bar{I} - 1 \right\}, L^{\mathrm{A}}_{w} > 0 \right\} \quad (4.63)$$

$$\sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{w\tilde{w}}=1 \right\}} Z^{\mathrm{D}}_{w\tilde{w}} = \sum_{\tilde{w} \in L^{\mathrm{D}}_{w}} Y^{\mathrm{C}}_{\tilde{w}} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{\tilde{w}w}=1 \right\}} Z^{\mathrm{D}}_{\tilde{w}w} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{\tilde{w}w}>1 \right\}} Z^{\mathrm{D}}_{\tilde{w},E^{\mathrm{D}}_{\tilde{w}w}}$$
$$\forall\ w \in \left\{ w \middle| w \in \left\{ 1,\dots,2^{\bar{I}} - \bar{I} - 1 \right\}, L^{\mathrm{A}}_{w} = 0 \right\} \quad (4.64)$$

In general, the same procedure is applied to the nodes connected to items. But so far we neglected the aspect of item damaging during the disassembly process.[34] All items that are separated to single items might get damaged during the disassembly process with a probability of θ_i. This is not an item condition, because an item gets not damaged if the core is not disassembled. However, this damaging has to be considered. Of course, damaging happens with all single items, regardless if they are intended for distribution, recycling, or disposal. But only for item distribution this is relevant, because a damaged item can still be recycled or disposed of. Thus, we have to differentiate between damaged and undamaged items.

If $\widetilde{X}_i^{\mathrm{I}}$ functioning (and genuine) items are the result of the disassembly process, they are separated into functioning undamaged X_i^{I} and functioning damaged X_i^{A} items, i.e.

$$\widetilde{X}_i^{\mathrm{I}} = X_i^{\mathrm{I}} + X_i^{\mathrm{A}} \quad \forall\, i\,. \tag{4.65}$$

Furthermore, at least a fraction of $0 \leq \theta_i \leq 1$ of the functioning items is damaged.

$$X_i^{\mathrm{A}} \geq \theta_i\, \widetilde{X}_i^{\mathrm{I}} = \theta_i \left(X_i^{\mathrm{I}} + X_i^{\mathrm{A}}\right) \qquad \Leftrightarrow \qquad (1-\theta_i)\, X_i^{\mathrm{A}} \geq \theta_i\, X_i^{\mathrm{I}} \quad \forall\, i \tag{4.66}$$

The variable $\widetilde{X}_i^{\mathrm{I}}$ is the output of the distribution graph and can be substituted with $X_i^{\mathrm{I}} + X_i^{\mathrm{A}}$. The damaged items (or assumed damaged items)[35] increase the items to recycle or dispose of. Assuming, that the output of the recycling and disposal graph is $\widetilde{X}_i^{\mathrm{R}}$ and $\widetilde{X}_i^{\mathrm{D}}$, respectively, the equation

$$X_i^{\mathrm{R}} + X_i^{\mathrm{D}} = \widetilde{X}_i^{\mathrm{R}} + \widetilde{X}_i^{\mathrm{D}} + X_i^{\mathrm{A}} \qquad \Leftrightarrow \qquad \widetilde{X}_i^{\mathrm{R}} = X_i^{\mathrm{R}} + X_i^{\mathrm{D}} - \widetilde{X}_i^{\mathrm{D}} - X_i^{\mathrm{A}} \quad \forall\, i \tag{4.67}$$

must apply. In addition, the quantity of items to dispose of has to be greater than or equal the output of the disposal graph.

$$X_i^{\mathrm{D}} \geq \widetilde{X}_i^{\mathrm{D}} \quad \forall\, i \tag{4.68}$$

This does not have to apply to the recycling, because any item that can be recycled can also be disposed of. Now we can formulate the constraints for the three graphs regarding the nodes representing the single items. Note that these nodes have no outgoing edges. Thus, the only output is the variable for the planned quantity of items to distribute, i.e., $\widetilde{X}_i^{\mathrm{I}}$, $\widetilde{X}_i^{\mathrm{R}}$, and $\widetilde{X}_i^{\mathrm{D}}$. Thereby,

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[34] We assume that only single items separated from a module can be damaged. All items in a module get not damaged.

[35] Because of the greater than or equal relationship in Eq. (4.66) more than the exact damaged fraction of items can be marked as damaged. To avoid this, an extra constraint in the form of $X_i^{\mathrm{A}} < \frac{\theta_i}{1-\theta_i} X_i^{\mathrm{I}} + 1$ could be added, because we work with integral values.

$\widetilde{X}_i^{\mathrm{I}}$ and $\widetilde{X}_i^{\mathrm{R}}$ will be substituted by the right hand side of Eqs. (4.65) and (4.67). The item index i can be derived from the node index w by the entry in L_w^{A}. For the distribution graph it means that

$$X_{L_w^{\mathrm{A}}}^{\mathrm{I}} + X_{L_w^{\mathrm{A}}}^{\mathrm{A}} = \sum_{\tilde{w} \in L_w^{\mathrm{I}}} Y_{\tilde{w}}^{\mathrm{C}} + \sum_{\tilde{w} \in \left\{\tilde{w} \middle| E_{\tilde{w}w}^{\mathrm{D}}=1\right\}} Z_{\tilde{w}w}^{\mathrm{I}} + \sum_{\tilde{w} \in \left\{\tilde{w} \middle| E_{\tilde{w}w}^{\mathrm{D}}>1\right\}} Z_{\tilde{w},E_{\tilde{w}w}^{\mathrm{D}}}^{\mathrm{I}}$$
$$\forall\ w \in \left\{2^{\bar{I}} - \bar{I}, \ldots, 2^{\bar{I}} - 1\right\}. \quad (4.69)$$

The same applies to the recycling and disposal graph so that we have

$$X_{L_w^{\mathrm{A}}}^{\mathrm{R}} + X_{L_w^{\mathrm{A}}}^{\mathrm{D}} - \widetilde{X}_{L_w^{\mathrm{A}}}^{\mathrm{D}} - X_{L_w^{\mathrm{A}}}^{\mathrm{A}}$$
$$= \sum_{\tilde{w} \in L_w^{\mathrm{R}}} Y_{\tilde{w}}^{\mathrm{C}} + \sum_{\tilde{w} \in \left\{\tilde{w} \middle| E_{\tilde{w}w}^{\mathrm{D}}=1\right\}} Z_{\tilde{w}w}^{\mathrm{R}} + \sum_{\tilde{w} \in \left\{\tilde{w} \middle| E_{\tilde{w}w}^{\mathrm{D}}>1\right\}} Z_{\tilde{w},E_{\tilde{w}w}^{\mathrm{D}}}^{\mathrm{R}}$$
$$\forall\ w \in \left\{2^{\bar{I}} - \bar{I}, \ldots, 2^{\bar{I}} - 1\right\} \quad (4.70)$$

and

$$\widetilde{X}_{L_w^{\mathrm{A}}}^{\mathrm{D}} = \sum_{\tilde{w} \in L_w^{\mathrm{D}}} Y_{\tilde{w}}^{\mathrm{C}} + \sum_{\tilde{w} \in \left\{\tilde{w} \middle| E_{\tilde{w}w}^{\mathrm{D}}=1\right\}} Z_{\tilde{w}w}^{\mathrm{D}} + \sum_{\tilde{w} \in \left\{\tilde{w} \middle| E_{\tilde{w}w}^{\mathrm{D}}>1\right\}} Z_{\tilde{w},E_{\tilde{w}w}^{\mathrm{D}}}^{\mathrm{D}}$$
$$\forall\ w \in \left\{2^{\bar{I}} - \bar{I}, \ldots, 2^{\bar{I}} - 1\right\}. \quad (4.71)$$

Because of the integration of the Eqs. (4.65) and (4.67) into the constraints, only the two constraints

$$(1 - \theta_i)\, X_i^{\mathrm{A}} \geq \theta_i\, X_i^{\mathrm{I}} \quad \forall\ i \quad (4.72)$$

and (4.68) are necessary in addition. Lastly, in all three graphs the unused flow variables can be set to zero.

$$Z_{w\tilde{w}}^{\mathrm{I}} = Z_{w\tilde{w}}^{\mathrm{R}} = Z_{w\tilde{w}}^{\mathrm{D}} = 0 \quad \forall\ w, \tilde{w} \in \left\{\tilde{w} \middle| E_{w\tilde{w}}^{\mathrm{D}} \neq 1\right\} \quad (4.73)$$

Taking a look at the here used decision variables Y_m^{R}, Y_m^{D}, X_i^{R}, and X_i^{D}, we notice the lack of indices for the recycling and disposal target, i.e., index r and d. Of course, the core index c is also missing, but this index has to be added to every variable, because for each core c the above condition and damaging consideration applies individually. Coming back to the indices r and d. Once a unit of an item or module comes out of the recycling graph, a decision is necessary to which r the item has to be assigned to. Since this decision appears after the output of the recycling graph, the above used variables Y_m^{R}, Y_m^{D}, X_i^{R}, and X_i^{D} can easily be substituted by the expressions $\sum_r Y_{mr}^{\mathrm{R}}$, $\sum_d Y_{md}^{\mathrm{D}}$, $\sum_r X_{ir}^{\mathrm{R}}$, and $\sum_d X_{id}^{\mathrm{D}}$, respectively.[36] Note that the index c has to be added to all variables in a further step. This can be seen in the next section where the complete model formulation is presented.

But before we get to this we take a step back to the core graph. The coefficients ρ_v of the quantity of cores Q^{C} in Eq. (4.59) can have values between one and zero. The sum of them per core equals one. Depending on the given data, many coefficient values can be almost zero, e.g., values of 10^{-12}. Such values are numerically problematic when solving the model. In addition, assuming a value of up to 1,000 for Q^{C} the input of this specific node would be 10^{-9} at the most. This is negligible. Of course, the sum of all ρ_v has to equal one in order to get the same quantity out of the core graph as is put in. Hence, the equation

$$\sum_v V_v^{\mathrm{C}} = Q^{\mathrm{C}} \tag{4.74}$$

must be valid.

To avoid numerical problems we modify coefficients and split Eq. (4.59). The modified coefficients are denoted by $\tilde{\rho}_v$ and are calculated in the following way, when we assume values of less than 10^{-7} being insignificant.

$$\tilde{\rho}_v = \begin{cases} \rho_v & \rho_v > 10^{-7} \\ 0 & \text{else} \end{cases} \qquad \forall, v \in \{1, \ldots, 3^{\bar{I}} - 1\} \tag{4.75}$$

$$\tilde{\rho}_{3^{\bar{I}}} = 1.00001 - \sum_{v=1}^{3^{\bar{I}}-1} \tilde{\rho}_v \tag{4.76}$$

Furthermore, the node representing the condition that all items of a core are functioning is the one with the number $3^{\bar{I}}$. The coefficient of this node is increased so that the sum over all coefficients $\tilde{\rho}_v$ is greater than or equal to one. We choose a sum value of more than one, e.g., 1.00001. It must be assured that no extra output unit of a core is generated. For example, a sum value of 1.01 with an expected $Q^{\mathrm{C}} = 1{,}000$ should be avoided, because the output could be 1,010 with an input of 1,000. This case does not happen, because Eq. (4.74) is added as constraint, too. To realise this input and output equality in the case of $\sum_v \tilde{\rho}_v > 1$, the equation for node $v = 1$ must be changed into an inequality. In addition, no outgoing edges from node 1 exist so that the constraint is

[36] The variables X_i^{A} and $\widetilde{X}_i^{\mathrm{D}}$ are not affected.

$$V_1^{\mathrm{C}} \leq \tilde{\rho}_1 \, Q^{\mathrm{C}} + \sum_{\tilde{v} \in \{\tilde{v} | \tilde{v} > 1, E^{\mathrm{C}}_{\tilde{v},1} = 1\}} Z^{\mathrm{C}}_{\tilde{v},1} \, . \tag{4.77}$$

For the remaining nodes $v \in \{2, \ldots, 3^{\bar{I}}\}$ the coefficient ρ_v is substituted by $\tilde{\rho}_v$ in constraint (4.59). All these above discussed aspects need to be expressed by a mathematical formulation, which follows in the next section.

4.2.3 Model formulation

The calculation of the profit is unchanged, i.e., it is the difference between the revenues and the cost.

$$\text{Maximise} \quad P = R - C \tag{4.78}$$

The first change is extending the revenues by the ones of the distributed modules. The individual price for a demanded module f is denoted by r^{M}_f and the quantity that is distributed by Q^{M}_f.

$$R = \sum_e r^{\mathrm{I}}_e Q^{\mathrm{I}}_e + \sum_f r^{\mathrm{M}}_f Q^{\mathrm{M}}_f + \sum_r r^{\mathrm{R}}_r Q^{\mathrm{R}}_r \tag{4.79}$$

In the cost function the cost savings are modelled in the following way. When a module is distributed, recycled, or disposed of, the connections keeping together the consisting items are not separated. In addition, the cost to disassemble the module completely is known. This was already assumed in the basic model. Furthermore, when a core is disassembled into several modules and items not only one saving occurs. Each module represents a saving of a fraction of the core. Knowing the disassembly cost of the complete core and of each module, which can be generated out of this core, the resulting disassembly cost is the complete disassembly cost minus the saved cost.

Without loss of generality, we assume that the complete core is denoted by the index $m = 1$, as is done in the disassembly state and and/or graph above. Hence, the cost parameter notation is changed from c^{J}_c to $c^{\mathrm{J}}_{c,1}$, but the value is identical. The cost that is saved is the sum of all modules m of all cores c, that are distributed Y^{M}_{cm}, recycled in all bins Y^{R}_{cmr}, and disposed of Y^{D}_{cmd}.

$$C = \sum_c \left(c_c^{\mathrm{A}} + c_{c,1}^{\mathrm{J}}\right) Q_c^{\mathrm{C}} - \sum_c \sum_{m=1}^{\overline{M}_c} c_{cm}^{\mathrm{J}} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}}\right) + \sum_d c_d^{\mathrm{D}} Q_d^{\mathrm{D}} \quad (4.80)$$

The constraints are again grouped in the categories

- item and module flow,
- core condition,
- purity, and
- limits,

so that they are comparable with the basic model more easily.

Item and module flow constraints

In addition to all the single items, all corresponding items in modules plus the single items must equal the items available through the quantity of cores. Thereby, the quantities of modules (for distribution, recycling, and disposal) are added according to the consisting items. This information contains the modules definition matrix δ_{cmi}. In order to assure that the quantity of item A contained in the core equals the quantity on the output side of the process the quantity of distributed items X_{ci}^{I}, recycled items X_{cir}^{R}, disposed items X_{cid}^{D}, as well as all modules only those quantities of modules need to be added, which contain an item A. And this information is given in the module definition matrix by a value of 1 if item $i = \mathrm{A}$ is in module m. Thereby, only up to the last index $\overline{M}_c$ of the modules of core c need to be considered in the summation.

$$Q_c^{\mathrm{C}} = X_{ci}^{\mathrm{I}} + \sum_r X_{cir}^{\mathrm{R}} + \sum_d X_{cid}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}}\right) \quad \forall\, c, i \in \{1, \ldots, \bar{I}_c\} \quad (4.81)$$

For determining the weight of the items and modules in the recycling boxes r and disposal bins d each item (either as single item or in a module) is multiplied with its weight and added.

$$Q_r^{\mathrm{R}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X_{cir}^{\mathrm{R}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmr}^{\mathrm{R}}\right) \quad \forall\, r \quad (4.82)$$

$$Q_d^{\mathrm{D}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X_{cid}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmd}^{\mathrm{D}} \right) \quad \forall\ d \tag{4.83}$$

The consideration of additional items along with certain modules is a new aspect. For each item i (up to $\bar{I}_c$) of a core c the wanted solution has to fulfil the property that at least the quantity of additional items exist as single items somewhere (therefore the sum on the left hand side) when certain modules with their particular additional items α_{cmi} are in the solution. Here, it does not matter what the module is intended for.

$$X_{ci}^{\mathrm{I}} + \sum_r X_{cir}^{\mathrm{R}} + \sum_d X_{cid}^{\mathrm{D}} \geq \sum_{m=1}^{\overline{M}_c} \alpha_{cmi} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}} \right) \quad \forall\ c, i \in \{1, \ldots, \bar{I}_c\} \tag{4.84}$$

For the single items nothing has changed regarding the commonality and multiplicity for demanded items.

$$Q_e^{\mathrm{I}} = \sum_{(c,i) \in \mathcal{P}_e} X_{ci}^{\mathrm{I}} \quad \forall\ e \tag{4.85}$$

If an item is not demanded, the corresponding decision variable is set to zero.

$$X_{ci}^{\mathrm{I}} = 0 \quad \forall\ (c, i) \notin \bigcup_e \mathcal{P}_e \tag{4.86}$$

The same applies to modules. Here, commonalities might also exist and therefore a relationship between demanded modules f and the modules of a particular core m is expressed by the sets $\mathcal{R}_f$ for each demanded module f. The set contains core module combinations (c, m) that satisfy the demand.

$$Q_f^{\mathrm{M}} = \sum_{(c,m) \in \mathcal{R}_f} Y_{cm}^{\mathrm{M}} \quad \forall\ f \tag{4.87}$$

If a module is not demanded at all, the corresponding variable is set to zero.

$$Y_{cm}^{\mathrm{M}} = 0 \quad \forall\ (c, m) \notin \bigcup_f \mathcal{R}_f \tag{4.88}$$

Condition constraints

The condition constraints are the major impact of the model extension to the flexible disassembly planning. To model the flexibility, a core graph with nodes representing all condition permutations of the cores in conjunction with three graphs representing the modules to distribute, recycle, and dispose of is added. The coefficients for the core graph are determined by

$$\tilde{\rho}_{cv} = \begin{cases} \rho_{cv} & \rho_{cv} < 10^{-7} \\ 0 & \text{else} \end{cases} \qquad \forall\ c, v \in \{1, \dots, 3^{\bar{I}_c} - 1\} \tag{4.89}$$

$$\tilde{\rho}_{c,3^{\bar{I}_c}} = \left(1.00001 - \sum_{v=1}^{3^{\bar{I}_c}-1} \tilde{\rho}_{cv}\right) \qquad \forall\ c \tag{4.90}$$

based on the given data ρ_{cv}. To check whether the chosen value of 1.00001 is not to big, the model can be solved without the integrality constraints. The resulting solution of the continuous model shows if the quantity of cores Q_c^{C} does not exceed 10,000. The node number $v = 1$ is treated specially, because it has no outgoing edges and the flow through the node is an inequality. Thereby, the flow through the edges from node v to node $\tilde{v}$ of core c is denoted by $Z_{cv\tilde{v}}^{\mathrm{C}}$. The information if an edge exists is given by $E_{cv\tilde{v}}^{\mathrm{C}}$ equalling one. In addition, the output of a node of this graph to the other three graphs is represented by the variable V_{cv}^{C}.

$$V_{c,1}^{\mathrm{C}} \leq \tilde{\rho}_{c,1}\, Q_c^{\mathrm{C}} + \sum_{\tilde{v} \in \{\tilde{v} \mid \tilde{v} > 1, E_{c,\tilde{v},1}^{\mathrm{C}} = 1\}} Z_{c,\tilde{v},1}^{\mathrm{C}} \qquad \forall\ c \tag{4.91}$$

The flow through the remaining nodes of the core graph is an equality of input and output. Here, the outgoing edges are included, too.

$$\sum_{\tilde{v} \in \{\tilde{v} \mid \tilde{v} < v, E_{cv\tilde{v}}^{\mathrm{C}} = 1\}} Z_{cv\tilde{v}}^{\mathrm{C}} + V_{cv}^{\mathrm{C}} = \tilde{\rho}_{cv}\, Q_c^{\mathrm{C}} + \sum_{\tilde{v} \in \{\tilde{v} \mid \tilde{v} > v, E_{c\tilde{v}v}^{\mathrm{C}} = 1\}} Z_{c\tilde{v}v}^{\mathrm{C}} \qquad \forall\ c, v \in \{2, \dots, 3^{\bar{I}_c} - 1\} \tag{4.92}$$

The output of the core graph V_{cv}^{C} has to equal the input Q_c^{C}, i.e.,

$$\sum_v V_{cv}^{\mathrm{C}} = Q_c^{\mathrm{C}} \quad \forall\ c\,. \tag{4.93}$$

Lastly, all edge variables $Z^{\mathrm{C}}_{cv\tilde{v}}$ without an existing edge flow do not exist and are set to zero.

$$Z^{\mathrm{C}}_{cv\tilde{v}} = 0 \quad \forall\, c, v, \tilde{v} \in \left\{ \tilde{v} \,\middle|\, E^{\mathrm{C}}_{cv\tilde{v}} = 0 \right\} \tag{4.94}$$

The output of the core graph goes into the distribution, the recycling, and the disposal graph. The structure of these three is identical, i.e., the edge definition is given by a single value $E^{\mathrm{D}}_{cw\tilde{w}}$. The nodes of these three graphs are labelled with w instead of v. For each graph, three sets of constraints exist: one for nodes representing an existing module (Eq. (4.95)), one for nodes representing no existing module (Eq. (4.96)), and one for the item representing nodes (Eq. (4.97)). The flow variable of the distribution graph is $Z^{\mathrm{I}}_{cw\tilde{w}}$. The output of the distribution graph is the quantity of modules to distribute Y^{M}_{cm} as well as the quantity of items to distribute X^{I}_{ci} together with the damaged items X^{A}_{ci}. The mapping between the nodes of the distribution graph and the core graph is given by L^{I}_{cw} and the one to the module and item index by L^{A}_{cw}.

$$\begin{aligned}
&\sum_{\tilde{w} \in \left\{ \tilde{w} \,\middle|\, E^{\mathrm{D}}_{cw\tilde{w}} = 1 \right\}} Z^{\mathrm{I}}_{cw\tilde{w}} + Y^{\mathrm{M}}_{c, L^{\mathrm{A}}_{cw}} \\
&\quad = \sum_{v \in L^{\mathrm{I}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w} \in \left\{ \tilde{w} \,\middle|\, E^{\mathrm{D}}_{c\tilde{w}w} = 1 \right\}} Z^{\mathrm{I}}_{c\tilde{w}w} + \sum_{\tilde{w} \in \left\{ \tilde{w} \,\middle|\, E^{\mathrm{D}}_{c\tilde{w}w} > 1 \right\}} Z^{\mathrm{I}}_{c, \tilde{w}, E^{\mathrm{D}}_{c\tilde{w}w}} \\
&\qquad \forall\, c, w \in \left\{ w \,\middle|\, w \in \left\{ 1, \ldots, 2^{\bar{I}_c} - \bar{I}_c - 1 \right\}, L^{\mathrm{A}}_{cw} > 0 \right\}
\end{aligned} \tag{4.95}$$

$$\begin{aligned}
&\sum_{\tilde{w} \in \left\{ \tilde{w} \,\middle|\, E^{\mathrm{D}}_{cw\tilde{w}} = 1 \right\}} Z^{\mathrm{I}}_{cw\tilde{w}} \\
&\quad = \sum_{v \in L^{\mathrm{I}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w} \in \left\{ \tilde{w} \,\middle|\, E^{\mathrm{D}}_{c\tilde{w}w} = 1 \right\}} Z^{\mathrm{I}}_{c\tilde{w}w} + \sum_{\tilde{w} \in \left\{ \tilde{w} \,\middle|\, E^{\mathrm{D}}_{c\tilde{w}w} > 1 \right\}} Z^{\mathrm{I}}_{c, \tilde{w}, E^{\mathrm{D}}_{c\tilde{w}w}} \\
&\qquad \forall\, c, w \in \left\{ w \,\middle|\, w \in \left\{ 1, \ldots, 2^{\bar{I}_c} - \bar{I}_c - 1 \right\}, L^{\mathrm{A}}_{cw} = 0 \right\}
\end{aligned} \tag{4.96}$$

$$\begin{aligned}
&X^{\mathrm{I}}_{c, L^{\mathrm{A}}_{cw}} + X^{\mathrm{A}}_{c, L^{\mathrm{A}}_{cw}} \\
&\quad = \sum_{v \in L^{\mathrm{I}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w} \in \left\{ \tilde{w} \,\middle|\, E^{\mathrm{D}}_{c\tilde{w}w} = 1 \right\}} Z^{\mathrm{I}}_{c\tilde{w}w} + \sum_{\tilde{w} \in \left\{ \tilde{w} \,\middle|\, E^{\mathrm{D}}_{c\tilde{w}w} > 1 \right\}} Z^{\mathrm{I}}_{c, \tilde{w}, E^{\mathrm{D}}_{c\tilde{w}w}} \\
&\qquad \forall\, c, w \in \left\{ 2^{\bar{I}_c} - \bar{I}_c, \ldots, 2^{\bar{I}_c} - 1 \right\}
\end{aligned} \tag{4.97}$$

The recycling graph is more or less identical to the distribution graph, only the flow variables are $Z^{\mathrm{R}}_{cw\tilde{w}}$ and the core graph node mapping is given by L^{R}_{cw}. In addition, the output of the item nodes is directly formulated by the variables X^{R}_{cir}, X^{D}_{cid}, X^{A}_{ci}, and $\widetilde{X}^{\mathrm{D}}_{ci}$. The latter is the output of the disposal graph. The three constraint sets are:

$$\sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{cw\tilde{w}}=1\right\}} Z^{\mathrm{R}}_{cw\tilde{w}} + \sum_{r} Y^{\mathrm{R}}_{c,L^{\mathrm{A}}_{cw},r} = \sum_{v\in L^{\mathrm{R}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{R}}_{c\tilde{w}w} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{R}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\ c, w \in \left\{w \middle| w \in \left\{1,\ldots,2^{\bar{I}_c}-\bar{I}_c-1\right\}, L^{\mathrm{A}}_{cw}>0\right\}, \quad (4.98)$$

$$\sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{cw\tilde{w}}=1\right\}} Z^{\mathrm{R}}_{cw\tilde{w}} = \sum_{v\in L^{\mathrm{R}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{R}}_{c\tilde{w}w} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{R}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\ c, w \in \left\{w \middle| w \in \left\{1,\ldots,2^{\bar{I}_c}-\bar{I}_c-1\right\}, L^{\mathrm{A}}_{cw}=0\right\}, \quad (4.99)$$

and

$$\sum_{r} X^{\mathrm{R}}_{c,L^{\mathrm{A}}_{cw},r} + \sum_{d} X^{\mathrm{D}}_{c,L^{\mathrm{A}}_{cw},d} - \widetilde{X}^{\mathrm{D}}_{c,L^{\mathrm{A}}_{cw}} - X^{\mathrm{A}}_{c,L^{\mathrm{A}}_{cw}} = \sum_{v\in L^{\mathrm{R}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{R}}_{c\tilde{w}w} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{R}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\ c, w \in \left\{2^{\bar{I}_c}-\bar{I}_c,\ldots,2^{\bar{I}_c}-1\right\}. \quad (4.100)$$

For the disposal graph the same substitutions are applied. The flow variables are $Z^{\mathrm{D}}_{cw\tilde{w}}$ and the mapping L^{D}_{cw}. The output of the item nodes is $\widetilde{X}^{\mathrm{D}}_{ci}$.

$$\sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{cw\tilde{w}}=1\right\}} Z^{\mathrm{D}}_{cw\tilde{w}} + \sum_{d} Y^{\mathrm{D}}_{c,L^{\mathrm{A}}_{cw},d} = \sum_{v\in L^{\mathrm{D}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{D}}_{c\tilde{w}w} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{D}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\ c, w \in \left\{w \middle| w \in \left\{1,\ldots,2^{\bar{I}_c}-\bar{I}_c-1\right\}, L^{\mathrm{A}}_{cw}>0\right\} \quad (4.101)$$

$$\sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{cw\tilde{w}}=1 \right\}} Z^{\mathrm{D}}_{cw\tilde{w}} = \sum_{v \in L^{\mathrm{D}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{c\tilde{w}w}=1 \right\}} Z^{\mathrm{D}}_{c\tilde{w}w} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{c\tilde{w}w}>1 \right\}} Z^{\mathrm{D}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\ c, w \in \left\{ w \middle| w \in \left\{ 1, \ldots, 2^{\bar{I}_c} - \bar{I}_c - 1 \right\}, L^{\mathrm{A}}_{cw} = 0 \right\} \quad (4.102)$$

$$\widetilde{X}^{\mathrm{D}}_{c,L^{\mathrm{A}}_{cw}} = \sum_{v \in L^{\mathrm{D}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{c\tilde{w}w}=1 \right\}} Z^{\mathrm{D}}_{c\tilde{w}w} + \sum_{\tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{c\tilde{w}w}>1 \right\}} Z^{\mathrm{D}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\ c, w \in \left\{ 2^{\bar{I}_c} - \bar{I}_c, \ldots, 2^{\bar{I}_c} - 1 \right\} \quad (4.103)$$

All edge flow variables without an existing edge in the three graphs are set zero.

$$Z^{\mathrm{I}}_{cw\tilde{w}} = Z^{\mathrm{R}}_{cw\tilde{w}} = Z^{\mathrm{D}}_{cw\tilde{w}} = 0 \quad \forall\ c, w, \tilde{w} \in \left\{ \tilde{w} \middle| E^{\mathrm{D}}_{cw\tilde{w}} \neq 1 \right\} \quad (4.104)$$

Important is that the quantity of disposed items is at least the output of the disposal graph, because otherwise the disposal quantity could be less. In Eq. (4.100) exists no lower limitation.

$$\sum_{d} X^{\mathrm{D}}_{cid} \geq \widetilde{X}^{\mathrm{D}}_{ci} \quad \forall\ c, i \quad (4.105)$$

The last aspect of the condition—even though it is not really a condition—is the item damaging. The damaged items are denoted by X^{A}_{ci}, the functioning ones by X^{I}_{ci}, and the damaging probability by θ_{ci}. According to Eq. (4.72)

$$(1 - \theta_{ci})\, X^{\mathrm{A}}_{ci} \geq \theta_{ci}\, X^{\mathrm{I}}_{ci} \quad \forall\ (c, i) \in \bigcup_{e} \mathcal{P}_e \quad (4.106)$$

must apply. In addition, when no demand for distribution exists for an item, the damaging is irrelevant so that we can state

$$X^{\mathrm{A}}_{ci} = 0 \quad \forall\ (c, i) \notin \bigcup_{e} \mathcal{P}_e \quad (4.107)$$

analogously to Eq. (4.86).

Purity constraints

In addition to the items the modules in the recycling boxes are added to the purity constraint.

$$\omega_r Q_r^{\mathrm{R}} \leq \sum_c \sum_{i=1}^{\bar{I}_c} \pi_{cir} w_{ci} \left(X_{cir}^{\mathrm{R}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmr}^{\mathrm{R}} \right) \quad \forall\ r \tag{4.108}$$

The treatment of hazardous items stays unchanged.

$$X_{cir}^{\mathrm{R}} = 0 \quad \forall\ (c,i) \in \mathcal{H}, r \tag{4.109}$$

$$X_{cid}^{\mathrm{D}} = 0 \quad \forall\ (c,i) \in \mathcal{H}, d \in \{1\} \tag{4.110}$$

This treatment is adapted to the modules. Modules with hazardous items must not be placed into recycling boxes or non-hazardous disposal bins.

$$Y_{cmr}^{\mathrm{R}} = 0 \quad \forall\ (c,m) \in \big\{(c,m)|\delta_{cmi} = 1, (c,i) \in \mathcal{H}, m \in \{1,\ldots,\overline{M}_c\}\big\}, r \tag{4.111}$$

$$Y_{cmd}^{\mathrm{D}} = 0 \quad \forall\ (c,m) \in \big\{(c,m)|\delta_{cmi} = 1, (c,i) \in \mathcal{H}, m \in \{1,\ldots,\overline{M}_c\}\big\}, d \in \{1\} \tag{4.112}$$

Limits constraints

The demand for modules D_f^{M} and the lower distribution limit $\underline{Q}_f^{\mathrm{M}}$ limit the values for the distribution quantity Q_f^{M}. The other limits for cores, items, recycling material, and disposal are unchanged.

$$\underline{Q}_c^{\mathrm{C}} \leq Q_c^{\mathrm{C}} \leq \overline{Q}_c^{\mathrm{C}} \quad \forall\ c \tag{4.113}$$

$$\underline{Q}_e^{\mathrm{I}} \leq Q_e^{\mathrm{I}} \leq D_e^{\mathrm{I}} \quad \forall\ e \tag{4.114}$$

$$\underline{Q}_f^{\mathrm{M}} \leq Q_f^{\mathrm{M}} \leq D_f^{\mathrm{M}} \quad \forall\ f \tag{4.115}$$

$$\underline{Q}_r^{\mathrm{R}} \leq Q_r^{\mathrm{R}} \leq D_r^{\mathrm{R}} \quad \forall\ r \tag{4.116}$$

$$\underline{Q}_d^{\mathrm{D}} \leq Q_d^{\mathrm{D}} \leq \overline{Q}_d^{\mathrm{D}} \quad \forall\ d \tag{4.117}$$

Furthermore, the saved disassembly time t_{cm}^{J} for each kept module is subtracted from the complete disassembly time $t_{c,1}^{\mathrm{J}}$ and must not exceed the available labour time $\overline{L}$.

$$\sum_{c} t^{\mathrm{J}}_{c,1} Q^{\mathrm{C}}_{c} - \sum_{c} \sum_{m=1}^{\overline{M}_c} t^{\mathrm{J}}_{cm} \left(Y^{\mathrm{M}}_{cm} + \sum_{r} Y^{\mathrm{R}}_{cmr} + \sum_{d} Y^{\mathrm{D}}_{cmd} \right) \leq \bar{L} \qquad (4.118)$$

Lastly, the domain of the added variables Y^{M}_{cm}, Y^{R}_{cmr}, and Y^{D}_{cmd} is integer numbers and that of the quantity Q^{M}_{f} real numbers.

$$X^{\mathrm{I}}_{ci}, X^{\mathrm{A}}_{ci}, \widetilde{X}^{\mathrm{D}}_{ci}, X^{\mathrm{R}}_{cir}, X^{\mathrm{D}}_{cid}, Y^{\mathrm{M}}_{cm}, Y^{\mathrm{R}}_{cmr}, Y^{\mathrm{D}}_{cmd} \in \mathbb{Z}^*$$
$$\forall\ c, i \in \{1, \ldots, \bar{I}_c\}, m \in \{1, \ldots, \overline{M}_c\}, r, d \qquad (4.119)$$

With these variables the remaining variables (Q^{C}_{c}, Q^{I}_{e}, Q^{M}_{f}, Q^{R}_{r}, Q^{D}_{d}, P, R, and C) are automatically in the correct domain.

Model size

To illustrate the model size the number of decision variables and constraints are listed in Table 4.12under "flexible disassembly planning". The basis of the determination of the numbers is the compact model, i.e., the model formulation in appendix C.3. To avoid confusion with existing variables the number of indices is denoted by the index itself. This means that the c in the table must be read as $\sum_c 1$. Hence, when writing $c \cdot r$, the interpretation is to calculate the number of cores times the number of recycling boxes. The X^{I}_{ci} only have values different than zero for elements of the set $\bigcup_e \mathcal{P}_e$, which leads to the entry $|\bigcup_e \mathcal{P}_e|$ in the table. The variables X^{R}_{cir} and X^{D}_{cid} occur $\sum_c \bar{I}_c$ times r and d, respectively. But, Eqs. (4.109) and (4.110) set the value of core item combinations of hazardous items equal to zero. Thus, these variables are excluded from the consideration. The resulting number of decision variables for a model with one hazardous disposal bin is depicted in the table.

To compare the size with the basic model of the complete disassembly, the content of Table 3.2 is listed here, too. The increase of integer variables results from the demanded modules f, the modules m in the recycling boxes and disposal bins, i.e., $\left|\bigcup_f \mathcal{R}_f\right| + (r+d)\sum_c \overline{M}_c$, and the damaged items X^{A}_{ci} as well as the output of the disposal graph $\widetilde{X}^{\mathrm{D}}_{ci}$, i.e., $|\bigcup_e \mathcal{P}_e| + \sum_c \bar{I}_c$. In addition, all the real variables are added, too. This increase is enormous, because of the term $\bar{I}_c \cdot 3^{\bar{I}_c}$ (simplified here). A similar increase can also be noticed with the constraints. The difference is $\sum_c \left(3^{\bar{I}_c} + 3 \cdot 2^{\bar{I}_c} + \bar{I}_c\right) - 2\,c + 2\,f$. The number of variables and constraints increases exponentially for the number of items. Besides, the number of modules also depends exponentially

Table 4.12 Number of decision variables and constraints

flexible disassembly planning	
real variables	$\sum_c \left(\left(\frac{2}{3}\bar{I}_c + \frac{5}{2}\right) 3^{\bar{I}_c} - 3 \cdot 2^{\bar{I}_c} \right) + \frac{3}{2}c$
integer variables	$2\left|\bigcup_e \mathcal{P}_e\right| + \left|\bigcup_f \mathcal{R}_f\right| + (r+d)\left(\sum_c \left(\bar{I}_c + \overline{M}_c\right) - \lvert\mathcal{H}\rvert\right) + \sum_c \bar{I}_c + \lvert\mathcal{H}\rvert$
constraints	$\sum_c \left(3^{\bar{I}_c} + 3 \cdot 2^{\bar{I}_c} + 3\,\bar{I}_c\right) + \left|\bigcup_e \mathcal{P}_e\right| - c + 3\,r + 2\,(e+f+d) + 1$
complete disassembly planning	
integer variables	$\left|\bigcup_e \mathcal{P}_e\right| + \left(\sum_c \bar{I}_c - \lvert\mathcal{H}\rvert\right)(r+d) + \lvert\mathcal{H}\rvert$
constraints	$2\sum_c \bar{I}_c + \left|\bigcup_e \mathcal{P}_e\right| + c + 3r + 2d + 2e + 1$

(in the worst case) on the number of items, which leads to an exponential increase of the number of variables and constraints with respect to the number of items a core consists of.

To illustrate this, let us assume a problem with two cores ($c = 2$), four and six items in one of these cores ($\bar{I}_1 = 4$, $\bar{I}_2 = 6$), 15 and 37 modules in one of these cores ($\overline{M}_1 = 15$, $\overline{M}_2 = 37$), three recycling boxes ($r = 3$), two disposal bins ($d = 2$), two hazardous items ($\lvert\mathcal{H}\rvert = 2$), two demanded items ($e = 2$), five core item combinations the demand can be met with ($\lvert\bigcup_{e=1}^{2} \mathcal{P}_e\rvert = 5$), three demanded modules ($f = 3$), and four core module combinations the demand can be met with ($\lvert\bigcup_{f=1}^{3} \mathcal{R}_f\rvert = 4$). With these values the number of real variables is limited by $((\frac{2}{3} \cdot 4 + \frac{5}{2})3^4 - 3 \cdot 2^4) + ((\frac{2}{3} \cdot 6 + \frac{5}{2})3^6 - 3 \cdot 2^6) + \frac{3}{2} \cdot 2 = 4920$, the integer variables by $2 \cdot 5 + 4 + (3 + 2)(4 + 15 + 6 + 37 - 2) + 4 + 6 + 2 = 326$, and the number of constraints by $(3^4 + 3 \cdot 2^4 + 3 \cdot 4) + (3^6 + 3 \cdot 2^6 + 3 \cdot 6) + 5 - 2 + 3 \cdot 3 + 2\,(2 + 3 + 2) + 1 = 1107$.

4.2.4 Numerical example

4.2.4.1 Data

The exemplary data to illustrate the planning is based on the one in the basic model (see Sect. 3.1.3). The three forklift trucks with their eight items each are the available cores. The demand for items, the availability of cores, the condition, and so on apply to the example here, too. The Tables 3.3–3.8 contain most of the data. In addition, the item $i = \mathrm{H}$ of core $c = 1$ is

Table 4.13 Cost and time of separating connections

		joint time							joint cost						
		item i							item i						
c	i	B	C	D	E	F	G	H	B	C	D	E	F	G	H
	A	.	.	.	.	.	0.125	.	.	.	.	.	.	3.75	.
	B		.	.	.	.	0.125	.		.	.	.	.	3.75	.
	C			.	.	.	0.125	.			.	.	.	3.75	.
1	D				.	.	0.125	.				.	.	3.75	.
	E					0.5	.	.					15	.	.
	F						3	.						90	.
	G							6							180
	A	.	.	.	.	.	0.125	.	.	.	.	.	.	3.75	.
	B		.	.	.	.	0.125	.		.	.	.	.	3.75	.
	C			.	.	.	0.125	.			.	.	.	3.75	.
2	D				.	.	0.125	.				.	.	3.75	.
	E					0.5	.	.					15	.	.
	F						2.5	.						75	.
	G							5.5							175
	A	.	.	.	.	.	0.125	.	.	.	.	.	.	3.75	.
	B		.	.	.	.	0.125	.		.	.	.	.	3.75	.
	C			.	.	.	0.125	.			.	.	.	3.75	.
3	D				.	.	0.125	.				.	.	3.75	.
	E					0.5	.	.					15	.	.
	F						2.5	.						75	.
	G							4.5							155

A dot denotes a value of zero.

hazardous. The labour hours are limited to $\bar{L} = 2{,}200\,\text{h}$. Again, the disposal bin $d = 2$ holds the hazardous items. The data for the modules is already developed in Sect. 4.2.1. The module definition matrix δ_{cmi} as well as the additional item matrix α_{cmi} in Table 4.2 are important. In this example setting these are identical for all three cores which means that $\delta_{1,m,i} = \delta_{2,m,i} = \delta_{3,m,i}$ and $\alpha_{1,m,i} = \alpha_{2,m,i} = \alpha_{3,m,i}$.

In addition, the saved disassembly cost c^{J}_{cm} and saved time t^{J}_{cm} for each module is necessary. These values are calculated on the basis of time and cost for each existing connection that holds the core together. The upper triangle matrices holding this information are listed in Table 4.13. Thereby, an entry of $1/8$ in row B and column G of the joint time matrix means that a time of 0.125 h is planned to separate the connection between item B and G, i.e., to take of a wheel. Adding all entries of the matrix for the corresponding core leads to the disassembly time of the complete core (i.e., 10 h, 9 h, and 8 h) as used in the basic model. Based on the time the cost is simply calculated by multiplying the time with a factor 30. In addition, the cost for separating the connection between G and H in core 2 is furthermore

Table 4.14 Saved cost and time of modules

	saved time t^{J}_{cm}			saved cost c^{J}_{cm}				saved time t^{J}_{cm}			saved cost c^{J}_{cm}		
m	$c=1$	$c=2$	$c=3$	$c=1$	$c=2$	$c=3$	m	$c=1$	$c=2$	$c=3$	$c=1$	$c=2$	$c=3$
1	10	9	8	300	280	260	26	6.375	5.875	4.875	191.25	186.25	166.25
2	9.875	8.875	7.875	296.25	276.25	256.25	27	9.25	8.25	7.25	277.5	257.5	237.5
3	9.875	8.875	7.875	296.25	276.25	256.25	28	3.75	3.25	3.25	112.5	97.5	97.5
4	9.875	8.875	7.875	296.25	276.25	256.25	29	9.5	8.5	7.5	285	265	245
5	9.875	8.875	7.875	296.25	276.25	256.25	30	9.125	8.125	7.125	273.75	253.75	233.75
6	9.5	8.5	7.5	285	265	245	31	9.125	8.125	7.125	273.75	253.75	233.75
7	9.75	8.75	7.75	292.5	272.5	252.5	32	6.25	5.75	4.75	187.5	182.5	162.5
8	9.75	8.75	7.75	292.5	272.5	252.5	33	9.125	8.125	7.125	273.75	253.75	233.75
9	9.75	8.75	7.75	292.5	272.5	252.5	34	3.625	3.125	3.125	108.75	93.75	93.75
10	9.375	8.375	7.375	281.25	261.25	241.25	35	9.125	8.125	7.125	273.75	253.75	233.75
11	9.75	8.75	7.75	292.5	272.5	252.5	36	3.625	3.125	3.125	108.75	93.75	93.75
12	9.75	8.75	7.75	292.5	272.5	252.5	37	6.25	5.75	4.75	187.5	182.5	162.5
13	9.375	8.375	7.375	281.25	261.25	241.25	38	6.25	5.75	4.75	187.5	182.5	162.5
14	9.75	8.75	7.75	292.5	272.5	252.5	39	3.25	2.75	2.75	97.5	82.5	82.5
15	9.375	8.375	7.375	281.25	261.25	241.25	40	9	8	7	270	250	230
16	9.375	8.375	7.375	281.25	261.25	241.25	41	3.5	3	3	105	90	90
17	9.625	8.625	7.625	288.75	268.75	248.75	42	6.125	5.625	4.625	183.75	178.75	158.75
18	9.625	8.625	7.625	288.75	268.75	248.75	43	6.125	5.625	4.625	183.75	178.75	158.75
19	9.25	8.25	7.25	277.5	257.5	237.5	44	3.125	2.625	2.625	93.75	78.75	78.75
20	9.625	8.625	7.625	288.75	268.75	248.75	45	6.125	5.625	4.625	183.75	178.75	158.75
21	9.25	8.25	7.25	277.5	257.5	237.5	46	3.125	2.625	2.625	93.75	78.75	78.75
22	9.25	8.25	7.25	277.5	257.5	237.5	47	6	5.5	4.5	180	175	155
23	9.625	8.625	7.625	288.75	268.75	248.75	48	3	2.5	2.5	90	75	75
24	9.25	8.25	7.25	277.5	257.5	237.5	49	0.5	0.5	0.5	15	15	15
25	9.25	8.25	7.25	277.5	257.5	237.5	50	0.125	0.125	0.125	3.75	3.75	3.75

increased by 10 € and in core 3 by 20 €. Thus, adding all entries of the cost matrices leads to the values 300 €, 280 €, and 260 € for core 1, 2, and 3, respectively, as used in the basic model with complete disassembly.

Based on these individual values the saved cost and time of modules can be determined. Thereby, the cost and times of the still existing connections are added. For example, in module $m = 21$ (BDFGH) of core $c = 1$ the connections B-G, D-G, F-G, and G-H exist. Adding the times results in $0.125+0.125+3+6 = 9.25\,\mathrm{h}$ and the cost is $3.75+2.75+90+180 = 277.5$ €. Following this procedure, the values (times and cost) for all 50 modules can now be determined. These are listed in Table 4.14.

Missing data is the demand of modules. This means that we need to know which modules f are demanded, which modules of cores we can take to meet the demand ($\mathcal{R}_f$), the lower distribution limits $\underline{Q}^{\mathrm{M}}_f$, the demanded quantity D^{M}_f, and the price of such a module r^{M}_f. Let us assume two modules are demanded. The first is module GH ($m = 47$) of core 3 and the second module EF ($m = 49$) of core 1 and 2, because the items E and F as well as the resulting module EF are identical in core 1 and 2. Hence, module GH is

Table 4.15 Demanded modules with distribution limits

f	$\underline{Q}_f^{\mathrm{M}}$	D_f^{M}	r_f^{M}	$\mathcal{R}_f$
1	0	30	2900	$\{(3,47)\}$
2	0	90	600	$\{(1,49),(2,49)\}$

Table 4.16 Usage probability

		item i							
core	usage	A	B	C	D	E	F	G	H
	disposal	0	0	0.0015	0.0015	0	0	0	0
$c=1$	disposal & recycling	0.55	0.55	0.5735	0.5735	0.01	0.05	0.01	0.05
	disp. & rec. & distribution	0.45	0.45	0.425	0.425	0.99	0.95	0.99	0.95
	disposal	0	0	0	0	0	0	0	0
$c=2$	disposal & recycling	0.55	0.55	0.55	0.55	0.01	0.05	0.01	0.05
	disp. & rec. & distribution	0.45	0.45	0.45	0.45	0.99	0.95	0.99	0.95
	disposal	0	0	0.0015	0.0015	0	0	0	0
$c=3$	disposal & recycling	0.55	0.55	0.5735	0.5735	0.01	0.05	0.01	0.01
	disp. & rec. & distribution	0.45	0.45	0.425	0.425	0.99	0.95	0.99	0.99

unique in core 3 whereas module EF is common across core 1 and 2. This information is stored in the sets $\mathcal{R}_f$ as is depicted in Table 4.15. The set $\mathcal{R}_f$ contains core module combinations like the set $\mathcal{P}_e$ for items. Obviously, the module index (in our example $m = 49$) does not have to be identical across the cores the modules are taken from to meet the demand and even two or more modules from the same core can be used to meet the demand. This represents the commonality and multiplicity, respectively, with respect to modules. The lower distribution limits, the prices, and the demand as upper limit are given in the table, too.

Furthermore, the condition coefficients for the nodes of the core graphs need to be determined. They are based on the condition probability to gain the values ρ_{cv}. And these are the basis for the modified values $\tilde{\rho}_{cv}$ used in the model. To make the calculation of the values ρ_{cv} more comprehensible, we can first create a table that contains the probabilities that an item can be used for disposal only, disposal and recycling, as well as disposal, recycling, and distribution. According to Fig. 3.3 an item can be used for disposal only with a probability of $\zeta_{ci}\iota_{ci}$. On the other hand, an item can be used for all three cases with a probability of $(1-\zeta_{ci})(1-\eta_{ci})$, leaving the damaging aside. The remaining probability for disposal and recycling is $1-\zeta_{ci}\iota_{ci}-(1-\zeta_{ci})(1-\eta_{ci})$. For item D of core 1 the three values are 0.0015, 0.5735, and 0.4250, respectively. All values can be found in Table 4.16. The value of ρ_{cv} is then calculated in the following way. Let us take node $v = 6512$. This node represents a core with the following condition: item C has to be disposed of, items A, B, and D can be recycled or be disposed of, and items E through H can be used for anything. According to the three-base system developed on page 194, the node 6512 is converted to 11012222 which represents the core condition of $\underline{\text{A}}\underline{\text{B}}\underline{\underline{\text{C}}}\underline{\text{D}}\text{EFGH}$. With this coding $\rho_{1,6512}$ is calculated by multiplying the following values in Table 4.16 of core $c = 1$. Each number added by one (i.e., 22123333) is the row index the values have to be taken from. This means we get $\rho_{1,6512} = 0.55 \cdot 0.55 \cdot 0.0015 \cdot 0.5735 \cdot 0.99 \cdot 0.95 \cdot 0.99 \cdot 0.95 = 0.00023018$. This is calculated for all 6561 nodes per core. Doing so, we get 576, 256, and 576 values being greater than zero for core 1, 2, and 3, respectively. With these values we determine the modified values $\tilde{\rho}_{cv}$. The result is that only 364, 240, and 326 values greater than zero remain of the above for the cores 1, 2, and 3, respectively. A listing of the values is skipped here, because of the amount of data. (An excerpt is listed in Table C.2 in appendix C.4.)

The edges of the core graphs coded in $E^{\mathrm{C}}_{cv\tilde{v}}$ and $E^{\mathrm{D}}_{cw\tilde{w}}$ are calculated as described further above. The same applies to the mapping between the distribution, recycling and disposal graph with the core graph, i.e., L^{I}_{cw}, L^{R}_{cw}, and L^{D}_{cw}, respectively. The last mapping between the distribution, recycling and disposal graph and the module and item index is L^{A}_{cw}. It is also automatically determined using the coding of w and the module definition δ_{cmi} as well as the item index i. Excerpts of the data (i.e., $E^{\mathrm{C}}_{cv\tilde{v}}$, $E^{\mathrm{D}}_{cw\tilde{w}}$, L^{I}_{cw}, L^{R}_{cw}, L^{D}_{cw}, and L^{A}_{cw}) can be found in appendix C.4. Given all the data, the optimal solution can be determined.

4.2.4.2 Solution

Based on the model with the given exemplary data a solution shall be generated. Unfortunately, this model is already so big, that a formulation with LINGO is not possible, because of missing memory when building the model. Therefore, the model is directly formulated in R to be solved with GUROBI. Solving the model results in a maximal profit of $P = 30{,}739$ €. Thereby, the revenues and cost are $R = 826{,}995.4$ € and $C = 796{,}256.4$ €, respectively. The values of the variables are listed in Table 4.17and the item and module flow with the quantities of the solution is depicted in Fig. 4.11. Note that all non-listed item and module variables have a value that equals zero. This means that no module is disposed of and, as expected, no items or modules are allocated into the boxes for material recycling of rubber and plastics, because the revenues are to low or the purity is not given. With the

Table 4.17 Optimal solution of the flexible planning

variables representing the interfaces									
Q_1^{C}	30	Q_1^{I}	242	Q_1^{M}	30	Q_1^{R}	35,627	Q_4^{R}	0
Q_2^{C}	218	Q_2^{I}	154	Q_2^{M}	90	Q_2^{R}	72,041	Q_1^{D}	32
Q_3^{C}	31	Q_3^{I}	215			Q_3^{R}	0	Q_2^{D}	6,000

integer variables															
	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
				$r=1$			$r=2$			$d=1$			$d=2$		
	c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	9	98	13	.	117	12	.	1	5	.	.	.	.	.	.
B	12	97	13	.	117	17	.	1	.	.	.	.	.	.	.
C	.	.	.	23	215	30	6	.	.	1	.	1	.	.	.
D	.	.	.	.	215	30	29	.	.	1	.	1	.	.	.
E	29	125	.	.	.	30	.	.	.	.	.	.	.	.	.
F	.	.	.	.	123	30	.	.	.	.	.	.	.	.	.
G	.	215	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	215	.	.	.	.	30	.	.

		Y_{cm}^{M}			Y_{cmr}^{R}					
					$r=1$			$r=2$		
		c			c			c		
m		1	2	3	1	2	3	1	2	3
1	(ABCDEFGH)	.	.	.	.	.	.	.	1	.
6	(ABCDFGH)	.	.	.	.	.	.	.	1	.
10	(BCDFGH)	.	.	.	.	.	.	.	1	.
14	(ABEFGH)	.	.	.	.	.	.	.	.	1
28	(ABEFG)	.	.	.	.	.	.	1	.	.
39	(ABFG)	.	.	.	.	.	.	11	.	.
44	(BFG)	.	.	.	.	.	.	6	.	.
46	(AFG)	.	.	.	.	.	.	9	.	.
47	(GH)	.	.	30	.	.	.	.	.	.
48	(FG)	.	.	.	.	.	.	3	.	.
49	(EF)	.	90	.	.	2	.	.	.	.

A dot denotes a value of zero.

explicit demand of a module of core 3 it is now beneficial to acquire more $Q_3^{\mathrm{C}} = 31$ than just the lower limit $\underline{Q}_3^{\mathrm{C}} = 25$ of core 3. Core 1 is still not beneficial, probably because of the hazardous item H. All three demanded items are distributed much more than the lower limits. The demand of item $e = 1$ with $D_1^{\mathrm{I}} = 250$ units is almost met with $Q_1^{\mathrm{I}} = 242$. The demand of modules is completely met with 30 and 90 units of the modules $f = 1$ and

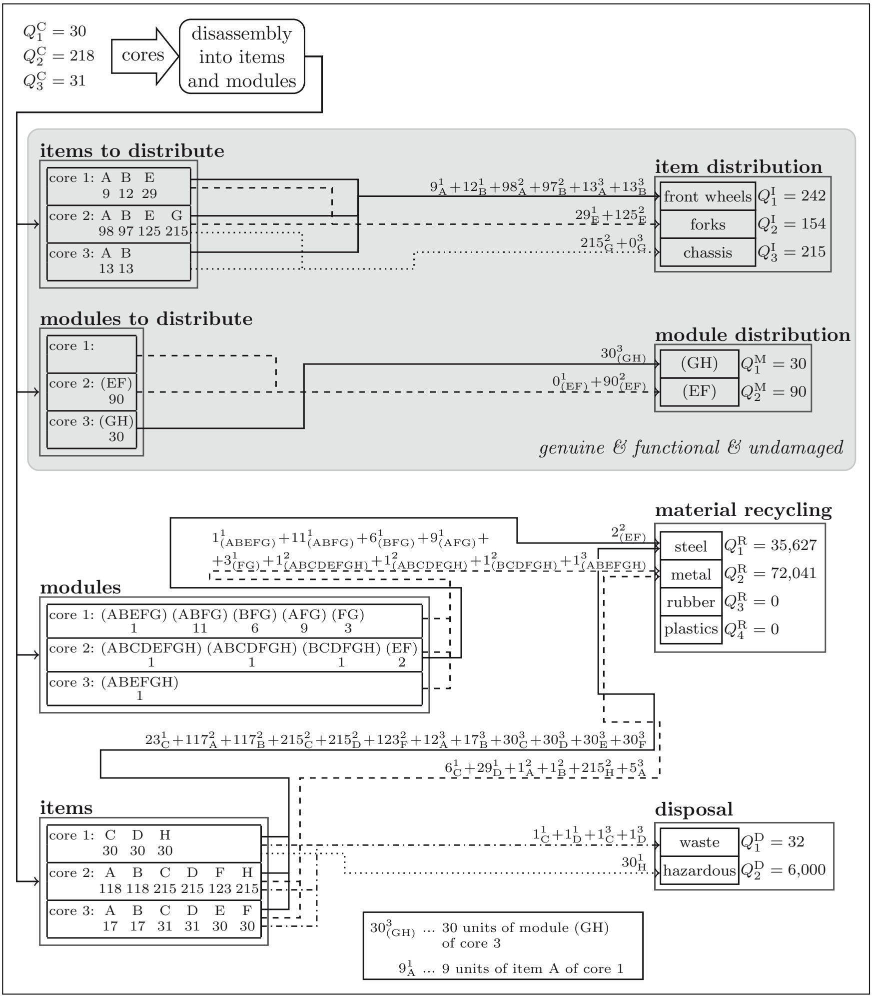

Fig. 4.11 Optimal module and item flow

$f = 2$, respectively. The material to recycle is 35,627 kg and 72,041 kg for steel and metal, respectively. The hazardous disposal is only used for the hazardous item H in core 1, i.e., $X^D_{1,H,2} w_{1,H} = 30 \cdot 200 = 6{,}000$ kg. On the contrary, the regular disposal is only used for one unit of item C and D in core 1 and 3, i.e., $2 \cdot 8 + 2 \cdot 8 = 32$ kg.

To check whether all items that are contained in cores are allocated, the sum of each item row together with the sum of the modules that also contain the items must equal the quantity of the acquired cores. For example, $X^{\mathrm{I}}_{2,\mathrm{A}} = 98$ units of item A of core 2 are distributed as single items. In addition, $X^{\mathrm{R}}_{2,\mathrm{A},1} = 117$ and $X^{\mathrm{R}}_{2,\mathrm{A},2} = 1$ units go into the material box of steel and metal, respectively, for recycling. Lastly, one unit each of the modules ABCDEFGH ($m = 1$) and ABCDFGH ($m = 6$) are allocated into the recycling box of metal (i.e., $Y^{\mathrm{R}}_{2,1,2} = 1$ and $Y^{\mathrm{R}}_{2,6,2} = 1$). All other modules containing item A are not gained out of core 2. Hence, together we have $98 + 117 + 1 + 1 + 1 = 218$ units of item A allocated in various ways out of 218 units of core 2.

To verify whether the required purity is achieved, we need to add the beneficial weight of all items and divide it by the weight of all items. This quotient has to be greater than or equal the given purity level ω_r. For recycling box $r = 1$ the beneficial weight of the items and the modules is

$$
\begin{aligned}
&\sum_{c}\sum_{i=1}^{\bar{I}_c} \pi_{c,i,1} w_{ci} \left(X^{\mathrm{R}}_{c,i,1} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y^{\mathrm{R}}_{c,m,1} \right) \\
&= \pi_{1,\mathrm{C},1}\, w_{1,\mathrm{C}}\, X^{\mathrm{R}}_{1,\mathrm{C},1} + \pi_{2,\mathrm{A},1}\, w_{2,\mathrm{A}}\, X^{\mathrm{R}}_{2,\mathrm{A},1} + \pi_{2,\mathrm{B},1}\, w_{2,\mathrm{B}}\, X^{\mathrm{R}}_{2,\mathrm{B},1} \\
&\quad + \pi_{2,\mathrm{C},1}\, w_{2,\mathrm{C}}\, X^{\mathrm{R}}_{2,\mathrm{C},1} + \pi_{2,\mathrm{D},1}\, w_{2,\mathrm{D}}\, X^{\mathrm{R}}_{2,\mathrm{D},1} + \pi_{2,\mathrm{F},1}\, w_{2,\mathrm{F}}\, X^{\mathrm{R}}_{2,\mathrm{F},1} \\
&\quad + \pi_{3,\mathrm{A},1}\, w_{3,\mathrm{A}}\, X^{\mathrm{R}}_{3,\mathrm{A},1} + \pi_{3,\mathrm{B},1}\, w_{3,\mathrm{B}}\, X^{\mathrm{R}}_{3,\mathrm{B},1} + \pi_{3,\mathrm{C},1}\, w_{3,\mathrm{C}}\, X^{\mathrm{R}}_{3,\mathrm{C},1} \\
&\quad + \pi_{3,\mathrm{D},1}\, w_{3,\mathrm{D}}\, X^{\mathrm{R}}_{3,\mathrm{D},1} + \pi_{3,\mathrm{E},1}\, w_{3,\mathrm{E}}\, X^{\mathrm{R}}_{3,\mathrm{E},1} + \pi_{3,\mathrm{F},1}\, w_{3,\mathrm{F}}\, X^{\mathrm{R}}_{3,\mathrm{F},1} \\
&\quad + \left(\pi_{2,\mathrm{E},1}\, w_{2,\mathrm{E}} + \pi_{2,\mathrm{F},1}\, w_{2,\mathrm{F}}\right) Y^{\mathrm{R}}_{2,49,1} \\
&= 0.5 \cdot 8 \cdot 23 + 0.5 \cdot 11 \cdot 117 + 0.5 \cdot 11 \cdot 117 + 0.5 \cdot 7 \cdot 215 + 0.5 \cdot 7 \cdot 215 \\
&\quad + 0.99 \cdot 180 \cdot 123 + 0.5 \cdot 11 \cdot 12 + 0.5 \cdot 11 \cdot 17 + 0.5 \cdot 8 \cdot 30 + 0.5 \cdot 8 \cdot 30 \\
&\quad + 1 \cdot 36 \cdot 30 + 0.99 \cdot 180 \cdot 30 + (1 \cdot 40 + 0.99 \cdot 180)\, 2 \\
&= 32{,}064.5\,\mathrm{kg}.
\end{aligned}
\tag{4.120}
$$

This beneficial weight is compared with the material weight of $Q^{\mathrm{R}}_1 = 35{,}627\,\mathrm{kg}$, which results in a percentage of $\frac{32{,}064.5}{35{,}627} = 90.001\,\%$. This exceeds the required level of 90 % a little bit and is therefore feasible. It can be assumed that this purity constraint is limiting, i.e., when reducing the purity limit the solution changes. This is interesting, because this material has a higher price so that there should be an incentive to place more material here than in the metal mix box. Obviously, the purity requirements are relatively high compared to the incoming cores. The purity level of box $r = 2$ is $\frac{61{,}897.3}{72{,}041} = 85.920\,\%$, i.e., well above 85 %.

An interesting fact is that module $m = 1$ of core $c = 2$ is allocated in the recycling box $r = 2$ for metal. This nicely illustrates a possible and an economic beneficial decision of recycling a whole core without spending any resources on disassembling it. Of course, the question is if a forklift truck can be recycled as a whole. But it illustrates the flexibility of the modelling. If a complete core must not be recycled, the model can be modified by setting the decision variable zero, e.g., $Y^{\mathrm{R}}_{c,1,r} = 0 \; \forall \; r$. Furthermore, we notice that quite a few modules of core 1 are allocated into box $r = 2$, but none of them contains item H. This is of course prohibited by the fact that item H of core 1 is hazardous.

The inclusion of the condition of the cores is a further interesting aspect—especially with modules. To discuss the result we focus on core $c = 2$, because 90 units of module EF are sold for reuse. This presupposes that both items E and F in those modules are genuine and functioning. From the 218 units of core 2 we expect $(1-\zeta_{2,\mathrm{E}})(1-\eta_{2,\mathrm{E}})(1-\zeta_{2,\mathrm{F}})(1-\eta_{2,\mathrm{F}})Q^{\mathrm{C}}_2 = (1-0)(1-0.01)(1-0)(1-0.05) \cdot 218 = 205.03$ to be with a genuine and functioning item combination of E and F. This is clearly no limitation for the current solution. In total $(1-0)(1-0.01) \cdot 218 = 215.82$ and $(1-0)(1-0.05) \cdot 218 = 207.1$ units of item E and F, respectively, are expected to be genuine and functioning. But when disassembling the single items damage might occur. The percentages of damaging the items are given with $\theta_{2,\mathrm{E}} = 0$ and $\theta_{2,\mathrm{F}} = 0.01$. Of the 205.03 possible genuine and functioning modules 90 are chosen. Hence, 115.03 units remain. Reducing the numbers of available items E and F by the 90 units contained in the module results in 125.82 and 117.1 remaining items, respectively. In order to meet the demand for the item E, the items need to be disassembled. Here, they might be damaged, but not item E ($\theta_{2,\mathrm{E}} = 0$). Thus, the $X^{\mathrm{I}}_{2,\mathrm{E}} = 125$ units of item E can be distributed. For item F no explicit demand exists. Hence, no question of damaging during the disassembly process arises, because this is irrelevant for recycling or disposing.

If there had existed a demand, the upper bound of items F of core 2 could have been calculated as follows. The possible 117.1 genuine and functioning units are taken off modules. Thereby, one per cent is damaged ($\theta_{2,\mathrm{F}} = 0.01$). This leaves us with $117.1 \cdot (1-0.01) = 115.93$, which means that at most further 115 units of item F could be used for item distribution. The difference of $117 - 115 = 2$ units then needs to be recycled or disposed of. But, since no demand exists, no item F will be distributed.

Expanding the view to the material recycling leads to three further modules used of core 2. These three modules are ABCDEFGH, ABCDFGH, and BCDFGH. These three together with EF are planned for material recycling in different material boxes ($Y^{\mathrm{R}}_{2,49,1} = 2$, $Y^{\mathrm{R}}_{2,1,2} = 1$, $Y^{\mathrm{R}}_{2,6,2} = 1$, and

$Y^{\mathrm{R}}_{2,10,2} = 1$). In order to recycle the modules they must not contain the so-called wrong material. Otherwise, they can only be disposed of. Wrong material appears only with non-genuine items. Thus, when all consisting items of a module are genuine and not of the wrong material at the same time, the module can be recycled. An item i is non-genuine and of the wrong material with a probability of $\zeta_{ci}\,\iota_{ci}$. Therefore, with a probability of $(1-\zeta_{ci}\,\iota_{ci})$, it is not non-genuine and of the wrong material at the same time. This probability multiplied for all items in the module gives the expected number of modules that can be used for recycling. In the example solution regarding module ABCDFGH this is 100 %, because in core 2 no wrong material does exist (see Table 3.5). Obviously, all modules ABCDFGH of core 2 could be recycled. To illustrate the aspect, we take module ABCDEFGH ($m = 1$) of core 3. The expected fraction of completely recyclable modules is $\prod_{i\in\{\mathrm{A,B,C,D,E,F,G,H}\}}(1-\zeta_{2,i}\,\iota_{2,i}) = (1-0.1\cdot 0)(1-0.1\cdot 0)(1-0.15\cdot 0.01)(1-0.15\cdot 0.01)(1-0\cdot 0)(1-0\cdot 0)(1-0\cdot 0)(1-0\cdot 0) = 0.997$. This means that at most $\lfloor 0.997\cdot 31\rfloor = 30$ units could be recycled of module 1 of core 3.

Coming back to core 2, any of the modules ABCDEFGH, ABCDFGH, BCDFGH and EF as well as all items can be used for recycling, because of no occurrence of wrong material in core 2. The distribution of items E and F is already discussed above. The items A, B, C, and D are not present in modules to be distributed. Hence, the limitation equals the one of the complete disassembly planning. Of all 218 units we can expect $(1-0.1)(1-0.5)(1-0)218 = 98.1$ units to be genuine, functioning, and undamaged for each of the four items. Taking a look at the solution we find 98 and 97 units for item A and B, respectively, planned for distribution, because of an existing demand. Item C and D are not demanded, hence no distribution is planned.

Item H is also not demanded for distribution. But item G is demanded. The quantity of item G to be distributed is limited by a defective rate of one per cent. Hence, $(1-0)(1-0.01)(1-0)218 = 215.82$ units are expected to be genuine, functioning, and undamaged. This is exactly the number we find in the solution ($X^{\mathrm{I}}_{2,\mathrm{G}} = 215$). All remaining single items can be used and are used for recycling, i.e., either for steel or metal material recycling. In addition, the planned workload results in 2,200 h which equals exactly the given limitation $\bar{L}$.

This first illustrative example for the flexible disassembly shows the possibilities of this approach. Given several cores the optimal disassembly depths and quantities are determined under the above discussed assumptions to achieve the maximal profit out of the item and module revenues as well as material revenues minus the acquisition, disassembly, and disposal cost. The obtained solution does not reveal the precise disassembly state for each unit of a core, yet. This is discussed in Sect. 4.4. But prior to this we take a closer look at two things. One is the gain of the profit introduced by the flexible disassembly planning compared to the complete disassembly planning. The other is the gain of the flexible disassembly planning compared to a two-stage approach.

4.3 Benefit of flexible disassembly planning

4.3.1 Flexible vs. complete disassembly planning

After discussing the solution of the flexible planning with demanded modules, a first comparison with the complete disassembly planning shall follow next. To demonstrate the benefit of the flexible planning, we solve the model again, but this time without any demand for modules to be distributed, i.e., with $\underline{Q}_f^{\mathrm{M}}$ and D_f^{M} set to zero. Thus, no extra revenues can be generated because of the modules. This means that benefits can only be caused by saved disassembly operations for material to recycle and disposal.

The resulting revenues are 739,036.7 €, the cost 731,650.15 €, and thus the profit 7,386.55 €. This is remarkably greater than the profit of the complete disassembly planning with only 2,632.1 € (see Sect. 3.1.3.2). This increasing of the profit is not only caused by decreasing cost. On the contrary, the cost increases from 700,846.4 € to 731,650.15 €. Consequently, the revenues increase even more. This is possible because of the time savings in combination with the limited labour time.

The solution is given in Table 4.18. Thereby, the values in round brackets are the solution values of the complete disassembly planning (see Table 3.9), as long as they are different from this solution. In addition, all decision variables (i.e., X_{ci}^{I}, X_{cir}^{R}, X_{cid}^{D}, Y_{cmr}^{R}, Y_{cmd}^{D}) not explicitly listed equal zero. Note that Y_{cm}^{M} is not a decision variable anymore, because no demand exists. Thus, the value of the variables is set to zero prior solving. From this solution we notice an increased acquisition of core $c = 2$ by 12 units. This rise leads to an increased item distribution and material recycling, because the quantity of disposal is unchanged. The distribution quantity Q_e^{I} of each of the three demanded items (A or B, E, and G) is increased by 12 units. Taking a look at the values of the variables $X_{2,i}^{\mathrm{I}}$ in Table 4.18 we notice that the quantity of item A and B is increased by six units each. The remaining six units of items A and B as well as 12 units of C, D, F, and H result in an increased material recycling.

Table 4.18 Optimal solution of flexible planning without module demand

variables representing the interfaces									
Q_1^{C}	30	Q_1^{I}	228 (216)	Q_1^{M}	0	Q_1^{R}	49,488 (52,610)	Q_4^{R}	0
Q_2^{C}	200 (188)	Q_2^{I}	227 (215)	Q_2^{M}	0	Q_2^{R}	67,882 (60,500)	Q_1^{D}	32
Q_3^{C}	25	Q_3^{I}	222 (210)			Q_3^{R}	0	Q_2^{D}	6,000

integer variables															
	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
				$r=1$			$r=2$			$d=1$			$d=2$		
	c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13	90 (84)	11	1 (17)	109 (104)	13 (14)	.	.	.	.	.	.	.	.	.
B	13	90 (84)	11	1 (17)	110 (104)	13 (14)	.	.	.	.	.	.	.	.	.
C	.	.	.	29	198 (188)	24 (24)	.	.	.	1	.	1	.	.	.
D	.	.	.	29	195 (188)	24 (24)	.	3 (0)	.	1	.	1	.	.	.
E	29	198 (186)	.	0 (1)	0 (2)	0 (25)	.	.	.	.	.	.	.	.	.
F	.	.	.	0 (30)	198 (188)	0 (25)	.	.	.	.	.	.	.	.	.
G	.	198 (186)	24	0 (1)	.	.	1 (29)	0 (2)	0 (1)	.	.	.	.	.	.
H	.	.	.	.	3	.	.	195 (185)	24 (25)	.	.	.	30	.	.

	Y_{cmr}^{R}					
	$r=1$			$r=2$		
	c			c		
m	1	2	3	1	2	3
14 (ABEFGH)	.	.	1	.	.	.
26 (ACDGH)	.	.	.	.	1	.
32 (CDGH)	.	.	.	.	1	.
39 (ABFG)	.	.	.	9	.	.
44 (BFG)	.	.	.	7	.	.
46 (AFG)	.	.	.	7	.	.
48 (FG)	.	.	.	6	.	.
49 (EF)	1	2	24	.	.	.

A dot denotes a value of zero.

Having a look at the decision variables for the modules we notice that of all modules only the modules 14, 26, 32, 39, 44, 46, 48, and 49 are used. Obviously, not all of them are used in all cores. Module $m = 49$ occurs in all cores and all other modules in a particular core. Adding the saved time t_{cm}^{J} of all 59 modules in the solution results in

$$\sum_c \sum_m t^{\mathrm{J}}_{cm} \sum_r Y^{\mathrm{R}}_{cmr} = 7.75 \cdot 1 + 5.875 \cdot 1 + 5.75 \cdot 1 + 3.250 \cdot 9 + 3.125 \cdot 7 + 3.125 \cdot 7 + 3 \cdot 6 + 0.5 \cdot 1 + 0.5 \cdot 2 + 0.5 \cdot 24 = 123.875\,\mathrm{h}. \tag{4.121}$$

To disassemble one unit of core 2 completely, 9 h are necessary. Subtracting the saved 123.875 h from the 2,192 h of the basic model and adding the 9 h for the 12 units results in a workload of 2,176.125 h. This leaves a gap of 23.875 h to the limit of 2,200 h such that the labour time seems to be not limiting anymore, because the gap is greater than the time to disassemble any of the cores completely. But, when increasing the available labour time to $\bar{L} = 2{,}230$ h a solution with the workload of 2,230 h and an increased profit (7,810.1 €) results. Thus, the labour time is still limiting and probably in combination with the core condition.

This solution presented in Table 4.18 is again a good example for flexible disassembling. Taking core 1 five different modules are included in the solution. These are 39, 44, 46, 48, and 49. Taking a look at the module definition we notice that these modules are mutual exclusive, i.e., not two of them can be gained simultaneously out of one unit of the core. This is a degree of freedom that cannot be reached by a two-stage approach of first determining the disassembly sequence and then determining the optimal quantities.

4.3.2 Flexible vs. incomplete two-stage disassembly planning

The second comparison is between two approaches for incomplete disassembly. The two approaches are the flexible disassembly planning (presented here) and a two-stage approach, with first determining the optimal disassembly sequence (and state) and afterwards the optimal quantities.[37] The problem with the two-stage approach is to decide what the criterion for an

[37] Remember, several sequences can lead to one disassembly state, because the state is the result of the disassembly, i.e., the gained modules and items. Hence, when a sequence is determined the state is automatically determined, too. But the opposite is not necessarily true. One example of a similar two-stage approach is the determination of a disassembly state using goal programming in the first stage and the multi-core planning maximising the profit in the second phase. Cf. Xanthopoulos / Iakovou (2009): *On the optimal design of disassembly.*

optimal disassembly sequence is. Several criteria exist in the literature, e.g., disassembly time, profit, removals, cost, and orientation changes.[38]

But among these approaches, we cannot select one that results coercively in the optimal disassembly state for our problem. This is because not all of the aspects we consider are integrated in these approaches found in the literature. Therefore, we integrate the state selection into the above presented model. Thus, it is certain that the optimal single state for our problem is found. Hence, all other existing sequence planning approaches will most likely result in a suboptimal result with regard to our planning problem.

The modification is realised by an inclusion of binary variables to assure that only one of the possible states per core is selected. Thereby, we have to keep in mind, that a state might not contain only one module. The binary variable has to assure that only one node of the disassembly state graph is selected. One such node is the one representing the complete disassembly. Hence, the lower bound of the solution is that of the complete disassembly. Of course, we expect finding a solution with a higher profit, than of the complete disassembly planning. Otherwise, the extra effort of determining the sequence would not be beneficial.

The favoured design to model this aspect is by extending the earlier developed model. Thus, all constraints can stay unchanged. To incorporate the limitation to a state a binary variable is introduced. This is denoted U_{cs} for every core c and state s of the disassembly state graph. Each state is characterised by a unique module and single item combination. Using the disassembly state graph in Fig. 4.4 and numbering the states in increasing order beginning at the top we have state $s = 1$ representing the whole core, i.e., no disassembling takes place. State $s = 18$ represents the disassembly sequence that results in the combination "A.B(EF)(CDGH)". The last state $s = 60$ does not contain any modules. That means it stands for the complete disassembly. For each particular core an individual optimal disassembly state can exist and therefore the binary variable can be set for every core c individually. The number of states per core is denoted by $\bar{S}_c$. In our example this is $\bar{S}_c = 60 \ \forall \ c$.

The above mentioned unique combination of modules and items of a state needs to be stored to be used in the optimisation. We achieve this with two arrays $\gamma^{\mathrm{I}}_{cis}$ and $\gamma^{\mathrm{M}}_{cms}$ for items and modules, respectively. For each state s and core c a value of one represents that the item i and module

[38] Cf. Go et al. (2012): *Genetically optimised disassembly sequence*, Tripathi et al. (2009): *Disassembly modeling and sequencing*, Srinivasan / Gadh (2002): *Selective disassembly sequence with geometric constraints*, Moore / Güngör / Gupta (2001): *Petri net approach to disassembly process planning*, and Lu et al. (2008): *A multi-objective disassembly planning with ACO*, respectively.

Table 4.19 Module state combinations for the three cores

m	states s	m	states s	m	states s	m	states s
1	1	14	15	27	31	40	48
2	2	15	16	28	32	41	49
3	3	16	17	29	34	42	33, 50
4	4	17	19	30	35	43	36, 51
5	5	18	20	31	37	44	52
6	6	19	21	32	18, 38	45	41, 53
7	7	20	22	33	39	46	54
8	8	21	23	34	40	47	47, 56
9	9	22	24	35	42	48	57
10	10	23	26	36	43	49	11, 18, 25, 28, 33,
11	12	24	27	37	25, 44		36, 41, 47, 55, 58
12	13	25	29	38	28, 45	50	55, 59
13	14	26	11, 30	39	46		

Module $m = 49$ is present in ten states.

m is the result of the disassembly in this state. The state $s = 18$ results in the combination "A.B(EF)(CDGH)". Thus, the values $\gamma^{\mathrm{I}}_{c,\mathrm{A},18}$, $\gamma^{\mathrm{I}}_{c,\mathrm{B},18}$, $\gamma^{\mathrm{M}}_{c,32,18}$, and $\gamma^{\mathrm{M}}_{c,49,18}$ have a value of one. All other values for $\gamma^{\mathrm{M}}_{cms}$ and $\gamma^{\mathrm{I}}_{cis}$ for this state of the particular core c are 0. The complete information for the modules is listed in Table 4.19. Since all three cores are in the example structurally identical, the module state pairs (m, s) apply to all three cores identically. All other entries of $\gamma^{\mathrm{M}}_{cms}$ not indicated by a given pair have a zero value. Only the explicitly listed pairs in the table have a value equal one. Given the module state combinations, the values of $\gamma^{\mathrm{I}}_{cis}$ can automatically be calculated with the formula

$$\gamma^{\mathrm{I}}_{cis} = \begin{cases} 1 & \sum_{m\in\{m|\gamma^{\mathrm{M}}_{cms}=1\}} \delta_{cmi} = 0 \\ 0 & \text{else} \end{cases} \qquad \forall\ c, i \in \{1, \ldots, \bar{I}_c\}, s \in \{1, \ldots, \bar{S}_c\}\,. \tag{4.122}$$

This means that $\gamma^{\mathrm{I}}_{cis}$ equals one if the item i is not in any of the modules in state s. Let us illustrate this with state $s = 18$ and core $c = 1$. For all items we have to look whether $\sum_{m\in\{m|\gamma^{\mathrm{M}}_{cms}=1\}} \delta_{cmi}$ equals zero or not. According to Table 4.19 state 18 contains the modules $m = 32$ and $m = 49$, because $\gamma^{\mathrm{M}}_{1,32,18} = 1$ and $\gamma^{\mathrm{M}}_{1,49,18} = 1$. Adding the two vectors of the module definition matrix $\delta_{1,32,i}$ and $\delta_{1,49,i}$ results in

$$\begin{array}{r} \delta_{1,32,i} = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \\ +\ \delta_{1,49,i} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \\ \hline = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}\,. \end{array} \tag{4.123}$$

We see that only the first two entries representing items A and B equal zero. Hence, the values $\gamma^{\mathrm{I}}_{1,\mathrm{A},18}$ and $\gamma^{\mathrm{I}}_{1,\mathrm{B},18}$ equal one and the remaining six $\gamma^{\mathrm{I}}_{1,\mathrm{C}\ldots\mathrm{H},18}$ equal zero.

To add the necessary constraints to the existing model two possible ways shall be discussed in the sequel. The first approach limits all decision variables for quantities of modules and items that are not in the state to zero. This is via the sum over the relevant variables and not for every variable individually. This means that for any selected state s the sum over all non-contained modules plus the non-contained items has to be zero.

$$\sum_{m\in\{m|\gamma^{\mathrm{M}}_{cms}=0\}}\left(Y^{\mathrm{M}}_{cm}+\sum_{r}Y^{\mathrm{R}}_{cmr}+\sum_{d}Y^{\mathrm{D}}_{cmd}\right) + \sum_{i\in\{i|\gamma^{\mathrm{I}}_{cis}=0\}}\left(X^{\mathrm{I}}_{ci}+\sum_{r}X^{\mathrm{R}}_{cir}+\sum_{d}X^{\mathrm{D}}_{cid}\right)=0 \quad (4.124)$$

The fact that an item or module is contained in the state is stored in the parameter $\gamma^{\mathrm{I}}_{cis}$ and $\gamma^{\mathrm{M}}_{cms}$. The modules and items contained in the state are not limited here. The state selection is achieved by the binary variable U_{cs}. A selected state is denoted by $U_{cs}=1$. Hence, when the above sum must be zero, we can also write

$$\sum_{m\in\{m|\gamma^{\mathrm{M}}_{cms}=0\}}\left(Y^{\mathrm{M}}_{cm}+\sum_{r}Y^{\mathrm{R}}_{cmr}+\sum_{d}Y^{\mathrm{D}}_{cmd}\right) + \sum_{i\in\{i|\gamma^{\mathrm{I}}_{cis}=0\}}\left(X^{\mathrm{I}}_{ci}+\sum_{r}X^{\mathrm{R}}_{cir}+\sum_{d}X^{\mathrm{D}}_{cid}\right)\leq(1-U_{cs})\,. \quad (4.125)$$

This constraint is also applied to the unselected states with $U_{cs}=0$. The problem that then arises is an unwanted limitation in the following manner. Assuming state 18 is selected ($U_{1,18}=1$) and all other states not. In addition, we expect a quantity of 100 units to be distributed of item A ($X^{\mathrm{I}}_{1,\mathrm{A}}=100$). Taking a look at the constraint for state $s=1$, only module $m=1$ exists and all other quantities are limited by

$$\sum_{m \in \{m | \gamma^{\mathrm{M}}_{1,m,1}=0\}} \left(Y^{\mathrm{M}}_{1,m} + \sum_r Y^{\mathrm{R}}_{1,m,r} + \sum_d Y^{\mathrm{D}}_{1,m,d} \right)$$
$$+ \sum_{i \in \{i | \gamma^{\mathrm{I}}_{1,i,1}=0\}} \left(X^{\mathrm{I}}_{1,i} + \sum_r X^{\mathrm{R}}_{1,i,r} + \sum_d X^{\mathrm{D}}_{1,i,d} \right) \leq (1-U_{1,1}) = (1-0) = 1 \,. \tag{4.126}$$

With this constrained the quantity is limited to only one unit, if all other quantities are zero. This is not desired. Therefore, the limitation of one has to be increased so that it is not limiting anymore. This is done by multiplying the right hand side with an arbitrary value M. This value should be at least 100 to have the 100 units. To be more precise, the value of M should be $\max_c\{\bar{I}_c \, \overline{Q}^{\mathrm{C}}_c\}$ at least. In our example the cores are not limited by an upper limit, which leaves us with educated guessing. In the sequel we set $M = 3{,}000$.

Including this M in the constraint and assuring that only one state per core can be selected the set of constraints is as follows.

$$\sum_{m \in \{m | \gamma^{\mathrm{M}}_{cms}=0\}} \left(Y^{\mathrm{M}}_{cm} + \sum_r Y^{\mathrm{R}}_{cmr} + \sum_d Y^{\mathrm{D}}_{cmd} \right)$$
$$+ \sum_{i \in \{i | \gamma^{\mathrm{I}}_{cis}=0\}} \left(X^{\mathrm{I}}_{ci} + \sum_r X^{\mathrm{R}}_{cir} + \sum_d X^{\mathrm{D}}_{cid} \right) \leq M(1 - U_{cs})$$
$$\forall \; c, s \in \{1, \ldots, \bar{S}_c\} \tag{4.127}$$

$$\sum_s U_{cs} = 1 \quad \forall \; c \tag{4.128}$$

$$U_{cs} \in \{0, 1\} \quad \forall \; c, s \in \{1, \ldots, \bar{S}_c\} \,. \tag{4.129}$$

The number of added constraints is $\sum_c \left(\bar{S}_c + 1\right)$, neglecting the domain constraints of U_{cs}.

The second mentioned approach is as follows. The quantity of modules and items can only be greater than zero if a state containing this module or item is selected. To give an example, module $m = 32$ is the focused one. In order to have a quantity greater than zero for this module in core $c = 1$ the state $s = 18$ has to be selected. If only state $s = 17$ is selected, the module $m = 32$ cannot be gained. We see that the selection variable U_{cs} functions as a switch, again. This can be expressed by

$$Y^{\mathrm{M}}_{1,32} + \sum_r Y^{\mathrm{R}}_{1,32,r} + \sum_d Y^{\mathrm{D}}_{1,32,d} \leq M \sum_{s\in\{s|\gamma^{\mathrm{M}}_{1,32,s}=1\}} U_{1,s}\,. \tag{4.130}$$

As soon as a state is selected that contains the focused module the quantities can be greater than zero. Taking module $m = 49$ in the focus we find that we can choose state 18, 33, or any other state module 49 exists in to set the quantity limit for the decision variables $Y^{\mathrm{M}}_{1,49}$, $Y^{\mathrm{R}}_{1,49,r}$, and $Y^{\mathrm{D}}_{1,49,d}$ greater than zero. The same applies to the items. The adequate value for M can be found in the same way as discussed above. The corresponding constraints are

$$Y^{\mathrm{M}}_{cm} + \sum_r Y^{\mathrm{R}}_{cmr} + \sum_d Y^{\mathrm{D}}_{cmd} \leq M \sum_{s\in\{s|\gamma^{\mathrm{M}}_{cms}=1\}} U_{cs} \quad \forall\, c, m \in \{1,\ldots,\overline{M}_c\} \tag{4.131}$$

and

$$X^{\mathrm{I}}_{ci} + \sum_r X^{\mathrm{R}}_{cir} + \sum_d X^{\mathrm{D}}_{cid} \leq M \sum_{s\in\{s|\gamma^{\mathrm{I}}_{cis}=1\}} U_{cs} \quad \forall\, c, i \in \{1,\ldots,\bar{I}_c\}\,. \tag{4.132}$$

To limit the number of selected states to just one, Eq. (4.128) can be used right away. But with this approach the flexibility of modelling is increased in a way that the decision maker can specify the number of states to select, i.e., to limit the number of disassembly states. The allowed number of states shall be given by $\tilde{S}_c$ for each core. The resulting constraint is then

$$\sum_s U_{cs} \leq \tilde{S}_c \quad \forall\, c\,. \tag{4.133}$$

Setting all $\tilde{S}_c = 1$ we have the same constraint as in Eq. (4.128). The limitation of the domain of U_{cs} to binary values completes the second set of constraints.

$$U_{cs} \in \{0,1\} \quad \forall\, c, s \in \{1,\ldots,\bar{S}_c\} \tag{4.134}$$

The number of added constraints is $\sum_c \left(\overline{M}_c + \bar{I}_c + 1\right)$, when using this second set and neglecting the domain constraints of the variables.

In our example the number of modules and items of each core is $3\,(50 + 8) = 174$ whereas the number of states is $3 \cdot 60 = 180$. Hence, the second set of constraints results in fewer constraints to be added than the first set. In general this is a good reason to choose the second constraint set. A further reason for the second set—independent of the number of constraints—is the possibility to adjust $\tilde{S}_c$ to any desired number of allowed disassembly sequences per core. This will be discussed in more detail in Sect. 4.3.3.

Table 4.20 Optimal solution of best two-stage planning

variables representing the interfaces									
Q_1^{C}	30	Q_1^{I}	232 (242)	Q_1^{M}	30	Q_1^{R}	50,948 (35,627)	Q_4^{R}	0
Q_2^{C}	200 (218)	Q_2^{I}	198 (154)	Q_2^{M}	28 (90)	Q_2^{R}	60,502 (72,041)	Q_1^{D}	32
Q_3^{C}	31	Q_3^{I}	198 (215)			Q_3^{R}	0	Q_2^{D}	6,000

integer variables															
	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
				$r=1$			$r=2$			$d=1$			$d=2$		
	c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13 (9)	90 (98)	13	17 (0)	110 (117)	18 (12)	.	0 (1)	0 (5)	.	.	.	.	.	.
B	13 (12)	90 (97)	13	17 (0)	110 (117)	18 (17)	.	0 (1)	.	.	.	.	.	.	.
C	.	.	.	29 (23)	200 (215)	30	0 (6)	.	.	1	.	1	.	.	.
D	.	.	.	29 (0)	200 (215)	30	0 (29)	.	.	1	.	1	.	.	.
E	0 (29)	198 (125)	.	.	.	29 (30)	.	2 (0)	2 (0)	.	.	.	.	.	.
F	.	.	.	.	200 (123)	31 (30)	.	.	.	.	.	.	.	.	.
G	.	198 (215)	.	1 (0)	.	.	29 (0)	2 (0)	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	200 (215)	.	.	.	.	30	.	.

	Y_{cm}^{M}			Y_{cmr}^{R}					
				$r=1$			$r=2$		
	c			c			c		
m	1	2	3	1	2	3	1	2	3
47 (GH)	.	.	30	.	.	.	.	.	1 (0)
49 (EF)	28 (0)	0 (90)	.	2 (0)	0 (2)	.	.	.	.

A dot denotes a value of zero.

Solving the model—including the module demand as given in Table 4.15—with the first or second (with $\tilde{S}_c = 1$) constraint set added to the flexible planning model, results in a solution where only one disassembly state per core is selected. The resulting profit is 22,505.3 €. The selected states are 58, 60, and 56 for the cores 1, 2, and 3, respectively. (The corresponding state selection variables $U_{1,58}$, $U_{2,60}$, and $U_{3,56}$ are one and all other zero.) The solution is listed in the known form in Table 4.20. The values in brackets are the ones from the flexible planning as long as they are different to the values of this solution. The revenues are 771,616.7 € and the cost 749,111.4 €. The resulting profit of 22,505.3 € is significantly larger than that of just complete disassembly. Note that the demanded modules increase the profit compared with just complete disassembly where there exist no modules. In comparison to the flexible planning we notice an expected decrease from 30,739 €

to 22,505.3 €. Note that this difference is the smallest possible between the flexible and any two-stage planning for this example.

When comparing the two solutions in more detail we see that less cores are acquired. Also, the quantities of distributed items are slightly increased and a shift towards item position $e = 2$ can be noticed. Whereas the item distribution is increased, the module distribution is drastically reduced from 90 to 28 units of module demand position $f = 2$. Taking a look at the material recycling quantities a shift from material metal ($r = 2$) to steel ($r = 1$) occurs. The metal recycling quantity is reduced by about 11,000 kg and at the same time the quantity for the steel recycling is increased by about 15,000 kg. This is interesting, because the steel recycling requires a higher purity than the metal one and more items are more beneficial with respect to the purity requirement. The quantities for disposal stay identical with 32 kg and 6,000 kg.

The state 58 of core 1 contains the module EF and the single items A, B, C, D, G, and H. Thus, of core 1 no other module than EF can appear anywhere in the solution. For core 2 the complete disassembly is chosen, which means that no module of core 2 can exist. Hence, the 90 modules of the flexible planning of core 2 cannot be used anymore. Thus, we notice a shift from core 2 with 90 units of module EF to core 1 with 28 units of this module. For core 3 the state 56 is chosen, which leads to module GH. In the solution we find 30 units to be distributed and one unit to be recycled. Summarising the findings, we see that the flexible planning is more profitable compared to any two-stage approach, where one disassembly state per core is selected and the quantities determined afterwards. In the numerical example the absolute difference is 8,233.7 €, which is a decrease of about 27 %.

4.3.3 Optimal flexible disassembly planning with minimal number of sequences

With the inclusion of the constraints (4.131) through (4.134) into the flexible disassembly planning model the number of states per core can be limited. In the section above the limitation was set to one per core, to determine the upper bound of any two-stage incomplete disassembly planning. In the optimal solution of the flexible planning five, five, and two states are necessary for core 1, 2, and 3, respectively. This can be derived from the solution in Table 4.17 in conjunction with the state determination in the subsequent Sect. 4.4. Starting with core 1 the modules 28, 39, 44, 46, and 48 are included in the solution with the quantities 1, 11, 6, 9, and 3, respectively. These modules are mutual exclusive, because they all contain the item G. Hence, not two of them can occur parallel. This gives $1+11+6+9+3=30$ units which equals the acquired quantity of $Q_1^{\mathrm{C}}=30$. Thus, no extra state with complete disassembly is planned. The corresponding states for the selected modules are 32, 46, 52, 54, and 57.

For core 2 we have the modules 1, 6, 10, and 49 in the solution. The corresponding quantities of these are 1, 1, 1, and 92 units. They are mutual exclusive, too. Therefore, the states 1, 6, and 10 are necessary one time each and state 58 (for module 49) 92 times. Comparing these $92+3=95$ with the acquired quantity of 218, results in a difference of 123. This difference is the quantity of state 60, i.e., the complete disassembly. Core 3 is listed with the modules 14 and 47 with quantities of one and 30. The sum of the quantities equals the quantity of the acquired core. Hence, only state 15 and 56 are selected. We see that five, five, and two states of the cores 1, 2, and 3, respectively, are required.

In general, there exist more than one optimal solutions for MILP of this size. Given the model extension, we would like to know how we can achieve the optimal solution with the least number of sequences. With this requirement the problem has two objective functions—one minimisation and one maximisation. This is the simplest case of multi-criteria or multi-objective programming. Several approaches exist to solve such problems. But the two basic ones suffice to answer the question above. One of them is the lexicographic (or preemptive) method.[39] With this method the objectives have to be ordered according to their importance to the decision maker. The most important objective is the profit maximisation and the less important the minimal number of states required for this profit maximal solution. According to the lexicographic order, the problem with the most important objective is solved neglecting the less important objectives. For this, no state limiting constraints are necessary and the optimal solution of the flexible disassembly planning with 30,739 € is the result (see Sect. 4.2.4).

With this information of the optimal value of the first objective we focus on the objective with the next lower priority, i.e., the number of sequences. Thereby, the profit P is fixed to the value $P=30{,}739$ in all following optimisations with lower priority objectives. The objective function is changed to the one of the next less important objective, i.e., to minimise the number of states.

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[39] Cf. Taha (2011): *Oprations Research*, pp. 342 et seq., Suhl / Mellouli (2009): *Optimierungssysteme*, pp. 120 et seq., Domschke / Drexl (2011): *Einführung in OR*, pp. 57 et seq., and Yager / Gumrah / Reformat (2011): *PET for lexicographic multi-criteria service selection*, p. 932.

$$\text{Minimise } \sum_{c} \tilde{S}_c \tag{4.135}$$

The model to solve is that with the limiting number of states, i.e., the flexible planning model including the constraints (4.131)–(4.134). Thereby, the parameter $\tilde{S}_c$ is changed into a decision variable that appears in the objective function.[40] Consequently, the number of states is no longer limited to one per core.

When relaxing the limitation of sequences to more than one, an additional constraint should be introduced. For specifying just the number of sequences it is not necessary, but it is helpful for interpreting the solution. Let us consider state 18 with “AB(EF)(CDGH)” and state 60 with “ABCDEFGH”. When these states are selected with a value of one of the binary variable U_{cs}, the quantities for the modules EF and CDGH as well as all items can be greater than zero. This however does not prevent a solution where the quantity of both or either module is zero. Of course, when both module quantities are zero and the number of sequences should be minimised, only state 60 is activated. Therefore, we assume that the quantities of module CDGH and EF are zero and ten, respectively, as an example. In addition, 20 units of the core are acquired. Hence, only module EF and single items are in the solution. But this solution is not gained by state 18 and 60. Because, if state 18 is activated and ten units of module EF are generated, ten units of module CDGH are generated, too. Thus, a state without the module CDGH and only with module EF, i.e., state 58, together with state 60 is the correct state combination to generate the module and item quantities. We see, that it might be helpful to force an “activation” of each module in a selected state. This can be achieved by adding

$$Y^{\mathrm{M}}_{cm} + \sum_{r} Y^{\mathrm{R}}_{cmr} + \sum_{d} Y^{\mathrm{D}}_{cmd} \geq \sum_{s \in \{s | \gamma^{\mathrm{M}}_{cms} = 1\}} U_{cs} \quad \forall\, c, m \in \{1, \ldots, \overline{M}_c\}\,. \tag{4.136}$$

In combination with the minimisation of the number of sequences the solution now contains activated states, which are necessary. Solving this model leads to an optimal value of 11. This means that in total 11 states for all three cores are required and the profit is still the optimal one.[41]

[40] Alternatively, the objective function can be modelled without the $\tilde{S}_c$ by just using $\sum_c \sum_{s=1}^{\tilde{S}_c} U_{cs}$ to be minimised.

[41] This approach can also be seen as goal programming, because the less important objectives are optimised with the goal to maintain the optimal value of the more important objectives.

A further approach is the programming with weighted objectives.[42] The advantage is the simultaneous consideration of all objectives. On the other hand, the drawback is the determination of adequate weights. When we assume that the focus is clearly on the profit and that only a magnitude up to 10^{-2} is relevant we can easily set a fairly small weight to the number of sequences. Thus, the influence on the profit is negligible if the weight is set to, e.g., 10^{-4}. Only with more than 100 states we would have an influence on the profit by 10^{-2}.[43] The modified objective function for the weighted programming of our example is

$$\text{Maximise } P - 0.0001 \sum_{c} \tilde{S}_c \tag{4.137}$$

or

$$\text{Maximise } P - 0.0001 \sum_{c} \sum_{s=1}^{\bar{S}_c} U_{cs}\,. \tag{4.138}$$

Otherwise, the model is equal to the one with the limiting number of states and $\tilde{S}_c$ being a decision variable. Solving this model to optimum the same solution results as given above with the lexicographical programming. For our problem the weighted programming returns the optimal result in less time. Mainly, because of the fact that the complex model has to be solved only once compared to the lexicographical approach. In addition, only the profit is fixed in the second run of the lexicographical approach, which leaves a lot of decision variables to be determined. Of course, this does not have to be the case for all problems.

The resulting minimal number of states for the optimal profit are five, four, and two. For core 1 the states 32, 46, 52, 54, and 57, for core 2 the states 1, 6, 58, and 60, and for core 3 the states 15 and 56 are selected. The resulting modules are 28, 39, 44, 46, and 48 of core 1, 1, 6, and 49 of core 2, as well as 14 and 47 of core 3. The optimal solution with only 11 states in total is listed in Table 4.21. The quantities of the interfaces are identical. In addition, a new module $m = 14$ instead of module 10 is included in the solution. Also, some changes of quantities of item flow variables can be noticed. In general this solution is to be preferred compared to the initial flexible planning result, because less states and sequences are necessary. But, the solving takes more time, because the model is more complex.

[42] Cf. Suhl / Mellouli (2009): *Optimierungssysteme*, pp. 120 et seq., and Domschke / Drexl (2011): *Einführung in OR*, pp. 58 et seq.

[43] To estimate the magnitude of the objectives, the flexible planning model can be solved without forcing integral values for the decision variables.

Table 4.21 Optimal solution with minimum number of states

variables representing the interfaces

Q_1^{C}	30	Q_1^{I}	242	Q_1^{M}	30	Q_1^{R}	35,627	Q_4^{R}	0
Q_2^{C}	218	Q_2^{I}	154	Q_2^{M}	90	Q_2^{R}	72,041	Q_1^{D}	32
Q_3^{C}	31	Q_3^{I}	215			Q_3^{R}	0	Q_2^{D}	6,000

integer variables

	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
				$r=1$			$r=2$			$d=1$			$d=2$		
	c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	11 (9)	97 (98)	13	.	108 (117)	13 (12)	.	10 (1)	4 (5)	.	.	.	.	.	.
B	11 (12)	97	13	.	117	17	.	1	.	.	.	.	.	.	.
C	.	.	.	29 (23)	215	30	. (6)	.	.	1	.	1	.	.	.
D	.	.	.	5 (0)	215	30	24 (29)	.	.	1	.	1	.	.	.
E	29	125	.	.	.	30	.	.	.	.	.	.	.	.	.
F	.	.	.	.	123	30	.	.	.	.	.	.	.	.	.
G	.	215	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	215	.	.	.	.	30	.	.

	Y_{cm}^{M}			Y_{cmr}^{R}					
				$r=1$			$r=2$		
	c			c			c		
m	1	2	3	1	2	3	1	2	3
1 (ABCDEFGH)	.	.	.	.	.	.	.	1	.
6 (ABCDFGH)	.	.	.	.	.	.	.	2 (1)	.
10 (BCDFGH)	.	.	.	.	.	.	.	. (1)	.
14 (ABEFGH)	.	.	.	.	.	.	.	.	1
28 (ABEFG)	.	.	.	.	.	.	1	.	.
39 (ABFG)	.	.	.	.	.	.	9 (11)	.	.
44 (BFG)	.	.	.	.	.	.	9 (6)	.	.
46 (AFG)	.	.	.	.	.	.	9	.	.
47 (GH)	.	.	30	.	.	.	.	.	.
48 (FG)	.	.	.	.	.	.	2 (3)	.	.
49 (EF)	.	90	.	.	2	.	.	.	.

A dot denotes a value of zero.

A benefit of this model is the choice of the decision maker, how relevant the number of states is. If for example the cost for each further state is 200 € the weight is set to 200 instead of 10^{-4} and the solution of the model determines the optimal number of states and quantities. Note that in this case the term $200 \sum_c \tilde{S}_c$ should be added to the cost and not as a tuning term in the objective function next to the profit. Solving the numerical example with this weight the optimal number of sequences would be four, instead of 11.

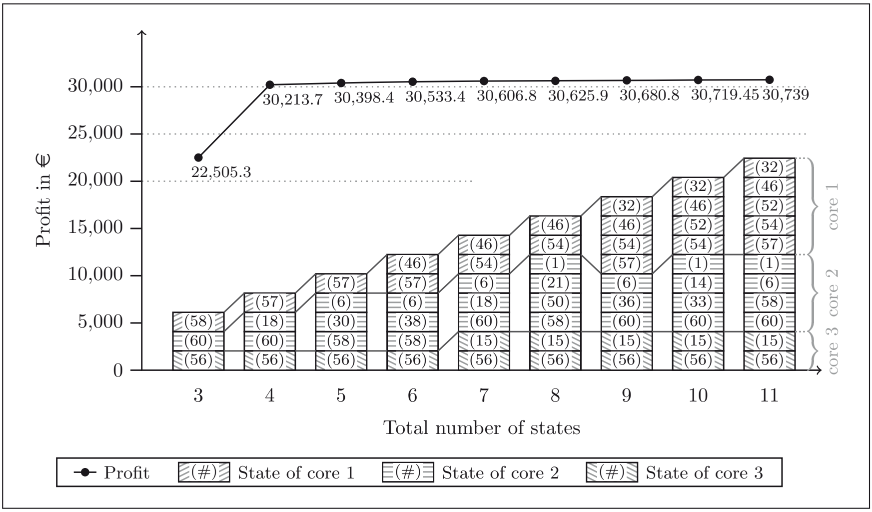

Fig. 4.12 Profit development depending on the allowed number of states

A further interesting insight for the decision maker is possible with this model. This is how the profit develops from three to 11 allowed states for the three cores. Therefore, we maximise the profit (without any extra terms), keep the $\tilde{S}_c$ as a decision variable, and add a further constraint that limits the sum of the $\tilde{S}_c$ from three through 11 for our example.

$$\sum_c \tilde{S}_c \leq 3, \ldots, 11 \tag{4.139}$$

The resulting profit and the allocation of the states is depicted in Fig. 4.12. The additional numbers of states from three to five are all used for core 2. The next extra state is used for core 1. From six to seven possible states each extra state goes into core 3 and so on. Looking at core 3, we notice that the used states are relatively constant. From the beginning (one state per core) up to the end (11 states overall) state 56 is part of the solution. Once state 15 is added (seven states allowed) it also stays part of the solution up to the end. In the other two cores the planned states fluctuate more when adding an extra possible state. A good example is from eight to nine possible states for the three cores, where even the number of states for core 2 decreases so that for core 1 two states are added. Note that there exist several optimal solutions per allowed number of states with most likely different state allocations.

Table 4.22 Solutions depending on allowed number of states

number of states	Q^{C}_c			Q^{I}_e			Q^{M}_f		Q^{R}_r				QD_d	
	c			e			f		r				d	
	1	2	3	1	2	3	1	2	1	2	3	4	1	2
3	30	200	31	232	198	198	30	28	50,948	60,605	0	0	32	6,000
4	30	216	31	246	152	213	30	90	36,213	70,679	0	0	32	6,000
5	30	217	31	244	153	214	30	90	35,772	71,508	0	0	32	6,000
6	30	217	31	246	153	214	30	90	35,993	71,265	0	0	32	6,000
7	30	218	31	233	154	215	30	90	35,727	72,040	0	0	32	6,000
8	30	218	31	235	154	215	30	90	35,677	72,068	0	0	32	6,000
9	30	218	31	238	154	215	30	90	35,677	72,035	0	0	32	6,000
10	30	218	31	241	154	215	30	90	35,627	72,052	0	0	32	6,000
11	30	218	31	242	154	215	30	90	35,627	72,041	0	0	32	6,000

What is also very interesting for the decision maker is the resulting maximal profit. It increases with the allowed number of states up to the optimal number of 11 states. One would expect, that the increase of the profit decreases with the allowed number of states, i.e., the profit increase from four to five is 30,398.4 − 30,213.7 = 184.7 € whereas the increase from five to six is smaller with 30,533.4 − 30,398.4 = 135 €. But, for example from seven to eight (19.1 €) compared to eight to nine (54.9 €) this is not the case. Nevertheless, the tendency of a decreasing profit increase can generally be assumed. The biggest profit increase appears from three to four allowed states. This results mainly from the possibility to distribute more demanded modules $m = 49$. With three states only $Q^{\mathrm{M}}_2 = 28$ units are distributed (see Table 4.20) and with four states already 90 units are distributed (see Table C.9 in appendix C.5). The solutions for the allowed number of states four through ten can be found in appendix C.5.

In Table 4.22the ingoing (Q^{C}_c) and outgoing quantities (Q^{I}_e, Q^{M}_f, Q^{R}_r, Q^{D}_d) of the company are summarised for the different allowed number of states. The disposal quantities stay unchanged. Always the same amount of items (never a module) is disposed of. Of core 1 and 3 the quantity of acquired cores stays identical with 30 and 31, respectively. Only core 2 shows increasing quantities of acquired cores. For a number of at least four states the demanded quantity of 90 modules is distributed. In general, for the items to distribute and the recycling quantity a trend of increasing item distribution with decreasing recycling quantity for an increasing allowed number of states can be detected.

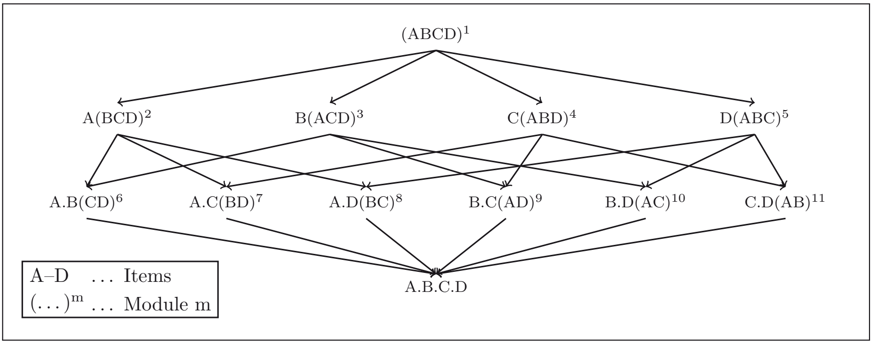

Fig. 4.13 Disassembly state graph of example one

4.4 Determining concrete disassembly path

4.4.1 Determining disassembly state quantities

4.4.1.1 Mathematical programming approach

With the solution of the flexible disassembly planning, i.e., knowing the optimal quantities of cores, items, modules, material, and waste, the planning task is not completed yet. What is missing is the specific information for the workers how to disassemble each incoming unit of a core in order to realise the optimal planning result. To get this information two approaches are presented in the sequel. Firstly, a mathematical model is developed, which can be solved with standard optimisation software for MILP. Secondly, based on the mathematical model, an algorithm is presented, which creates a smaller set of equations and avoids using solver software. But, before we start with the two approaches, two small examples shall be introduced to discuss the considerations.

The number of disassembly states of both exemplary cores is about equal. The first one represents a core where the items can only be taken off successively. This means that no two modules are created when separating a connection. It results in a disassembly state graph where in each state a unique module exists. (Again, the last state is an exception.) The graph is depicted in Fig. 4.13 and it consists of 12 states and 11 modules. The second core is highly modular. This means by separating a connection many modules are created. This can be seen in Fig. 4.14. This core can be disas-

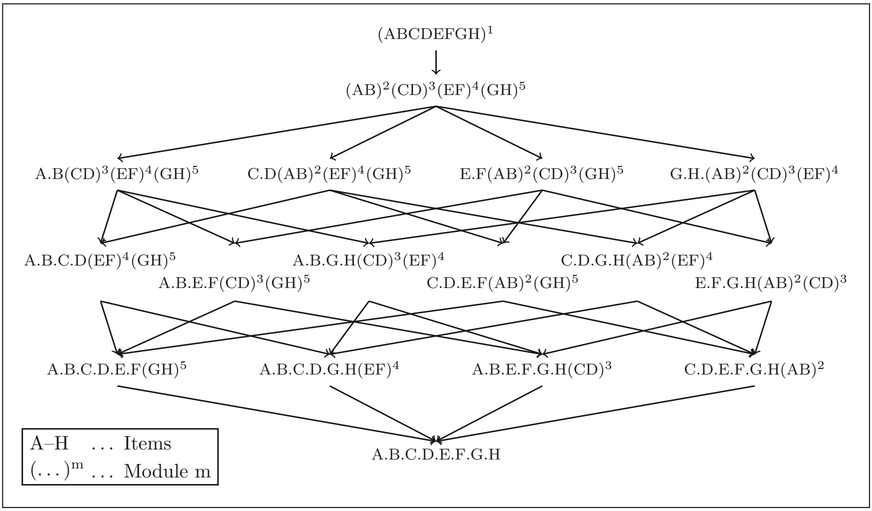

Fig. 4.14 Disassembly state graph of example two

sembled with 17 states and only five modules exist.[44] The explicit listing of the module definition matrix δ_{cmi}, module state combination matrix $\gamma^{\mathrm{M}}_{cms}$, and item state combination matrix $\gamma^{\mathrm{I}}_{cis}$ is displayed in appendix C.6. All the information can be directly extracted from the two graphs.

To determine the quantities for each state a variable Q^{S}_{cs} is introduced, which represents how often a core has to be disassembled to the specific state s. Thereby, the sum over all states that contain a particular module m or item i must equal the quantity that is planned for this module or item. The information whether an item or module results of a particular state can be seen in the graphs and is stored in $\gamma^{\mathrm{I}}_{cis}$ and $\gamma^{\mathrm{M}}_{cms}$. The set of equations for all items and modules is

$$\sum_{s\in\{s|\gamma^{\mathrm{I}}_{cis}=1\}} Q^{\mathrm{S}}_{cs} = X^{\mathrm{I}}_{ci} + \sum_{r} X^{\mathrm{R}}_{cir} + \sum_{d} X^{\mathrm{D}}_{cid} \quad \forall\ c, i \tag{4.140}$$

$$\sum_{s\in\{s|\gamma^{\mathrm{M}}_{cms}=1\}} Q^{\mathrm{S}}_{cs} = Y^{\mathrm{M}}_{cm} + \sum_{r} Y^{\mathrm{R}}_{cmr} + \sum_{d} Y^{\mathrm{D}}_{cmd} \quad \forall\ c, m\,. \tag{4.141}$$

Thereby, the quantities of the states have to be non-negative integer values.

[44] In comparison to the forklift truck example a lot less states and modules exist for the same number of items the core consists of.

$$Q^{\mathrm{S}}_{cs} \in \mathbb{Z}^{*} \quad \forall\ c, s \tag{4.142}$$

The right hand side of the equations (i.e., the quantities X^{I}_{ci}, X^{R}_{cir}, X^{D}_{cid}, Y^{M}_{cm}, Y^{R}_{cmr}, Y^{D}_{cmd}, and Q^{C}_{c}) are all given data, because they are the result of the flexible disassembly planning. The task is now to find a feasible solution. In some cases there might exist several solutions and any of them is as good as another one, in general. Because all relevant quantities are core specific, there exist no dependencies among the cores for determining the state quantities. Thus, the set of equations can be solved for each core individually, which reduces the complexity.

If the decision maker is indifferent between any of the feasible solutions, any can be chosen. If on the other hand the decision maker prefers a solution with fewer sequences, a little extension is necessary. This extension includes an objective function (to be minimised) and binary variables U_{cs}. Each time, the quantity of a state Q^{S}_{cs} is greater than zero the binary variable has to take a value of one. Otherwise, it can be zero. The extended mathematical model is as follows.

$$\text{Minimise} \qquad \sum_{s} U_{cs} \qquad \forall\ c \tag{4.143}$$

$$\text{s.t.} \qquad \sum_{s\in\{s|\gamma^{\mathrm{I}}_{cis}=1\}} Q^{\mathrm{S}}_{cs} = X^{\mathrm{I}}_{ci} + \sum_{r} X^{\mathrm{R}}_{cir} + \sum_{d} X^{\mathrm{D}}_{cid} \qquad \forall\ c, i \tag{4.144}$$

$$\sum_{s\in\{s|\gamma^{\mathrm{M}}_{cms}=1\}} Q^{\mathrm{S}}_{cs} = Y^{\mathrm{M}}_{cm} + \sum_{r} Y^{\mathrm{R}}_{cmr} + \sum_{d} Y^{\mathrm{D}}_{cmd} \qquad \forall\ c, m \tag{4.145}$$

$$Q^{\mathrm{S}}_{cs} \le Q^{\mathrm{C}}_{c}\, U_{cs} \qquad \forall\ c, s \tag{4.146}$$

$$U_{cs} \in \{0, 1\}, \quad Q^{\mathrm{S}}_{cs} \in \mathbb{Z}^{*} \qquad \forall\ c, s \tag{4.147}$$

Again, the solving is done for each core separately, because no dependencies of the state quantities between the cores exist. This fact is expressed by the term $\forall\ c$ in the objective function.

4.4.1.2 Successive approach

The resulting set of equations (according to Eqs. (4.140) and (4.141)) for the two examples is listed in the sequel. Because only one core is considered, we neglect the index c. For the first example the mentioned set of equations is

$$
\begin{pmatrix}
0&1&0&0&0&1&1&1&0&0&0&1\\
0&0&1&0&0&1&0&0&1&1&0&1\\
0&0&0&1&0&0&1&0&1&0&1&1\\
0&0&0&0&1&0&0&1&0&1&1&1\\
1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0
\end{pmatrix}
\begin{pmatrix}
Q_1^{\mathrm{S}}\\ Q_2^{\mathrm{S}}\\ Q_3^{\mathrm{S}}\\ Q_4^{\mathrm{S}}\\ Q_5^{\mathrm{S}}\\ Q_6^{\mathrm{S}}\\ Q_7^{\mathrm{S}}\\ Q_8^{\mathrm{S}}\\ Q_9^{\mathrm{S}}\\ Q_{10}^{\mathrm{S}}\\ Q_{11}^{\mathrm{S}}\\ Q_{12}^{\mathrm{S}}
\end{pmatrix}
=
\begin{pmatrix}
X_{\mathrm{A}}\\ X_{\mathrm{B}}\\ X_{\mathrm{C}}\\ X_{\mathrm{D}}\\ Y_1\\ Y_2\\ Y_3\\ Y_4\\ Y_5\\ Y_6\\ Y_7\\ Y_8\\ Y_9\\ Y_{10}\\ Y_{11}
\end{pmatrix}. \tag{4.148}
$$

Thereby, X_i and Y_m denote the sums $X_{ci}^{\mathrm{I}} + \sum_r X_{cir}^{\mathrm{R}} + \sum_d X_{cid}^{\mathrm{D}}$ and $Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}}$, respectively.

This system of equations has more equations than variables and is therefore over-determined. Checking the rank of the coefficients matrix[45] of the dimension $(m+i) \times s$ results in 12 for the first example.[46] Given the full rank of the coefficients matrix $\mathbf{A}$ equalling the number of states, we can calculate the Q_s^{S} right away. Based on the system of equations in Eq. (4.148) with coefficient matrix $\mathbf{A}$, quantity state vector $\mathbf{q}^{\mathrm{S}}$, and the right hand side, where the right hand side is a combination of $\mathbf{x}$ and $\mathbf{y}$, we can calculate

$$\mathbf{A}\,\mathbf{q}^{\mathrm{S}} = \begin{pmatrix}\mathbf{x}\\ \mathbf{y}\end{pmatrix} \tag{4.149}$$

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\,\mathbf{q}^{\mathrm{S}} = \mathbf{A}^{\mathrm{T}}\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\end{pmatrix} \tag{4.150}$$

$$\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{A}\,\mathbf{q}^{\mathrm{S}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\end{pmatrix} \tag{4.151}$$

$$\mathbf{q}^{\mathrm{S}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\end{pmatrix}, \tag{4.152}$$

where $\mathbf{A}^{\mathrm{T}}$ is the transposed of $\mathbf{A}$, $\left(\mathbf{x}\;\mathbf{y}\right)^{\mathrm{T}}$ is the vector that holds all X_i and Y_m, and $\mathbf{q}^{\mathrm{S}}$ is the vector of all Q_s^{S}.

[45] The coefficient matrix $\mathbf{A}$ is generated by putting the state definition for items $\gamma_{cis}^{\mathrm{I}}$ on top of the ones for modules $\gamma_{cms}^{\mathrm{M}}$. The result is $\mathbf{A} = \left(\gamma_{cis}^{\mathrm{I}}\;\gamma_{cms}^{\mathrm{M}}\right)^{\mathrm{T}}$.

[46] Similar to the model size calculation the term $(m+i) \times s$ denotes $(\sum_m 1 + \sum_i 1) \times \sum_s 1$.

Let us assume the solution gained by the flexible planning is $\mathbf{x} = (2\ 0\ 3\ 6)^{\mathrm{T}}$ and $\mathbf{y} = (1\ 2\ 0\ 3\ 6\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^{\mathrm{T}}$. For every zero in the vector $\mathbf{x}$ and $\mathbf{y}$, rows and columns of the system of equations can be eliminated. The reason is that the state quantities must be non-negative values. With this in mind the state quantities, having a corresponding coefficient equalling one in matrix $\mathbf{A}$, must be zero. Otherwise, the sum over the state quantities multiplied with the corresponding row does not equal zero. This means, that row two, seven, and ten through 15 can be eliminated, because the corresponding state quantities Q_s^{S} have to be zero. The corresponding states are three and six through 12. In addition, the columns with a state quantity of zero ($s \in \{3, 6, \ldots, 12\}$) are eliminated, too. Given the exemplary solution of the flexible planning, the resulting system of equations to determine the state quantities is reduced to

$$\begin{pmatrix} 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \\ 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \end{pmatrix} \begin{pmatrix} Q_1^{\mathrm{S}} \\ Q_2^{\mathrm{S}} \\ Q_4^{\mathrm{S}} \\ Q_5^{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} X_{\mathrm{A}} \\ X_{\mathrm{C}} \\ X_{\mathrm{D}} \\ Y_1 \\ Y_2 \\ Y_4 \\ Y_5 \end{pmatrix}. \tag{4.153}$$

The rank of the reduced coefficients matrix is four, which equals the number of considered states. Thus, the quantities can be determined directly with

$$\begin{pmatrix} Q_1^{\mathrm{S}} \\ Q_2^{\mathrm{S}} \\ Q_4^{\mathrm{S}} \\ Q_5^{\mathrm{S}} \end{pmatrix} = \left(\begin{pmatrix} 0&0&0&1&0&0&0 \\ 1&0&0&0&1&0&0 \\ 0&1&0&0&0&1&0 \\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} \right)^{-1} \begin{pmatrix} 0&0&0&1&0&0&0 \\ 1&0&0&0&1&0&0 \\ 0&1&0&0&0&1&0 \\ 0&0&1&0&0&0&1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 6 \\ 1 \\ 2 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 6 \end{pmatrix} \tag{4.154}$$

and we get the quantities $Q_1^{\mathrm{S}} = 1$, $Q_2^{\mathrm{S}} = 2$, $Q_4^{\mathrm{S}} = 3$, $Q_5^{\mathrm{S}} = 6$. All other equal zero. With systems of equations like the above only one solution exists that is feasible according to Eqs. (4.140) and (4.141), which makes any considerations with the number of states and the introduced variable U_{cs} obsolete.

The second example is different to the first one. The number of variables is higher than the equations as can be seen in the following system of equations.

$$\begin{pmatrix} 0&0&1&0&0&0&1&1&1&0&0&0&1&1&1&0&1\\ 0&0&1&0&0&0&1&1&1&0&0&0&1&1&1&0&1\\ 0&0&0&1&0&0&1&0&0&1&1&0&1&1&0&1&1\\ 0&0&0&1&0&0&1&0&0&1&1&0&1&1&0&1&1\\ 0&0&0&0&1&0&0&1&0&1&0&1&1&0&1&1&1\\ 0&0&0&0&1&0&0&1&0&1&0&1&1&0&1&1&1\\ 0&0&0&0&0&1&0&0&1&0&1&1&0&1&1&1&1\\ 0&0&0&0&0&1&0&0&1&0&1&1&0&1&1&1&1\\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&1&1&1&0&0&0&1&1&1&0&0&0&1&0\\ 0&1&1&0&1&1&0&1&1&0&0&1&0&0&1&0&0\\ 0&1&1&1&0&1&1&0&1&0&1&0&0&1&0&0&0\\ 0&1&1&1&1&0&1&1&0&1&0&0&1&0&0&0&0 \end{pmatrix} \begin{pmatrix} Q_1^{\mathrm{S}}\\ Q_2^{\mathrm{S}}\\ Q_3^{\mathrm{S}}\\ Q_4^{\mathrm{S}}\\ Q_5^{\mathrm{S}}\\ Q_6^{\mathrm{S}}\\ Q_7^{\mathrm{S}}\\ Q_8^{\mathrm{S}}\\ Q_9^{\mathrm{S}}\\ Q_{10}^{\mathrm{S}}\\ Q_{11}^{\mathrm{S}}\\ Q_{12}^{\mathrm{S}}\\ Q_{13}^{\mathrm{S}}\\ Q_{14}^{\mathrm{S}}\\ Q_{15}^{\mathrm{S}}\\ Q_{16}^{\mathrm{S}}\\ Q_{17}^{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} X_{\mathrm{A}}\\ X_{\mathrm{B}}\\ X_{\mathrm{C}}\\ X_{\mathrm{D}}\\ X_{\mathrm{E}}\\ X_{\mathrm{F}}\\ X_{\mathrm{G}}\\ X_{\mathrm{H}}\\ Y_1\\ Y_2\\ Y_3\\ Y_4\\ Y_5 \end{pmatrix} \tag{4.155}$$

Hence, the coefficient matrix does not have the rank equalling the number of states. Thus, a direct determination as described above is not possible and one of the presented MILP should be used to either find just a feasible solution or one with the minimal number of states necessary. Let us assume the solution of the flexible planning is $\mathbf{x} = (3\ 3\ 9\ 9\ 3\ 3\ 4\ 4)^{\mathrm{T}}$ and $\mathbf{y} = (2\ 6\ 0\ 6\ 5)^{\mathrm{T}}$. Thus, row 11 can be eliminated, when setting the corresponding quantity state variables for the states $s \in \{2, 3, 5, 6, 8, 9, 12, 15\}$ to zero. The columns with these indices s can be eliminated, too. The reduced system of equations is

$$\begin{pmatrix} 0&0&1&0&0&1&1&0&1\\ 0&0&1&0&0&1&1&0&1\\ 0&1&1&1&1&1&1&1&1\\ 0&1&1&1&1&1&1&1&1\\ 0&0&0&1&0&1&0&1&1\\ 0&0&0&1&0&1&0&1&1\\ 0&0&0&0&1&0&1&1&1\\ 0&0&0&0&1&0&1&1&1\\ 1&0&0&0&0&0&0&0&0\\ 0&1&0&1&1&0&0&1&0\\ 0&1&1&0&1&0&1&0&0\\ 0&1&1&1&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} Q_1^{\mathrm{S}}\\ Q_4^{\mathrm{S}}\\ Q_7^{\mathrm{S}}\\ Q_{10}^{\mathrm{S}}\\ Q_{11}^{\mathrm{S}}\\ Q_{13}^{\mathrm{S}}\\ Q_{14}^{\mathrm{S}}\\ Q_{16}^{\mathrm{S}}\\ Q_{17}^{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} X_{\mathrm{A}}\\ X_{\mathrm{B}}\\ X_{\mathrm{C}}\\ X_{\mathrm{D}}\\ X_{\mathrm{E}}\\ X_{\mathrm{F}}\\ X_{\mathrm{G}}\\ X_{\mathrm{H}}\\ Y_1\\ Y_2\\ Y_4\\ Y_5 \end{pmatrix} = \begin{pmatrix} 3\\ 3\\ 9\\ 9\\ 3\\ 3\\ 4\\ 4\\ 2\\ 6\\ 6\\ 5 \end{pmatrix}. \tag{4.156}$$

Table 4.23 State priorities

State s:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Priority:	1	2	3	3	3	3	4	4	4	4	4	4	5	5	5	5	6

The rank of the coefficients matrix is five and the number of remaining states is nine. They are not equal and therefore a feasible solution is determined with an MILP and not directly in closed form. A feasible solution to Eq. (4.156) with non-negative and integral Q_s^{S} is $Q_1^{\mathrm{S}} = 2$, $Q_4^{\mathrm{S}} = 4$, $Q_{13}^{\mathrm{S}} = 1$, $Q_{14}^{\mathrm{S}} = 2$, and $Q_{16}^{\mathrm{S}} = 2$. All other state quantities are zero. With this solution five sequences are necessary. Introducing the binary variable U_{cs} and solving the MILP (i.e., Eqs. (4.143)–(4.147)) an optimal solution with only four sequences is the result. The quantities are $Q_1^{\mathrm{S}} = 2$, $Q_4^{\mathrm{S}} = 2$, $Q_{11}^{\mathrm{S}} = 4$, and $Q_{13}^{\mathrm{S}} = 3$. (All other are zero.)

As mentioned above, a method shall be developed, that finds a solution for the state quantities without the use of a solver. This is illustrated with the second example, because for cases like the first example a closed form expression to find the solution exists. The flowchart of the method is depicted in Fig. 4.15. The starting point of the algorithm is Eq. (4.155), because the first step is the reduction to Eq. (4.156).[47] The necessary components are the coefficient matrix $\mathbf{A}$ of the dimension $j \times s$ and the right hand side vector $\mathbf{r}$ of length $j = i + m$, because it contains the solution X_i and Y_m of the flexible planning, i.e., $\mathbf{r} = (\mathbf{x}\ \mathbf{y})^{\mathrm{T}}$. Hence, this vector is given with $\mathbf{r} = (3\ 3\ 9\ 9\ 3\ 3\ 4\ 4\ 2\ 6\ 0\ 6\ 5)^{\mathrm{T}}$. The set of the states is $S = \{1, \ldots, 17\}$. In the initialisation the states need to be prioritised. A state with a bigger module has a higher priority than a state with a smaller module. (The size of a module is the number of containing items.) Exist more than one state with the same module size, then a state with more modules has a higher priority than a state with less modules. Thus, the highest priority state is always the one with module $m = 1$ (the complete core) and the lowest the state with no modules, i.e., representing the complete disassembly. The priority list for the 17 states of the example is listed in Table 4.23. We notice that quite a few states with the same priority exist.

After the initialisation the first check is whether the right hand side contains any zero values. The answer is yes, because in row $j = 11$ the vector $\mathbf{r}$ has a zero element. Accordingly, all these rows are selected in $\tilde{J} = \{11\}$. Furthermore, all states s where the coefficient matrix A_{js} has a value equalling one of these selected rows $\tilde{J}$ are selected, i.e., $\tilde{S} = \{2, 3, 5, 6, 8, 9, 12, 15\}$.

[47] Of course, one can also start with Eq. (4.156).

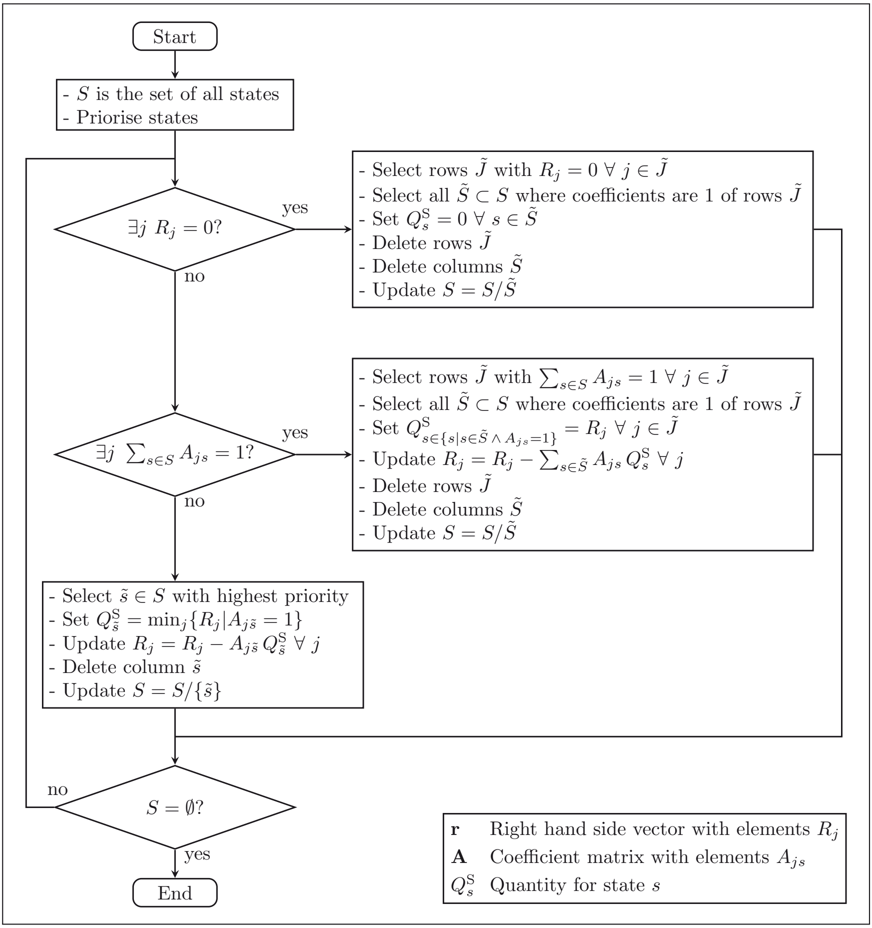

Fig. 4.15 Flowchart of state quantity algorithm

All the state quantity variables Q^{S}_s with $s \in \tilde{S}$ are set to zero, i.e., $Q^{\mathrm{S}}_2 = Q^{\mathrm{S}}_3 = Q^{\mathrm{S}}_5 = Q^{\mathrm{S}}_6 = Q^{\mathrm{S}}_8 = Q^{\mathrm{S}}_9 = Q^{\mathrm{S}}_{12} = Q^{\mathrm{S}}_{15} = 0$. The selected rows and columns are deleted from the system and the index set S is reduced by the selected states $\tilde{S}$. The resulting system is that of Eq. (4.156).

The set $S = \{1, 4, 7, 10, 13, 14, 16, 17\}$ is not empty, why we start in the beginning. The new vector $\mathbf{r}$ has no zero element. The next decision to be made is whether a row in the coefficient matrix exists, that has a row sum equalling one. This is the case, because row $j = 9$ is such a row.

Again, the selected rows are all those which meet this criterion. Hence, the set $\tilde{J} = \{9\}$ contains only one element. All states where this occurs are also selected, i.e., $\tilde{S} = \{1\}$. In a next step, for all selected quantity state variables the corresponding value of the vector $\mathbf{r}$ is assigned. In our example the value $R_9 = 2$ is assigned to Q_1^{S}, because the coefficient $A_{9,1}$ equals one. Afterwards, the right hand side is updated. This means that from all elements the amount gained by the selection of the selected states is subtracted. For now, only row nine is updated when $Q_1^{\mathrm{S}} = 2$. The resulting element is $R_9 = 0$. Lastly, the selected rows and columns are deleted from the system and the state set S is reduced by the selected states $\tilde{S}$.

$$\begin{pmatrix} 0&1&0&0&1&1&0&1 \\ 0&1&0&0&1&1&0&1 \\ 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \\ 0&0&1&0&1&0&1&1 \\ 0&0&1&0&1&0&1&1 \\ 0&0&0&1&0&1&1&1 \\ 0&0&0&1&0&1&1&1 \\ 1&0&1&1&0&0&1&0 \\ 1&1&0&1&0&1&0&0 \\ 1&1&1&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} Q_4^{\mathrm{S}} \\ Q_7^{\mathrm{S}} \\ Q_{10}^{\mathrm{S}} \\ Q_{11}^{\mathrm{S}} \\ Q_{13}^{\mathrm{S}} \\ Q_{14}^{\mathrm{S}} \\ Q_{16}^{\mathrm{S}} \\ Q_{17}^{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 9 \\ 9 \\ 3 \\ 3 \\ 4 \\ 4 \\ 6 \\ 6 \\ 5 \end{pmatrix} \tag{4.157}$$

The set S is still not empty. We start from the beginning and find no element $R_j = 0$ or a row with the row sum of one. Therefore, the state $\tilde{s}$ with the highest priority of all states in S is selected. We find that state $\tilde{s} = 4$ has the highest priority (see Table 4.23) in comparison to the states 7, 10, 11, 13, 14, 16, and 17. We set the quantity variable to the minimum of the right hand side values, where a coefficient of one exists in the selected column, i.e., $Q_4^{\mathrm{S}} = \min\{9, 9, 6, 6, 5\} = 5$. This way at least one row with the value zero is created and all elements of the right hand side will stay non-negative. The vector $\mathbf{r}$ is updated in the following way:

$$\begin{pmatrix} 3 \\ 3 \\ 9 \\ 9 \\ 3 \\ 3 \\ 4 \\ 4 \\ 6 \\ 6 \\ 5 \end{pmatrix} - 5 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 4 \\ 4 \\ 3 \\ 3 \\ 4 \\ 4 \\ 1 \\ 1 \\ 0 \end{pmatrix}. \tag{4.158}$$

Afterwards, the column $\tilde{s}$ is deleted and the set S is reduced by the selected state $\tilde{s}$.

$$\begin{pmatrix} 1&0&0&1&1&0&1 \\ 1&0&0&1&1&0&1 \\ 1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1 \\ 0&1&0&1&0&1&1 \\ 0&1&0&1&0&1&1 \\ 0&0&1&0&1&1&1 \\ 0&0&1&0&1&1&1 \\ 0&1&1&0&0&1&0 \\ 1&0&1&0&1&0&0 \\ 1&1&0&1&0&0&0 \end{pmatrix} \begin{pmatrix} Q_7^{\mathrm{S}} \\ Q_{10}^{\mathrm{S}} \\ Q_{11}^{\mathrm{S}} \\ Q_{13}^{\mathrm{S}} \\ Q_{14}^{\mathrm{S}} \\ Q_{16}^{\mathrm{S}} \\ Q_{17}^{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 4 \\ 4 \\ 3 \\ 3 \\ 4 \\ 4 \\ 1 \\ 1 \\ 0 \end{pmatrix} \tag{4.159}$$

The set S is not empty and we find a value of $R_{11} = 0$. Thus, row 11 is eliminated and the states 7, 10 and 13 get a value of $Q_7^{\mathrm{S}} = Q_{10}^{\mathrm{S}} = Q_{13}^{\mathrm{S}} = 0$. The corresponding row and columns are deleted and the set of states is reduced to $S = \{11, 14, 16, 17\}$.

$$\begin{pmatrix} 0&1&0&1 \\ 0&1&0&1 \\ 1&1&1&1 \\ 1&1&1&1 \\ 0&0&1&1 \\ 0&0&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \\ 1&0&1&0 \\ 1&1&0&0 \end{pmatrix} \begin{pmatrix} Q_{11}^{\mathrm{S}} \\ Q_{14}^{\mathrm{S}} \\ Q_{16}^{\mathrm{S}} \\ Q_{17}^{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 4 \\ 4 \\ 3 \\ 3 \\ 4 \\ 4 \\ 1 \\ 1 \end{pmatrix} \tag{4.160}$$

The next run starts and no zero value or row sum of one exists. Therefore, we select the state with the highest priority. This state is $\tilde{s} = 11$. The value is set to $Q^{\mathrm{S}}_{11} = \min\{4, 4, 4, 4, 1, 1\} = 1$. The right hand side is updated, the selected column deleted, and the set S reduced by the selected state.

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} Q^{\mathrm{S}}_{14} \\ Q^{\mathrm{S}}_{16} \\ Q^{\mathrm{S}}_{17} \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 0 \\ 0 \end{pmatrix} \tag{4.161}$$

The existing zero values in $\mathbf{r}$ result in setting $Q^{\mathrm{S}}_{14} = Q^{\mathrm{S}}_{16} = 0$. The two rows and two columns of the selected states are eliminated.

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} Q^{\mathrm{S}}_{17} \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \end{pmatrix} \tag{4.162}$$

The final run sets variable $Q^{\mathrm{S}}_{17} = 3$. The set S is now empty and the algorithm ends. The result is $Q^{\mathrm{S}}_{1} = 2$, $Q^{\mathrm{S}}_{4} = 5$, $Q^{\mathrm{S}}_{11} = 1$, and $Q^{\mathrm{S}}_{17} = 3$. All other state quantities are zero. In total four states are necessary. This solution is different from the one with the MILP as given above, but nonetheless with the same minimum number of states. Of course, the algorithm also works with the other example as can be seen in appendix C.7.

4.4.1.3 Determining state quantities for the numerical example

Now that we developed a procedure to find the corresponding quantities, we apply this to the forklift truck example. For each core we find 60 states, eight items, and 50 modules. This system of equations is too big to show it here. But, after the first step of reducing the number of equations by rows that have a zero value on the right hand side the size is drastically reduced. Thus, for core 1 the system of equations is

$$\begin{pmatrix} 0&0&1&0&1 \\ 0&0&0&1&1 \\ 1&1&1&1&1 \\ 1&1&1&1&1 \\ 0&1&1&1&1 \\ 1&1&1&1&1 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{pmatrix} \begin{pmatrix} Q^{\mathrm{S}}_{1,32} \\ Q^{\mathrm{S}}_{1,46} \\ Q^{\mathrm{S}}_{1,52} \\ Q^{\mathrm{S}}_{1,54} \\ Q^{\mathrm{S}}_{1,57} \end{pmatrix} = \begin{pmatrix} X_{1,\mathrm{A}} \\ X_{1,\mathrm{B}} \\ X_{1,\mathrm{C}} \\ X_{1,\mathrm{D}} \\ X_{1,\mathrm{E}} \\ X_{1,\mathrm{H}} \\ Y_{1,28} \\ Y_{1,39} \\ Y_{1,44} \\ Y_{1,46} \\ Y_{1,48} \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \\ 30 \\ 30 \\ 29 \\ 30 \\ 1 \\ 11 \\ 6 \\ 9 \\ 3 \end{pmatrix}, \tag{4.163}$$

when we take the optimal solution of the flexible planning with 12 states. The coefficient matrix has rank five, which implies that the solution can be gained by the closed form expression or in one step with the algorithm, because the last five rows determine the values for the state quantities. The values for the five quantities unequal zero are $Q^{\mathrm{S}}_{1,32} = 1$, $Q^{\mathrm{S}}_{1,46} = 11$, $Q^{\mathrm{S}}_{1,52} = 6$, $Q^{\mathrm{S}}_{1,54} = 9$, and $Q^{\mathrm{S}}_{1,57} = 3$ for core 1. As we can see here, a prioritising of the states is not necessary when the closed form expression can be used.

The same applies to core 2 and 3. After reducing the system of equations by rows with a zero value on the right hand side the resulting system of equations for core 2 is

$$\begin{pmatrix} 0&0&1&1&1 \\ 0&0&0&1&1 \\ 0&0&0&1&1 \\ 0&0&0&1&1 \\ 0&1&1&0&1 \\ 0&0&0&0&1 \\ 0&0&0&1&1 \\ 0&0&0&1&1 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \end{pmatrix} \begin{pmatrix} Q^{\mathrm{S}}_{2,1} \\ Q^{\mathrm{S}}_{2,6} \\ Q^{\mathrm{S}}_{2,10} \\ Q^{\mathrm{S}}_{2,58} \\ Q^{\mathrm{S}}_{2,60} \end{pmatrix} = \begin{pmatrix} X_{2,\mathrm{A}} \\ X_{2,\mathrm{B}} \\ X_{2,\mathrm{C}} \\ X_{2,\mathrm{D}} \\ X_{2,\mathrm{E}} \\ X_{2,\mathrm{F}} \\ X_{2,\mathrm{G}} \\ X_{2,\mathrm{H}} \\ Y_{2,1} \\ Y_{2,6} \\ Y_{2,10} \\ Y_{2,49} \end{pmatrix} = \begin{pmatrix} 216 \\ 215 \\ 215 \\ 215 \\ 125 \\ 123 \\ 215 \\ 215 \\ 1 \\ 1 \\ 1 \\ 92 \end{pmatrix}. \tag{4.164}$$

Table 4.24 State priorities for the three cores

state s:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
priority:	1	2	2	2	2	2	3	3	3	3	4	3	3	3	3	3	3	6	5	5
state s:	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
priority:	5	5	5	5	6	5	5	6	5	5	5	5	8	7	7	8	7	7	7	7
state s:	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
priority:	8	7	7	7	7	7	10	9	9	9	9	9	9	9	10	11	11	11	11	12

The rank of the matrix is five and the closed form expression or the algorithm delivers the solution in one step with $Q^{\mathrm{S}}_{2,1} = 1$, $Q^{\mathrm{S}}_{2,6} = 1$, $Q^{\mathrm{S}}_{2,10} = 1$, $Q^{\mathrm{S}}_{2,58} = 92$, and $Q^{\mathrm{S}}_{2,60} = 123$. The last is core 3. The reduced system of equations is

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} Q^{\mathrm{S}}_{3,15} \\ Q^{\mathrm{S}}_{3,56} \end{pmatrix} = \begin{pmatrix} X_{3,\mathrm{A}} \\ X_{3,\mathrm{B}} \\ X_{3,\mathrm{C}} \\ X_{3,\mathrm{D}} \\ X_{3,\mathrm{E}} \\ X_{3,\mathrm{F}} \\ Y_{3,14} \\ Y_{3,47} \end{pmatrix} = \begin{pmatrix} 30 \\ 30 \\ 31 \\ 31 \\ 30 \\ 30 \\ 1 \\ 30 \end{pmatrix} . \tag{4.165}$$

The rank of the matrix is two and the closed form expression or the algorithm delivers the solution in one step with $Q^{\mathrm{S}}_{3,15} = 1$ and $Q^{\mathrm{S}}_{3,56} = 30$.

Even though the solution can be gained using the closed form expression, the algorithm is once applied to illustrate the solution finding for core 2. The determination of priorities is not necessary here, but for completeness they are determined for the states and listed in Table 4.24. The priorities for the states 1, 6, 10, 58, and 60 are one, two, three, 11, and 12, respectively. After the reduction of the rows with $R_j = 0$ and the corresponding state columns the next loop of the algorithm according to Fig. 4.15 follows. We find five rows with a row sum equalling one. These are row six, nine, ten, 11, and 12, i.e., $\tilde{J} = \{6, 9, 10, 11, 12\}$, (belonging to $X_{2,\mathrm{F}}$, $Y_{2,1}$, $Y_{2,6}$, $Y_{2,10}$, $Y_{2,58}$, and $Y_{2,60}$). The corresponding state quantities ($\tilde{S} = \{1, 6, 10, 58, 60\}$) are set to $Q^{\mathrm{S}}_{2,1} = 1$, $Q^{\mathrm{S}}_{2,6} = 1$, $Q^{\mathrm{S}}_{2,10} = 1$, $Q^{\mathrm{S}}_{2,58} = 92$, and $Q^{\mathrm{S}}_{2,60} = 123$. The right hand side is updated and the rows $\tilde{J}$ as well as the columns one through five are eliminated. Hence, a system with no columns remains, i.e., $\tilde{S} = \emptyset$ and the algorithm terminates. With this the information which states and how often these states are necessary for a particular core is gained. Now, in

Table 4.25 Listing of incoming units of core 1

	item								item								item							
unit	A	B	C	D	E	F	G	H	unit	A	B	C	D	E	F	G	H	unit	A	B	C	D	E	F	G	H
1	○	○	●	○	●	●	●	●	11	○	●	○	○	●	●	●	○	21	○	●	●	○	●	●	●	●
2	○	●	○	●	●	●	●	●	12	●	●	●	●	●	●	●	●	22	●	○	○	○	●	○	●	●
3	○	○	○	○	●	●	●	●	13	○	●	○	●	●	●	●	●	23	○	○	○	○	●	●	●	●
4	●	●	○	●	●	●	●	●	14	●	○	●	○	●	●	●	●	24	●	○	●	●	●	●	●	●
5	○	○	●	○	●	●	●	●	15	○	●	○	○	●	●	●	●	25	○	●	●	○	●	●	●	●
6	●	●	○	○	●	●	●	●	16	●	○	○	×	●	●	●	●	26	○	○	○	○	○	○	●	○
7	○	○	○	●	●	●	●	●	17	○	○	●	●	●	●	●	●	27	○	○	○	○	●	●	●	●
8	○	●	○	○	●	●	●	●	18	●	●	○	○	●	●	●	●	28	●	●	●	○	●	●	●	●
9	○	●	●	●	●	●	●	●	19	●	○	●	○	●	●	●	●	29	●	○	○	●	●	●	●	●
10	●	○	○	○	●	●	●	○	20	●	○	○	○	●	●	●	●	30	●	○	○	●	●	●	●	●

The symbols ●, ○, and × denote the condition of an item that allows distribution, recycling, and disposal, recycling and disposal, as well as disposal only, respectively.

a further step, we need to specify which state an incoming core is going to be assigned to.

4.4.2 Assigning cores to selected states and recommended usage

4.4.2.1 Successive assignment

In the sequel we assume that the cores are first tested to determine their condition. Based on the tested condition the core is assigned to a particular disassembly state and accordingly disassembled. Then the next core is tested and assigned, etc. Thus, not the complete planned quantity of cores (e.g., all 30 units of core 1) is first tested (in a batch) and then assigned to the relevant states. The assignment is carried out step by step, i.e., unit by unit, for each type of core.

To illustrate the considerations a possible listing of incoming cores is displayed in Table 4.25. The optimal quantity of core 1 is 30 units. Among these 30 units certain fractions of conditions are expected. The first randomly chosen unit of core 1 that is tested contains items C, E, F, G, and H in a genuine and functioning condition, such that they can be used for distribution, recycling, or disposal. The other three items, i.e., A, B, and D, are either genuine and defective or non-genuine with the right material.

Table 4.26 Expected item and module usage quantities

usage	A	B	C	D	E	F	G	H	28	39	44	46	48
separate quantities													
	item i								module m				
× disposal	0	0	0.4	0.4	0	0	0	0	0	0	0	0	0
∘ disposal & recycling	16.5	16.5	17.21	17.21	0.3	1.5	0.3	1.5	24.34	24.29	17.30	17.30	1.78
• disposal & recycling & distribution	13.5	13.5	12.75	12.75	29.7	28.5	29.7	28.5	5.66	5.71	12.70	12.70	28.22
cumulative quantities													
disposal	30	30	30	30	30	30	30	30	30	30	30	30	30
disposal & recycling	30	30	29.96	29.96	30	30	30	30	30	30	30	30	30
disposal & recycling & distribution	13.5	13.5	12.75	12.75	29.7	28.5	29.7	28.5	5.66	5.71	12.70	12.70	28.22

Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Therefore, they cannot be used for distribution but only for recycling or disposal. The third condition is an item being non-genuine and of the wrong material. Such items have to be disposed of, e.g., item D of the 16^{th} unit.

The optimal quantity of cores is 30 units (see Table 4.17). Given this quantity and the usage probabilities (see Table 4.16) for core 1 the quantities presented in Table 4.26 are expected. Note that only the relevant modules, which can be gained by the disassembly states, need to be considered. In the upper half of the table the quantities represent the actual usage that corresponds to the core condition listed in Table 4.25. The values are gained by multiplying the usage probability with the core quantity, e.g., $0.45 \cdot 30 = 13.5$ for item A usable for distribution. For modules all containing items have to be usable for distribution. Otherwise, the module cannot be used for distribution. Module 44 consists of the items B, F, and G. With a probability of $0.45 \cdot 0.95 \cdot 0.99 = 0.4232$ one unit of core 1 can be used for distribution (see Table 4.16). This makes $0.4232 \cdot 30 = 12.70$ units of the 30 units. As soon as an item that can only be used for disposal appears in a module, the module has to be disposed of. The item disposal probability for the three items in module 44 is zero. Hence the probability for the module is $1-(1-0)(1-0)(1-0) = 0$. The missing probability for recycling usage is the difference between the distribution and disposal usage and 100 %, i.e., $1 - 0.4232 - 0 = 0.5768$ and in absolute terms $30 - 12.70 - 0 = 17.30$ units.

The lower half contains the quantities cumulated from bottom to top, because in the end all items and modules regardless of their condition can be used for disposal. In addition, all items that can be used for recycling can also be used for disposal but not for distribution. Counting the realisation of Table 4.25, we find for example 14 and 16 times item A being of distribution

Table 4.27 Realised item and module usage quantities

		item i								module m				
usage		A	B	C	D	E	F	G	H	28	39	44	46	48
separate quantities														
×	disposal	0	0	0	1	0	0	0	0	0	0	0	0	0
○	disposal & recycling	16	17	19	19	1	2	0	3	25	25	17	17	2
•	disposal & recycling & distribution	14	13	11	10	29	28	30	27	5	5	13	13	28
cumulative quantities														
	disposal	30	30	30	30	30	30	30	30	30	30	30	30	30
	disposal & recycling	30	30	30	29	30	30	30	30	30	30	30	30	30
	disposal & recycling & distribution	14	13	11	10	29	28	30	27	5	5	13	13	28

and recycling condition, respectively. The remaining values can be seen in Table 4.27. For the modules we find, for example, five units of module 44 among the 30 units with all three items being distributable, i.e., marked with "•" in Table 4.25.

The necessary states of core 1 of the optimal solution are 32, 46, 52, 54, and 57 with the corresponding quantities one, 11, six, nine, and three, respectively (see Sect. 4.4.1). When a unit of core 1 is tested the decision maker has to assign the particular unit to a disassembly state and the resulting modules and items to the corresponding usage, i.e., distribution, recycling, and disposal. Obviously, the decision depends on the condition of the unit—but not alone. In addition, the planned quantity in proportion to the available, i.e., expected, is a further important aspect. For example, 29 units of item E are planned for distribution. This is almost 30 units. Of course, 29.7 distributable units are expected, but one never knows if the realisation is 30 or 29 units. Therefore, the decision maker tends to use each distributable unit of item E for distribution. This proportion of the planned quantity to the expected we call (relative) scarcity in the sequel. Furthermore, when module constituent items allow a distribution of the module the constituent items can be distributed, too. Assuming that both the distribution of the module and the consisting items is planned, the module distribution should be preferred, because it is more likely that the item distribution can be met with other units of cores where not all module consisting items have the same condition.

Let us illustrate this with five units of an arbitrary three-item core. The quantity of two modules ABC and two items each of A, B, and C are planned. The expected quantities of the module ABC and items A, B, and C shall be four, each. Eventually, the realisation of the five units is • • •,

○●●, ●●○, ●●●, and ●○●. When confronted with the first unit the decision maker should choose the module usage, because even though four modules are expected only one more is coming. And if the single items would have been chosen for the first unit, the planned quantity of two modules ABC cannot be met.

When making a decision which state to choose the three aspects of

- core condition and corresponding usage,
- scarcity, and
- module or item

influence the choice. For each state, that can be chosen from, a priority value is determined. These values are compared and the state with the highest value is selected. The here developed priority value calculation is as follows.

The value is the sum of the priority values for each module and item that can be gained when applying a particular state. Let us consider state 32. According to this state, module 28 and the single items C, D, and H are the result. The first unit is the one listed in Table 4.25 with three recycling and five distribution items. The item C could be used for distribution, but it is not planned for distribution. Hence, it can be used for recycling. 29 units are planned of the 30 available. Thus, the scarcity is relatively high and in terms of numbers it is $\sigma^{\mathrm{I}}(\mathrm{C}, \mathrm{r}) = \frac{29}{29.96}$, i.e., planned quantity in relation to the expected quantity (see Table 4.26 lower half). The usage is recycling. This usage is represented by a certain value (usage weight), e.g., one. If the usage would be distribution, the usage weight should be higher, e.g., five. On the other hand, if the item can only be used for disposal—either because of the condition or the planned quantities—the usage weight should be less than that for recycling. Since any item condition satisfies the disposal usage, the usage weight can be set to zero, because of the fact that an item is disposable is not relevant for the planning. The three usage weights are given with: $\omega^{\mathrm{U}}_{\mathrm{i}} = 5$, $\omega^{\mathrm{U}}_{\mathrm{r}} = 1$, and $\omega^{\mathrm{U}}_{\mathrm{d}} = 0$, where i, r, and d denote distribution, recycling, and disposal, respectively.

In the case that the condition of an item is too bad to be used as planned, the corresponding priority value $\pi^{\mathrm{I}}(i, u)$ is as low as possible, i.e., $-\infty$, so that this state is not chosen. The planned quantity q^{I}_{iu} is taken from the optimal solution (see Table 4.17: X^{I}, X^{R}, X^{D}). The values are summarised in Table 4.28. The scarcity value is calculated as explained above. Thereby, not only the planned quantities of the items are of interest. The planned quantities of all modules, that contain this item (i.e., superordinate elements), have to be considered, too. The reason is that when only a certain quantity of distributable items exists, they might have to be split into single items

Table 4.28 Planned quantities

usage u		q^{I}_{iu} item i A	B	C	D	E	F	G	H	q^{M}_{mu} module m 28	39	44	46	48
×	disposal	0	0	1	1	0	0	0	30	0	0	0	0	0
∘	disposal & recycling	0	0	29	29	0	0	0	0	1	11	6	9	3
•	disposal & recycling & distribution	9	12	0	0	29	0	0	0	0	0	0	0	0

and items in modules. The function for the scarcity value for an item i and a determined usage u is given by

$$\sigma^{\mathrm{I}}(i,u) = \frac{q^{\mathrm{I}}_{iu} + \sum_{m\in\{m|\delta_{cmi}=1\}} q^{\mathrm{M}}_{mu}}{\epsilon^{\mathrm{I}}_{iu}} = \frac{1}{\epsilon^{\mathrm{I}}_{iu}} \left(q^{\mathrm{I}}_{iu} + \sum_{m\in\{m|\delta_{cmi}=1\}} q^{\mathrm{M}}_{mu} \right). \tag{4.166}$$

Thereby, $\epsilon^{\mathrm{I}}_{iu}$ denotes the expected quantity (see Table 4.26 lower half). The priority value $\pi^{\mathrm{I}}(i,u)$ is then calculated with

$$\pi^{\mathrm{I}}(i,u) = \begin{cases} \sigma^{\mathrm{I}}(i,u)\,\omega^{\mathrm{U}}_u & \text{if } q^{\mathrm{I}}_{iu} > 0 \\ -\infty & \text{else} \end{cases} . \tag{4.167}$$

The corresponding priority values for the three items C, D, and H are

$$\begin{aligned} \pi^{\mathrm{I}}(\mathrm{C},\mathrm{r}) = \sigma^{\mathrm{I}}(\mathrm{C},\mathrm{r})\,\omega^{\mathrm{U}}_{\mathrm{r}} &= \frac{\omega^{\mathrm{U}}_{\mathrm{r}}}{\epsilon^{\mathrm{I}}_{\mathrm{C,r}}} \left(q^{\mathrm{I}}_{\mathrm{C,r}} + \sum_{m\in\{m|\delta_{cmi}=1\}} q^{\mathrm{M}}_{m,\mathrm{r}} \right) \\ &= \frac{1}{29.96}\left(29 + \sum_{m\in\emptyset} q^{\mathrm{M}}_{m,\mathrm{r}}\right) = \frac{29}{29.96}\,, \end{aligned} \tag{4.168}$$

$\pi^{\mathrm{I}}(\mathrm{D},\mathrm{r}) = \frac{29}{29.96}$, and $\pi^{\mathrm{I}}(\mathrm{H},\mathrm{d}) = 0$, respectively. As we see in Eq. (4.168) item C does not exist in the modules 28, 39, 44, 46, and 48 why the sum over the planned quantities of relevant modules q^{M}_{mu} is not applied (i.e., $\sum_{m\in\emptyset} q^{\mathrm{M}}_{m,\mathrm{r}}$).

Analogously, the priority value $\pi^{\mathrm{M}}(m,u)$ for the modules is calculated using the information of the consisting items and is extended by module specific information. Firstly, the scarcity information of the consisting items is the basis for the module priority. This means that the scarcity $\sigma^{\mathrm{I}}(i,u)$ for the usage u the module is going to be assigned to is applied to all consisting items i. The information which item is a consisting one is given by δ_{cmi}.

On top of this, the modules can be even scarcer than the consisting items. Therefore, the scarcity is increased by the module scarcity. The module scarcity is a value in the interval of 0 and 1. This value can be added or multiplied with one plus this value in order to achieve a preference for the scarce modules. The latter approach, i.e., the multiplication, is chosen in this approach. The usage weight is multiplied with the so far determined value.

Finally, a module preference weight ω^{M} is appended to be able to favour the choosing of modules compared to items even more. Thereby, the scarcity of a module $\sigma^{\mathrm{M}}(m,u)$ is the proportion of the planned quantity q^{M}_{mu} of this module and all superordinate modules to the expected quantity $\epsilon^{\mathrm{M}}_{mu}$. A module $\tilde{m}$ is superordinate to a module m if it contains at least all the items module m consists of, i.e., $\delta_{c\tilde{m}i} \geq \delta_{cmi}$ with $\tilde{m} \neq m$. The scarcity value calculation can then be formulated with

$$\begin{aligned}\sigma^{\mathrm{M}}(m,u) =& \frac{1}{\epsilon^{\mathrm{M}}_{mu}} \left(q^{\mathrm{M}}_{mu} + \sum_{\tilde{m} \in \{\tilde{m} | \delta_{c\tilde{m}i} \geq \delta_{cmi},\, \tilde{m} \neq m\}} q^{\mathrm{M}}_{\tilde{m}u} \right) \\ =& \frac{1}{\epsilon^{\mathrm{M}}_{mu}} \sum_{\tilde{m} \in \{\tilde{m} | \delta_{c\tilde{m}i} \geq \delta_{cmi}\}} q^{\mathrm{M}}_{\tilde{m}u} \,. \end{aligned} \tag{4.169}$$

Here, the planned quantity of module m as well as the one of the superordinate modules is included why the indices of the sum can be combined. Given all the necessary data and functions the priority value of a module m for a given usage u can be determined.

$$\pi^{\mathrm{M}}(m,u) = \begin{cases} \left(\sum_{i \in \{i | \delta_{cmi} = 1\}} \sigma^{\mathrm{I}}(i,u) \right) \left(1 + \sigma^{\mathrm{M}}(m,u)\right) \omega^{\mathrm{U}}_{u}\, \omega^{\mathrm{M}} & \text{if } q^{\mathrm{M}}_{mu} > 0 \\ -\infty & \text{else} \end{cases} \tag{4.170}$$

For module $m = 28$ (of core 1) there exists no superordinate module with a planned quantity greater than zero. Hence, the scarcity value for recycling is

$$\sigma^{\mathrm{M}}(28,\mathrm{r}) = \frac{1}{\epsilon^{\mathrm{M}}_{28,\mathrm{r}}} \sum_{\tilde{m} \in \{\tilde{m} | \delta_{1,\tilde{m},i} \geq \delta_{1,28,i}\}} q^{\mathrm{M}}_{\tilde{m},\mathrm{r}} = \frac{1}{30} \sum_{\tilde{m} \in \{28\}} q^{\mathrm{M}}_{\tilde{m},\mathrm{r}} = \frac{1}{30}\, q^{\mathrm{M}}_{28,\mathrm{r}} = \frac{1}{30} \,. \tag{4.171}$$

The scarcity values of the consisting items A, B, E, F, and G for recycling are $^{21}/_{30}$, $^{18}/_{30}$, $^{1}/_{30}$, $^{30}/_{30}$, and $^{30}/_{30}$, respectively. The resulting priority value with a module preference weight equalling one is

$$\begin{aligned}\pi^{\mathrm{M}}(28,\mathrm{r}) &= \left(\sum_{i\in\{i|\delta_{1,28,i}=1\}} \sigma^{\mathrm{I}}(i,\mathrm{r})\right)\left(1+\sigma^{\mathrm{M}}(28,\mathrm{r})\right)\omega_{\mathrm{r}}^{\mathrm{U}}\,\omega^{\mathrm{M}} \\ &= \left(\sigma^{\mathrm{I}}(\mathrm{A},\mathrm{r})+\sigma^{\mathrm{I}}(\mathrm{B},\mathrm{r})+\sigma^{\mathrm{I}}(\mathrm{E},\mathrm{r})+\sigma^{\mathrm{I}}(\mathrm{F},\mathrm{r})+\sigma^{\mathrm{I}}(\mathrm{G},\mathrm{r})\right) \\ &\quad \cdot \left(1+\sigma^{\mathrm{M}}(28,\mathrm{r})\right)\omega_{\mathrm{r}}^{\mathrm{U}}\,\omega^{\mathrm{M}} \\ &= \left(\frac{7}{10}+\frac{3}{5}+\frac{1}{30}+1+1\right)\left(1+\frac{1}{30}\right)1\cdot 1 = \frac{31}{9}\,. \end{aligned} \tag{4.172}$$

The overall priority value $\pi^{\mathrm{S}}(s)$ of the state $s = 32$ is the sum of all item and module priority values, i.e., $\pi^{\mathrm{S}}(32) = \pi^{\mathrm{I}}(\mathrm{C},\mathrm{r}) + \pi^{\mathrm{I}}(\mathrm{D},\mathrm{r}) + \pi^{\mathrm{I}}(\mathrm{H},\mathrm{d}) + \pi^{\mathrm{M}}(28,\mathrm{r}) = \frac{29}{29.96} + \frac{29}{29.96} + 0 + \frac{31}{9} \approx 5.38$. Accordingly, the priority values for the states 46, 52, 54, and 57 are determined. These result in 11.33, $-\infty$, $-\infty$, and $-\infty$, respectively. The values for the states 52, 54, and 57 are minus infinity, because according to the state definition the single items A and/or B are gained. These items are planned for distribution only and in the first unit of the incoming core their condition is only valid for recycling. Thus, when applying these states the items could not be used as planned, which delivers this particular priority value.

The state with the maximum priority value is chosen. This is state $s = 46$. Disassembling the unit accordingly, gives the module $m = 39$ in recycling quality, the items C, E, and H, in distribution quality, and item D in recycling quality. Nevertheless, only item E is planned for distribution so that only this item is distributed. Item H is disposed of and all other items are recycled. Before the next unit can be assigned to a disassembly state, the quantities used for assigning have to be updated. This means the state quantities (q_s^{S}), the expected quantities ($\epsilon_{mu}^{\mathrm{M}}$ and $\epsilon_{iu}^{\mathrm{I}}$) as well as the planned quantities (q_{mu}^{M} and q_{iu}^{I}) are affected. The state quantities were determined in the section above, e.g., $q_{46}^{\mathrm{S}} = Q_{1,46}^{\mathrm{S}}$ for core 1.

Before the assignment, the state quantity for state $s = 46$ is $q_{46}^{\mathrm{S}} = 11$. After the assignment, it is reduced by one, i.e., $q_{46}^{\mathrm{S}} = 10$ for the next assignment. The other state quantities stay unchanged. Next, the planned quantities q_{mu}^{M} and q_{iu}^{I} are discussed. With state 46 the module 39 and the items C, D, E, and H are gained. These are used for recycling with the exception of item E and H. These two are used for distribution and disposal, respectively. For the distributed items a further step has to be included: the damaging. If an item is damaged during the disassembly process, it cannot be used for distribution anymore. It is then used for recycling or disposal, whatever options exist for this particular item. This has also to be considered in the quantity updating. In the case that an item is used for recycling instead of the assigned distribution the quantity $q_{i,\mathrm{r}}^{\mathrm{I}}$ is reduced by one in-

Table 4.29 Updated quantities

	planned quantities												
	q^{I}_{iu}								q^{M}_{mu}				
	item i								module m				
usage u	A	B	C	D	E	F	G	H	28	39	44	46	48
× disposal	0	0	1	1	0	0	0	29	0	0	0	0	0
∘ disposal & recycling	0	0	28	28	0	0	0	0	1	10	6	9	3
• disposal & recycling & distribution	9	12	0	0	28	0	0	0	0	0	0	0	0
	expected (cumulated) quantities												
	$\epsilon^{\mathrm{I}}_{iu}$								$\epsilon^{\mathrm{M}}_{mu}$				
disposal	29	29	29	29	29	29	29	29	29	29	29	29	29
disposal & recycling	29	29	28.96	28.96	29	29	29	29	29	29	29	29	29
disposal & recycling & distribution	13.5	13.5	11.75	12.75	28.7	27.5	28.7	27.5	5.66	5.71	12.70	12.70	27.22

stead of $q^{\mathrm{I}}_{i,\mathrm{i}}$. Since item E does not get damaged during the disassembly process (i.e., $\theta_{1,\mathrm{E}} = 0$) it is used for distribution. Hence, the corresponding values of Table 4.28 are reduced by one. The resulting quantities are depicted in Table 4.29.

Lastly, the expected quantities need to be updated, too. Thereby, only the cumulative expected quantities are of interest. For this updating the usage of the modules and/or items does not matter. Only the condition of the unit causes the corresponding updating. If an item or module could be used for distribution, all three quantities, i.e., for distribution, recycling, and disposal, are reduced by one. This applies to the items C, E, F, G, and H as well as module 48. For items and modules which could not be used for distribution but for recycling and disposal, the two quantities for recycling and disposal are reduced by one. For the remaining items A, B, and D as well as the modules 28, 39, 44, and 46 this is also necessary. Lastly, for items and modules only usable for disposal only the latter quantity has to be reduced by one unit. This is not the case for the first unit, but for the 16$^{\mathrm{th}}$ unit it is. The resulting quantities are depicted in Table 4.29.

The information gained with the developed algorithm for the decision maker is summarised in Table 4.30. The state priorities are for illustration. The selected state as well as the usage information are relevant. The items F and G are not listed in the table, because they always appear in modules and never as a single item. For exemplary orders of incoming units of core 2 and 3 the assignment information is listed in appendix C.8.

The structure of the above developed algorithm is depicted in Fig. 4.16. It starts with initialising the required quantities and weights. Once this is

Table 4.30 State assignment for core 1

	state priority $\pi^{\mathrm{S}}(s)$					selected state	item usage u						module usage u				
	s						i						m				
unit	32	46	52	54	57	s	A	B	C	D	E	H	28	39	44	46	48
1	5.38	11.33	$-\infty$	$-\infty$	$-\infty$	46			r	r	i	d		r			
2	5.36	11.22	$-\infty$	14.78	$-\infty$	54		i	r	r	i	d				r	
3	5.37	11.26	$-\infty$	$-\infty$	$-\infty$	46			r	r	i	d		r			
4	5.35	11.14	13.30	14.65	16.75	57	i	i	r	r	i	d					r
5	5.40	11.24	$-\infty$	$-\infty$	$-\infty$	46			r	r	i	d		r			
6	5.38	11.11	13.21	14.67	16.49	57	i	i	r	r	i	d					r
7	5.44	11.22	$-\infty$	$-\infty$	$-\infty$	46			r	r	i	d		r			
8	5.41	11.07	$-\infty$	14.68	$-\infty$	54		i	r	r	i	d				r	
9	5.43	11.13	$-\infty$	14.50	$-\infty$	54		i	r	r	i	d				r	
10	5.45	11.18	13.21	$-\infty$	$-\infty$	52	i		r	r	i	d			r		
11	5.47	11.25	$-\infty$	14.35	$-\infty$	54		i	r	r	i	d				r	
12	5.50	11.32	12.96	14.10	15.67	57	i	i	r	r	i	d					r
13	5.59	11.48	$-\infty$	14.02	$-\infty$	54		i	r	r	i	d				r	
14	5.62	11.58	12.89	$-\infty$	$-\infty$	52	i		r	r	i	d			r		
15	5.67	11.69	$-\infty$	13.73	$-\infty$	54		i	r	r	i	d				r	
16	4.78	10.88	11.60	$-\infty$	$-\infty$	52	i		r	d	i	d			r		
17	5.77	11.97	$-\infty$	$-\infty$	$-\infty$	46			r	r	i	d		r			
18	5.75	11.75	11.99	13.32	$-\infty$	54		i	r	r	i	d				r	
19	5.82	11.92	12.40	$-\infty$	$-\infty$	52	i		r	r	i	d			r		
20	5.90	12.13	11.65	$-\infty$	$-\infty$	46			r	r	i	d		r			
21	5.89	11.86	$-\infty$	12.67	$-\infty$	54		i	r	r	i	d				r	
22	5.99	12.10	12.15	$-\infty$	$-\infty$	52	i		r	r	i	d			r		
23	6.13	12.41	$-\infty$	$-\infty$	$-\infty$	46			r	r	i	d		r			
24	6.15	12.06	10.91	$-\infty$	$-\infty$	46			r	r	i	d		r			
25	6.17	11.59	$-\infty$	11.39	$-\infty$	46			r	r	i	d		r			
26	6.21	$-\infty$	$-\infty$	$-\infty$	$-\infty$	32			r	r		d	r				
27	$-\infty$	11.07	$-\infty$	$-\infty$	$-\infty$	46			r	r	i	d		r			
28	$-\infty$	9.93	11.04	12.38	$-\infty$	54		i	r	r	i	d				r	
29	$-\infty$	10.14	12.73	$-\infty$	$-\infty$	52	i		r	r	i	d			r		
30	$-\infty$	11.45	$-\infty$	$-\infty$	$-\infty$	46			d	r	i	d		r			

The values i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

done, the first incoming unit of the core in focus is tested to find out about its condition. With this information the priority values of all relevant states are determined. The state with the highest priority value is selected and as output, the selected state as well as the information, which item and module is to be assigned to what kind of usage, is given to the person or persons who do the disassembling. During the disassembly an item might be damaged.

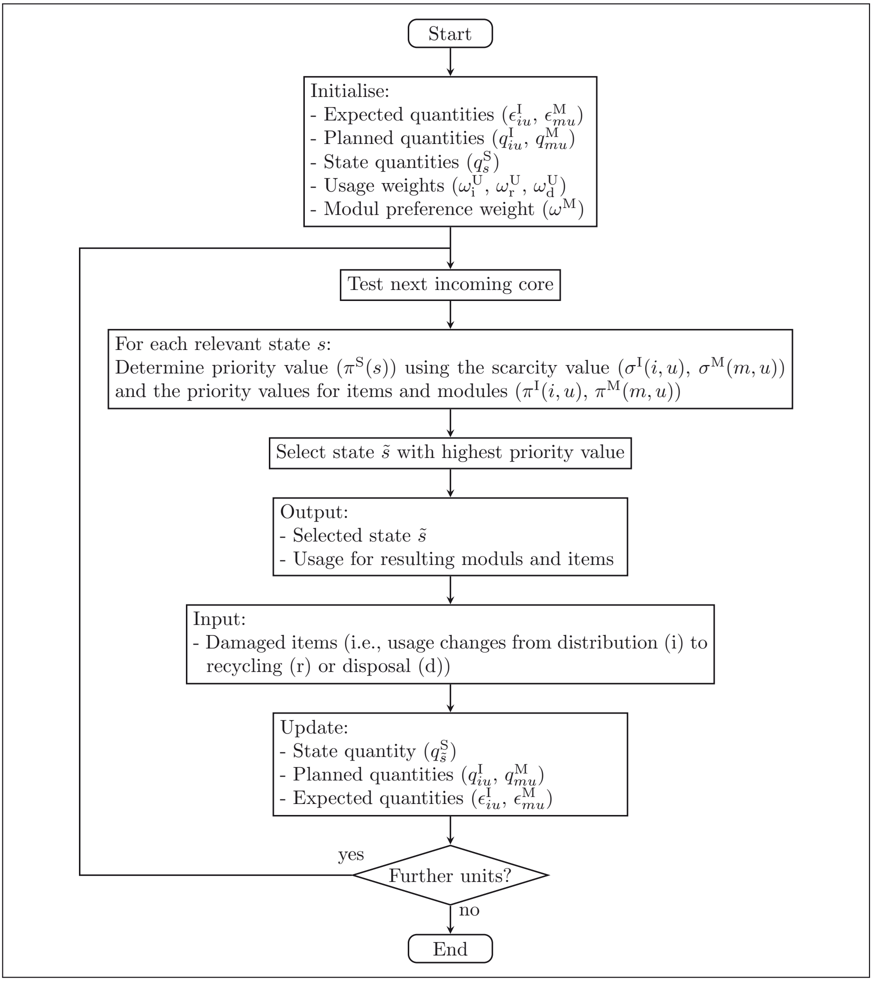

Fig. 4.16 Flowchart of state and usage assignment algorithm

If this happens to an item that should be distributed, it can only be used for recycling or disposal afterwards. The information of the damaging is passed to the decision maker, who assigns this particular item to a new usage. This is either recycling (r) or disposal (d), depending on the planned quantities. This information is again given to the disassembling person. The state quantity, planned quantities, and expected quantities are updated and, if further units of this core come in, they are tested and assigned. Once, all units are disassembled the assignment ends.

The applied weights for the usage ω_u^{U} as well as the module preference ω^{M} must be adjusted by the decision maker. For our numerical example the above given weights work well. For example, a usage weight of $\omega_{\mathrm{i}}^{\mathrm{U}} = 2$ results for some orders of incoming units of cores in infeasible assignments. To find adequate weight values a test set of several (e.g., 100) orders of units per core can be generated. Note that the incoming units of cores in the simulation should follow the expected quantities of items and modules.

4.4.2.2 Batch assignment

The above presented procedure for a step by step assignment can be utilised for batch testing without modification. Nevertheless, if all incoming units are first tested and in a next step assigned with the information of all units, a truly deterministic approach can be applied. One such approach is presented in the sequel. It is based on linear programming. The decision variables are the ones representing the state selection U_{ns} per unit n and relevant state s, the item usage X_{niu} of item i, and the module usage Y_{nmu} of the relevant module m. The term relevant emphasises that only the states and modules are considered that appear in the optimal solution. All unused states and modules are neglected. All decision variables are binary variables. The state quantities q_s^{S}, the condition of the items in a unit after testing t_{ni}, the item state relation γ_{is}^{I}, the modules state relation γ_{ms}^{M}, the module definition δ_{mi}, and the planned quantities of items q_{iu}^{I} and modules q_{mu}^{M} are given. The core index c is neglected for γ_{is}^{I}, γ_{ms}^{M}, and δ_{mi}, because this assigning is conducted for each core separately.

The sum over all units of the state selection variable has to equal the state quantities.

$$\sum_n U_{ns} = q_s^{\mathrm{S}} \quad \forall\ s \tag{4.173}$$

In addition, only one state selection variable can be chosen for a unit n.

$$\sum_s U_{ns} \leq 1 \quad \forall\ n \tag{4.174}$$

Depending on the selected state the corresponding decision variables representing the item and module usage assignment have to be greater than zero. Of course, per item and module only one of the usage assignments must be one.

$$\sum_u X_{niu} \geq U_{ns} \quad \forall\, n, s, i \in \{i|\gamma^{\mathrm{I}}_{is} = 1\} \tag{4.175}$$

$$\sum_u X_{niu} \leq 1 \quad \forall\, n, i \tag{4.176}$$

$$\sum_u Y_{nmu} \geq U_{ns} \quad \forall\, n, s, m \in \{m|\gamma^{\mathrm{M}}_{ms} = 1\} \tag{4.177}$$

$$\sum_u Y_{nmu} \leq 1 \quad \forall\, n, m \tag{4.178}$$

Furthermore, the assigned usage is limited by the unit condition. For items each item is to be considered separately. However, for modules the lowest condition of the items the module consists of determines the upper limit of the module usage. Thereby, the values 1, 2, and 3 represent the disposal, recycling, and distribution, respectively. This applies to t_{ni} and u.

$$\sum_u u\, X_{niu} \leq t_{ni} \quad \forall\, n, i \tag{4.179}$$

$$\sum_u u\, Y_{nmu} \leq \min_{i \in \{i|\delta_{mi}=1\}} \{t_{ni}\} \quad \forall\, n, m \tag{4.180}$$

Besides this, with each item and module assignment for a particular usage the planned quantities have to be met.

$$\sum_n X_{niu} = q^{\mathrm{I}}_{iu} \quad \forall\, i, u \tag{4.181}$$

$$\sum_n Y_{nmu} = q^{\mathrm{M}}_{mu} \quad \forall\, m, u \tag{4.182}$$

Lastly, the domain of the decision variables is limited to the values zero and one.

$$U_{ns}, X_{niu}, Y_{nmu} \in \{0, 1\} \quad \forall\, n, s, i, m, u \tag{4.183}$$

With this (pure) integer LP (ILP) a feasible assignment of the units to states and the usage of the resulting items and modules is gained. One of the existing 10,920 feasible solutions[48] is listed in Table 4.31 for core 1 and the same unit order as Table 4.25. (Solutions with this model for the other two cores can be found in appendix C.9.) The benefit of having all units tested in a batch and assigned to the states is that the units can be disassembled in an order that minimises the state changes. Hence, the person who disassembles

[48] The number of solutions is determined by successive adding of a constraint, which makes the last solution infeasible.

Table 4.31 State assignment for core 1 (LP)

unit	selected state s	item usage u						module usage u				
		i						m				
		A	B	C	D	E	H	28	39	44	46	48
1	46			r	r	i	d		r			
2	54		i	r	r	i	d				r	
3	46			r	r	i	d		r			
4	57	i	i	r	r	i	d					r
5	46			r	r	i	d		r			
6	54		i	r	r	i	d				r	
7	46			r	r	i	d		r			
8	54		i	r	r	i	d				r	
9	54		i	r	r	i	d				r	
10	52	i		r	r	i	d			r		
11	46			r	r	i	d		r			
12	54		i	r	r	i	d				r	
13	54		i	r	r	i	d				r	
14	52	i		r	r	i	d			r		
15	54		i	r	r	i	d				r	
16	52	i		r	d	i	d			r		
17	46			r	r	i	d		r			
18	57	i	i	r	r	i	d					r
19	46			r	r	i	d		r			
20	46			r	r	i	d		r			
21	54		i	r	r	i	d				r	
22	46			d	r	i	d		r			
23	46			r	r	i	d		r			
24	52	i		r	r	i	d			r		
25	54		i	r	r	i	d				r	
26	32			r	r		d	r				
27	46			r	r	i	d		r			
28	57	i	i	r	r	i	d					r
29	52	i		r	r	i	d			r		
30	52	i		r	r	i	d			r		

The values i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

does not have to change the state for each single unit in the worst case. But, still some uncertainty remains, because the damaging occurs during the disassembly and cannot be predicted. Therefore, the order of disassembling should be in a way that considers the damaging. For example, the units, where items to distribute are gained and damaging might occur, should be disassembled first. This way, there will be further units coming, which can be used to assign a functioning item to the distribution. In the reverse order it might happen that there are no more units to meet the planned quantities of items to distribute. Even with this approach no feasible solution might be found, if the conditions are not as expected. In addition, even with a feasible solution, the planned quantities cannot be met when damaging occurs more often than expected. This uncertainty is always present.

4.5 Discussion of alternative solution methods

4.5.1 Using the continuous solution

Following the above discussion of the optimal flexible disassembly planning solution in comparison to more limiting approaches, we find that the problem and thus the model is rather complex. This is mainly reflected in the model size and lastly in the solution time when solving the model. Even for such a small numerical example, the required time exceeds 4,000 seconds on a computer with two AMD Opteron 6282SE 2.6 GHz and eight threads used for GUROBI 5.0 solver. Even though, this is a clear motivation for applying a heuristic—be it a special one or a meta-heuristic—it is not the focus in the sequel. Instead, with the use of the presented model and some slight modifications of it, some ideas to speed up the solving with the drawback of gaining only suboptimal solutions are discussed in the following. The drawback of suboptimal solutions is a bit softened, because of the uncertainty that exists with regard to the condition realisation of the acquired cores. Thus, the decision maker might also be satisfied with a near optimal solution.

A first aspect is the continuous solution of the model. This means, all variables are real valued, i.e., the domain of the variables X^{I}_{ci}, X^{A}_{ci}, X^{R}_{cir}, X^{D}_{cid}, Y^{M}_{cm}, Y^{R}_{cmr}, and Y^{D}_{cmd} is changed from $\mathbb{Z}$ to $\mathbb{R}$. (This is also called the relaxed solution.) The non-negativity of the variables stays unchanged. Hence, the model changes from a MILP to a LP, which is generally solved much faster. The optimal continuous solution listed in Table 4.32 is gained in 29 seconds. Compared to the solution time of the MILP (4,046 s) this is almost 140 times faster. The resulting objective is 33,216.17 €.[49] Of course, this solution is infeasible, because integer values for particular decision variables are required. One could argument, that with large quantities the error of rounding the values is negligible. This is probably true for values that are very close to an integer value (e.g., $X^{\mathrm{R}}_{2,\mathrm{A},1} = 85.07$), but when the solution

[49] The optimal objective of the MILP is 30,739 €.

Table 4.32 Optimal continuous solution of the flexible planning

variables representing the interfaces									
Q_1^{C}	30	Q_1^{I}	249.68	Q_1^{M}	30	Q_1^{R}	35,635	Q_4^{R}	0
Q_2^{C}	216.80	Q_2^{I}	154.34	Q_2^{M}	90	Q_2^{R}	69,965.03	Q_1^{D}	1.45
Q_3^{C}	30.61	Q_3^{I}	214.94			Q_3^{R}	0	Q_2^{D}	6,000

item and module flow variables

	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
				$r=1$			$r=2$			$d=1$			$d=2$		
	c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13.50	97.56	13.77	.	85.07	16.67	.	32.98	.	.	.	.	.	.	.
B	13.50	97.56	13.77	.	118.05	16.67	.	.	.	.	.	.	.	.	.
C	.	.	.	29.96	214.64	30.26	.	.	.	0.04	.	0.05	.	.	.
D	.	.	.	29.96	214.64	30.26	.	.	.	0.04	.	0.05	.	.	.
E	29.70	124.64	.	.	.	30	.	.	.	.	.	.	.	.	.
F	.	.	.	.	122.49	30	.	.	.	.	.	.	.	.	.
G	.	214.64	0.30	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	214.64	0.30	.	.	.	30	.	.

		Y_{cm}^{M}			Y_{cmr}^{R}					
					$r=1$			$r=2$		
		c			c			c		
m		1	2	3	1	2	3	1	2	3
1	(ABCDEFGH)	.	.	.	.	.	.	.	0.00	0.09
2	(BCDEFGH)	.	.	.	.	.	.	.	0.01	0.07
3	(ACDEFGH)	.	.	.	.	.	.	.	0.01	0.07
6	(ABCDFGH)	.	.	.	.	.	.	.	0.65	.
7	(CDEFGH)	.	.	.	.	.	.	.	0.00	0.06
8	(BDEFGH)	.	.	.	.	.	.	.	.	0.00
9	(BCEFGH)	.	.	.	.	.	.	.	.	0.00
10	(BCDFGH)	.	.	.	.	.	.	.	0.53	.
11	(ADEFGH)	.	.	.	.	.	.	.	.	0.00
12	(ACEFGH)	.	.	.	.	.	.	.	.	0.00
13	(ACDFGH)	.	.	.	.	.	.	.	0.53	.
17	(DEFGH)	.	.	.	.	.	.	.	.	0.00
18	(CEFGH)	.	.	.	.	.	.	.	.	0.00
19	(CDFGH)	.	.	.	.	.	.	.	0.43	.
28	(ABEFG)	.	.	.	.	.	.	0.09	.	.
34	(BEFG)	.	.	.	.	.	.	0.07	.	.
36	(AEFG)	.	.	.	.	.	.	0.07	.	.
39	(ABFG)	.	.	.	.	.	.	8.98	.	.
41	(EFG)	.	.	.	.	.	.	0.06	.	.
44	(BFG)	.	.	.	.	.	.	7.35	.	.
46	(AFG)	.	.	.	.	.	.	7.35	.	.
47	(GH)	.	.	30	.	.	.	.	.	.
48	(FG)	.	.	.	.	.	.	6.01	.	.
49	(EF)	.	90	.	.	2.15	0.30	.	.	.

Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary. Dots denote a value of zero and 0.00 indicates a value of less than 0.005. All non-listed values are zero.

contains many values close to 0.5, i.e., in between two integer values, the question arises which integer value to round it to and trying to maintain feasibility. If it was just for one limitation (e.g., the purity) it might be fairly easy to find a decision rule, but with many contrary limitations this is difficult. To decide whether the value $Q_2^{\mathrm{C}} = 216.80$ is rounded down or up one has to consider that when rounding down the purity and/or lower distribution limitations make the solution infeasible. On the other hand, rounding up might lead to an infeasible workload with regard to the limited labour time. In addition, for a few variables the problem might be manageable, but for a large number of affected variables the little infeasibilities add up.

Rounding the continuous solution to the nearest integer value results in the values listed in Table 4.33. Thereby, only the variables X_{ci}^{I}, X_{cir}^{R}, X_{cid}^{D}, Y_{cm}^{M}, Y_{cmr}^{R}, and Y_{cmd}^{D} are rounded, because they represent the item flow and are the basis for the quantities coming in and going out of the disassembly process. They also determine the resulting profit. But when rounding these variables the first problem that arises is which value to choose for the acquired quantity Q_c^{C}, because the sum of the items and modules does not add up to the same quantity of items used in the disassembly process. For example, the rounded values for core 3 differing zero are $X_{3,\mathrm{A}}^{\mathrm{I}} = X_{3,\mathrm{B}}^{\mathrm{I}} = 14$, $X_{3,\mathrm{A},1}^{\mathrm{R}} = X_{3,\mathrm{B},1}^{\mathrm{R}} = 17$, $X_{3,\mathrm{C},1}^{\mathrm{R}} = X_{3,\mathrm{D},1}^{\mathrm{R}} = X_{3,\mathrm{E},1}^{\mathrm{R}} = X_{3,\mathrm{F},1}^{\mathrm{R}} = 30$, and $Y_{3,47}^{\mathrm{M}} = 30$. Adding these, results in 31 units for items A and B and 30 for the remaining. What is the correct value for the quantity of cores then?

For simplicity we take the maximum value, i.e., 30, 218, and 31 units for Q_1^{C}, Q_2^{C}, and Q_3^{C}, respectively. The differing items are disposed of. This way no item is lost, but the purity, workload, etc. limitations are still not considered. Using the values of the integer variables in Table 4.33, the values of the interfacing variables are calculated. The resulting workload is 2,212.75 h, which exceeds the limit of 2,200 h, and the profit 27,793.5 €. Thereby, the extra disposal of the 2,582 kg causes only 516.4 € disposal cost. The main driver comes from the increased quantity of acquired cores. The increase from 30, 216.80, and 30.61 to 30, 218, and 31 units for core 1, 2, and 3, respectively, leads to an increase of core acquisition cost of 4,251 € and of estimated disassembly cost of 437.4 €, assuming complete disassembly for the extra units. Note that most likely, the solution is still infeasible (e.g., because of the workload).

Another possibility of using the continuous solution to gain a solution with integral values is the following. Again, there exists no guaranteed integral solution with the following procedure, but the larger the quantities and the less restrict the constraints the more likely is a feasible solution. Based on the continuous solution each core is considered separately, which reduces

Table 4.33 Rounded integer solution of the flexible planning

variables representing the interfaces									
Q_1^{C}	30	Q_1^{I}	252	Q_1^{M}	30	Q_1^{R}	35,457	Q_4^{R}	0
Q_2^{C}	218	Q_2^{I}	155	Q_2^{M}	90	Q_2^{R}	69,511	Q_1^{D}	2,582
Q_3^{C}	31	Q_3^{I}	215			Q_3^{R}	0	Q_2^{D}	6,000

item and module flow variables

	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
				$r=1$			$r=2$			$d=1$			$d=2$		
	c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13	98	14	.	85	17	.	33	.	.	.	.	.	.	.
B	13	98	14	.	118	17	.	.	.	.	.	.	.	.	.
C	.	.	.	30	215	30	.	.	.	.	.	1	.	.	.
D	.	.	.	30	215	30	.	.	.	.	.	1	.	.	.
E	30	125	.	.	.	30	.	.	.	.	1	1	.	.	.
F	.	.	.	.	122	30	.	.	.	1	1	1	.	.	.
G	.	215	.	.	.	.	.	.	.	1	.	1	.	.	.
H	.	.	.	.	.	.	.	215	.	.	.	1	30	.	.

		Y_{cm}^{M}			Y_{cmr}^{R}					
					$r=1$			$r=2$		
		c			c			c		
m		1	2	3	1	2	3	1	2	3
6	(ABCDFGH)	.	.	.	.	.	.	.	1	.
10	(BCDFGH)	.	.	.	.	.	.	.	1	.
13	(ACDFGH)	.	.	.	.	.	.	.	1	.
39	(ABFG)	.	.	.	.	.	.	9	.	.
44	(BFG)	.	.	.	.	.	.	7	.	.
46	(AFG)	.	.	.	.	.	.	7	.	.
47	(GH)	.	.	30	.	.	.	.	.	.
48	(FG)	.	.	.	.	.	.	6	.	.
49	(EF)	.	90	.	.	2	.	.	.	.

A dot denotes a value of zero.

the complexity of the model a little bit. To avoid infeasibilities because of considering the cores separately, the values with direct influence on the variables of other cores are treated specially. For example, the quantity of items to distribute (e.g., demand position $e = 1$) is the sum of the decision variables of several cores (see Eq. (4.85)). In our numerical example the items A and B of all cores determine the quantity Q_1^{I}. All six variables adding up to Q^{I}_1 are no integer values (see Table 4.32). The quantity Q^{I}_1 is limited and goes into the objective function. When optimising the model just for one core, the contribution of the other cores to the core overlapping quantities is included in the optimisation. Solving the model for core 1 means that $X^{\mathrm{I}}_{1,\mathrm{A}}$ and $X^{\mathrm{I}}_{1,\mathrm{B}}$ are decision variables. The calculation of the distribution quantity is modified to

$$\begin{aligned} &Q^{\mathrm{I}}_1 = X^{\mathrm{I}}_{1,\mathrm{A}} + X^{\mathrm{I}}_{1,\mathrm{B}} + X^{\mathrm{I}}_{2,\mathrm{A}} + X^{\mathrm{I}}_{2,\mathrm{B}} + X^{\mathrm{I}}_{3,\mathrm{A}} + X^{\mathrm{I}}_{3,\mathrm{B}} \\ &Q^{\mathrm{I}}_1 = X^{\mathrm{I}}_{1,\mathrm{A}} + X^{\mathrm{I}}_{1,\mathrm{B}} + 97.56 + 97.56 + 13.77 + 13.77 = X^{\mathrm{I}}_{1,\mathrm{A}} + X^{\mathrm{I}}_{1,\mathrm{B}} + 222.66. \end{aligned} \tag{4.184}$$

Thus, the quantity limitations do not have to be changed. Just an adding of a fixed term and the removing of all variables of unfocussed cores are necessary. This results in removing sums over c and $\forall$ quantifiers regarding c. The core in focus is denoted with $\tilde{c}$. The **objective function** is to be maximised and modified to

$$\text{Maximise} \quad P = R - C \tag{4.185}$$

$$R = \sum_e r^{\mathrm{I}}_e Q^{\mathrm{I}}_e + \sum_f r^{\mathrm{M}}_f Q^{\mathrm{M}}_f + \sum_r r^{\mathrm{R}}_r Q^{\mathrm{R}}_r \tag{4.186}$$

$$C = \left(c^{\mathrm{A}}_{\tilde{c}} + c^{\mathrm{J}}_{\tilde{c},1}\right) Q^{\mathrm{C}}_{\tilde{c}} - \sum_{m=1}^{\overline{M}_{\tilde{c}}} c^{\mathrm{J}}_{\tilde{c}m} \left(Y^{\mathrm{M}}_{\tilde{c}m} + \sum_r Y^{\mathrm{R}}_{\tilde{c}mr} + \sum_d Y^{\mathrm{D}}_{\tilde{c}md} \right) + \sum_d c^{\mathrm{D}}_d Q^{\mathrm{D}}_d \,. \tag{4.187}$$

The values of the unfocussed values in the objective function are not necessary, because they are fixed and thus not relevant for decision making. The objective value is therefore different to the one for the model covering all three cores. The **item flow constraints** are changed to:

$$Q^{\mathrm{C}}_{\tilde{c}} = X^{\mathrm{I}}_{\tilde{c}i} + \sum_r X^{\mathrm{R}}_{\tilde{c}ir} + \sum_d X^{\mathrm{D}}_{\tilde{c}id} + \sum_{m=1}^{\overline{M}_{\tilde{c}}} \delta_{\tilde{c}mi} \left(Y^{\mathrm{M}}_{\tilde{c}m} + \sum_r Y^{\mathrm{R}}_{\tilde{c}mr} + \sum_d Y^{\mathrm{D}}_{\tilde{c}md} \right) \quad \forall\, i \in \{1, \ldots, \bar{I}_{\tilde{c}}\}, \tag{4.188}$$

$$
\begin{aligned}
Q_r^{\mathrm{R}} = {} & \sum_{i=1}^{\bar{I}_{\tilde{c}}} w_{\tilde{c}i} \left(X_{\tilde{c}ir}^{\mathrm{R}} + \sum_{m=1}^{\overline{M}_{\tilde{c}}} \delta_{\tilde{c}mi} Y_{\tilde{c}mr}^{\mathrm{R}} \right) \\
& + \left[\sum_{c \neq \tilde{c}} \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X_{cir}^{\mathrm{R}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmr}^{\mathrm{R}} \right) \right] \quad \forall\, r\,, \qquad (4.189) \\
Q_d^{\mathrm{D}} = {} & \sum_{i=1}^{\bar{I}_{\tilde{c}}} w_{\tilde{c}i} \left(X_{\tilde{c}id}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_{\tilde{c}}} \delta_{\tilde{c}mi} Y_{\tilde{c}md}^{\mathrm{D}} \right) \\
& + \left[\sum_{c \neq \tilde{c}} \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X_{cid}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmd}^{\mathrm{D}} \right) \right] \quad \forall\, d\,, \qquad (4.190)
\end{aligned}
$$

$$
X_{\tilde{c}i}^{\mathrm{I}} + \sum_r X_{\tilde{c}ir}^{\mathrm{R}} + \sum_d X_{\tilde{c}id}^{\mathrm{D}} \geq \sum_{m=1}^{\overline{M}_{\tilde{c}}} \alpha_{\tilde{c}mi} \left(Y_{\tilde{c}m}^{\mathrm{M}} + \sum_r Y_{\tilde{c}mr}^{\mathrm{R}} + \sum_d Y_{\tilde{c}md}^{\mathrm{D}} \right) \quad \forall\, i \in \{1, \ldots, \bar{I}_{\tilde{c}}\}, \qquad (4.191)
$$

$$
Q_e^{\mathrm{I}} = \sum_{\substack{(c,i) \in \mathcal{P}_e \\ c = \tilde{c}}} X_{ci}^{\mathrm{I}} + \left[\sum_{\substack{(c,i) \in \mathcal{P}_e \\ c \neq \tilde{c}}} X_{ci}^{\mathrm{I}} \right] \quad \forall\, e\,, \qquad (4.192)
$$

$$
X_{\tilde{c}i}^{\mathrm{I}} = 0 \quad \forall\, (\tilde{c}, i) \notin \bigcup_e \mathcal{P}_e\,, \qquad (4.193)
$$

$$
Q_f^{\mathrm{M}} = \sum_{\substack{(c,m) \in \mathcal{R}_f \\ c = \tilde{c}}} Y_{cm}^{\mathrm{M}} + \left[\sum_{\substack{(c,m) \in \mathcal{R}_f \\ c = \tilde{c}}} Y_{cm}^{\mathrm{M}} \right] \quad \forall\, f\,, \qquad (4.194)
$$

and

$$
Y_{\tilde{c}m}^{\mathrm{M}} = 0 \quad \forall\, (\tilde{c}, m) \notin \bigcup_f \mathcal{R}_f\,. \qquad (4.195)
$$

Thereby, the term in the squared brackets is the fixed term and $c \neq \tilde{c}$ denotes all indices of c with the exception of $\tilde{c}$. The **condition constraints** including the damaging are not displayed here, because they are formulated for each core individually already (see Eqs. (4.91)–(4.107)). Only the index c

is substituted by $\tilde{c}$. The **purity constraints** need to be modified again. The left hand side can stay unchanged, because the quantities of the unfocussed cores are already included in Eq. (4.189). Thus, the beneficial weight of the unfocussed cores has to be added to the right hand side of the relation.

$$\omega_r Q_r^{\mathrm{R}} \leq \sum_{i=1}^{\bar{I}_{\tilde{c}}} \pi_{\tilde{c}ir} w_{\tilde{c}i} \left(X_{\tilde{c}ir}^{\mathrm{R}} + \sum_{m=1}^{\overline{M}_{\tilde{c}}} \delta_{\tilde{c}mi} Y_{\tilde{c}mr}^{\mathrm{R}} \right) + \left[\sum_{c \neq \tilde{c}} \sum_{i=1}^{\bar{I}_c} \pi_{cir} w_{ci} \left(X_{cir}^{\mathrm{R}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmr}^{\mathrm{R}} \right) \right] \quad \forall\ r \tag{4.196}$$

$$X_{\tilde{c}ir}^{\mathrm{R}} = 0 \quad \forall\ (\tilde{c}, i) \in \mathcal{H}, r \tag{4.197}$$

$$X_{\tilde{c}id}^{\mathrm{D}} = 0 \quad \forall\ (\tilde{c}, i) \in \mathcal{H}, d \in \{1\} \tag{4.198}$$

$$Y_{cmr}^{\mathrm{R}} = 0 \quad \forall\ (c,m) \in \left\{(c,m) | c = \tilde{c}, \delta_{cmi} = 1, (c,i) \in \mathcal{H}, m \in \{1, \dots, \overline{M}_c\}\right\}, r \tag{4.199}$$

$$Y_{cmd}^{\mathrm{D}} = 0 \quad \forall\ (c,m) \in \left\{(c,m) | c = \tilde{c}, \delta_{cmi} = 1, (c,i) \in \mathcal{H}, m \in \{1, \dots, \overline{M}_c\}\right\}, d \in \{1\} \tag{4.200}$$

Only the **limit constraints** for the acquired quantities have to be modified. The others stay as they are.

$$\underline{Q}_{\tilde{c}}^{\mathrm{C}} \leq Q_{\tilde{c}}^{\mathrm{C}} \leq \overline{Q}_{\tilde{c}}^{\mathrm{C}} \tag{4.201}$$

$$\underline{Q}_e^{\mathrm{I}} \leq Q_e^{\mathrm{I}} \leq D_e^{\mathrm{I}} \quad \forall\ e \tag{4.202}$$

$$\underline{Q}_f^{\mathrm{M}} \leq Q_f^{\mathrm{M}} \leq D_f^{\mathrm{M}} \quad \forall\ f \tag{4.203}$$

$$\underline{Q}_r^{\mathrm{R}} \leq Q_r^{\mathrm{R}} \leq D_r^{\mathrm{R}} \quad \forall\ r \tag{4.204}$$

$$\underline{Q}_d^{\mathrm{D}} \leq Q_d^{\mathrm{D}} \leq \overline{Q}_d^{\mathrm{D}} \quad \forall\ d \tag{4.205}$$

On the contrary, in the workload and labour time limit constraint the amount of workload used by the unfocussed cores is added or subtracted from the limit $\bar{L}$.

$$t^{\mathrm{J}}_{\tilde{c},1} Q^{\mathrm{C}}_{\tilde{c}} - \sum_{m=1}^{\overline{M}_{\tilde{c}}} t^{\mathrm{J}}_{\tilde{c}m} \left(Y^{\mathrm{M}}_{\tilde{c}m} + \sum_r Y^{\mathrm{R}}_{\tilde{c}mr} + \sum_d Y^{\mathrm{D}}_{\tilde{c}md} \right) \leq$$
$$\bar{L} - \left[\sum_{c \neq \tilde{c}} \left(t^{\mathrm{J}}_{c,1} Q^{\mathrm{C}}_{c} - \sum_{m=1}^{\overline{M}_{c}} t^{\mathrm{J}}_{cm} \left(Y^{\mathrm{M}}_{cm} + \sum_r Y^{\mathrm{R}}_{cmr} + \sum_d Y^{\mathrm{D}}_{cmd} \right) \right) \right] \quad (4.206)$$

Lastly, the domain of the variables is changed to integer values only for the focussed core $\tilde{c}$.

$$X^{\mathrm{I}}_{\tilde{c}i}, X^{\mathrm{A}}_{\tilde{c}i}, X^{\mathrm{R}}_{\tilde{c}ir}, X^{\mathrm{D}}_{\tilde{c}id}, Y^{\mathrm{M}}_{\tilde{c}m}, Y^{\mathrm{R}}_{\tilde{c}mr}, Y^{\mathrm{D}}_{\tilde{c}md} \in \mathbb{Z}^*$$
$$\forall\ i \in \{1, \ldots, \bar{I}_{\tilde{c}}\}, m \in \{1, \ldots, \overline{M}_{\tilde{c}}\}, r, d \quad (4.207)$$

Focussing on core 1 and solving the above model fixes the integral variables for core 1. All these fixed variables are then used to replace the continuous values of the variable of core 1. After this update of variable values the next core is in focus, i.e., core 2. The integral variable values for core 2 are gained. Then the integral values substitute the continuous values of core 2 and the focus is shifted to core 3. The model is solved a third time and the last set of integral variable values is gained. The objective function variables, i.e., R, C, and P, are calculated based on the integral values. The solution is listed in Table 4.34.

The overall gained solution is feasible regarding all constraints. The objective results in $P = 30{,}580.1$ €. This is just a gap of 0.52 % compared to the optimal value of 30,739 €. The solution is gained in 662 s. Thereby, 29 s are used for the continuous model. For the three runs of determining the integral values for the cores 143 s, 115 s, and 375 s are necessary for our numerical example. This is about six times faster than the 4,046 s for the optimal solution and the gap is relatively small.[50]

4.5.2 Fixed solution time

Another rather easy to implement speed up of finding a solution is the limitation of the solution time. Choosing the right time in advance is not

[50] The question is, whether the optimal integral solution for each core is necessary. As in the dynamic planning, the overall solution with optimal solutions of the subproblems does not have to be better than the overall solution with suboptimal solutions of the subproblems. Therefore the so-called MIP gap could be set to a bigger value, e.g., 0.001, instead of the value used for solving all problems in this work, i.e., 10^{-7}.

Table 4.34 Core successive integer solution of the flexible planning

variables representing the interfaces

Q_1^{C}	30	Q_1^{I}	246	Q_1^{M}	30	Q_1^{R}	35,941	Q_4^{R}	0
Q_2^{C}	217	Q_2^{I}	153	Q_2^{M}	90	Q_2^{R}	71,317	Q_1^{D}	32
Q_3^{C}	31	Q_3^{I}	214			Q_3^{R}	0	Q_2^{D}	6,000

item and module flow variables

	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
				$r=1$			$r=2$			$d=1$			$d=2$		
	c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13	97	13	.	93	17	.	25	.	.	.	.	.	.	.
B	13	97	13	1	119	16	.	.	1	.	.	.	.	.	.
C	.	.	.	29	211	30	.	3	.	1	.	1	.	.	.
D	.	.	.	29	214	30	.	.	.	1	.	1	.	.	.
E	29	124	.	.	.	31	.	.	.	.	.	.	.	.	.
F	.	.	.	.	123	30	.	.	.	.	.	.	.	.	.
G	.	214	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	214	.	.	.	.	30	.	.

		Y_{cm}^{M}			Y_{cmr}^{R}					
					$r=1$			$r=2$		
		c			c			c		
m		1	2	3	1	2	3	1	2	3
10	(BCDFGH)	.	.	.	.	.	.	.	1	.
26	(ACDGH)	.	.	.	.	.	.	.	2	.
27	(ABFGH)	.	.	.	.	.	.	.	.	1
36	(AEFG)	.	.	.	.	.	.	1	.	.
39	(ABFG)	.	.	.	.	.	.	9	.	.
44	(BFG)	.	.	.	.	.	.	7	.	.
46	(AFG)	.	.	.	.	.	.	7	.	.
47	(GH)	.	.	30	.	.	.	.	.	.
48	(FG)	.	.	.	.	.	.	6	.	.
49	(EF)	.	90	.	.	3	.	.	.	.

A dot denotes a value of zero.

so easy, because the decision maker has to manage a trade-off between fast solving and a good solution or even a feasible solution. In general, when solving a flexible disassembly problem with limiting constraints there exists no feasible solution from the beginning. A first feasible solution might occur after seconds, minutes, or hours, depending on the problem, solver software, and hardware. Thus, when fixing the solution time the condition should be added, that the solving process should run as long as a first feasible solution is found. Note that one feasible solution is known to the decision maker in short time. This is the solution of the complete disassembly. Thus, as long as the objective of the suboptimal flexible planning is below the optimal objective of the complete disassembly planning, the solving should be continued or the complete disassembly used. Of course, one could use the optimal solution of the complete disassembly as initial solution.

To illustrate the above the log file of the solving with GUROBI is listed in Fig. 4.17. We see that the objective of the relaxed problem (i.e., without integer variables) is 33,216.17 €. After 444 s a first feasible solution with an objective of 10,182.6 € is found. After 534 s a solution with an objective of 27,506.15 € is found. The time limit set for this solving run is 662 s, because it equals the time needed to find the solution in the above Sect. 4.5.1. For this example the solution found with the time limitation is worse than that of the above presented method. The MIP gap, i.e., the gap between the actual best feasible solution and the best bound, is 15.67 %. The optimality gap, i.e., the gap between the best feasible solution and the optimum, is $\frac{30{,}739}{27{,}506.15} = 11.75\,\%$. Note that the optimality gap can only be determined when the optimal solution is known.

4.5.3 Reducing the disassembly state graph

Taking a look at the continuous solution (see Table 4.32) we notice that for core 1 not a single module with item H exists. This is expected, because the hazardous item H forces each module with this item to be disposed of at a higher unit cost. Depending on the core this leads to higher cost compared than separating all other items from the hazardous one. The exception is a case where a connected item causes less extra cost of disposal compared to the disassembly. In such a case, the decision maker should check, whether these two items should be modelled as a single one. When the continuous solution indicates that the hazardous items are always separate from anything else or the decision maker is confident that this is going to be a property of the solution for the actual problem, all decision variables representing modules that contain a hazardous item are set to zero. For the numerical example this reduces the number of modules of core 1 from 50 to ten. The remaining modules are $m \in \{28, 34, 36, 39, 41, 44, 46, 48, 49, 50\}$. Thereby, all the existing states stay unchanged.

This can also be considered from the beginning, i.e., when building the disassembly state graph. Thereby, the items C and D have to be taken off

```
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 33216.1650    0   64          - 33216.1650     -      -   30s
     0     0 32668.6115    0   68          - 32668.6115     -      -   71s
     0     0 32666.8357    0   64          - 32666.8357     -      -   93s
     0     0 32666.7376    0   64          - 32666.7376     -      -  119s
     0     0 32666.7376    0   64          - 32666.7376     -      -  132s
     0     2 32666.7376    0   64          - 32666.7376     -      -  281s
     4     5 32419.0317    4   66          - 32529.6314     -    134  285s
    15    15 32101.1308    9   63          - 32101.1308     -    161  292s
    22    23 31959.4642   11   63          - 32021.3098     -    377  305s
    31    29 31817.7976   13   63          - 32002.2862     -    409  311s
    55    41 31333.0632   15   63          - 32002.2862     -    499  325s
    68    46 31100.8376   16   63          - 32002.2862     -    516  333s
    83    48 30903.3544   17   44          - 32002.2862     -    496  336s
    98    55 30299.1913   18   54          - 32002.2862     -    441  340s
   114    72 29493.9736   20   54          - 32002.2862     -    416  361s
   381   234 11618.5706   74   26          - 32002.2862     -    407  374s
   498   322 11600.2229   77   18          - 32002.2862     -    451  389s
   629   414 11589.7643   80   14          - 32002.2862     -    476  401s
   725   489 11565.3000   83    8          - 32002.2862     -    512  413s
   859   597 11564.7577  100    8          - 32002.2862     -    508  430s
  1043   778 10182.6000  112    4          - 32002.2862     -    482  444s
* 1064   734             131    10182.599994 32002.2862   214%   476  444s
  1187   822     cutoff  132      10182.6000 31903.1066   213%   468  479s
  1198   821 31090.9070   12   40 10182.6000 31817.6140   212%   472  494s
* 1252   537             163    27005.750000 31817.6140  17.8%   464  494s
  1348   571 29482.3411   18   34 27005.7500 31817.6140  17.8%   447  511s
* 1376   296             179    27506.150000 31817.6140  15.7%   447  511s
  1479   307 29007.1778   30   39 27506.1500 31817.6140  15.7%   438  534s
  1562   345 27508.3500  108   64 27506.1500 31817.6140  15.7%   433  591s
  1564   346 30938.2751    9   64 27506.1500 31817.6140  15.7%   432  647s

Cutting planes:
  Gomory: 66

Explored 1564 nodes (792259 simplex iterations) in 662.88 seconds
Thread count was 8 (of 32 available processors)

Time limit reached
Best objective 2.750614999990e+04, best bound 3.181760000000e+04, gap 15.6745%
```

Fig. 4.17 GUROBI log file (excerpt)

prior to item H. This is depicted in the extended connection state graph. This graph is repeated here (see Fig. 4.18, right), because it still applies to the cores 2 and 3. The way with the least amounts of items to take off to separate item H, is taking off item C, D, and H. Therefore, the individual

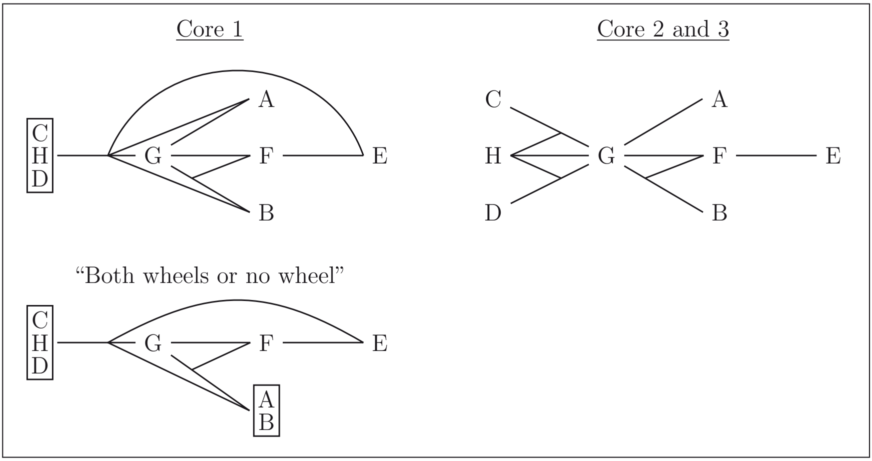

Fig. 4.18 Extended connection graph

connections they have are grouped to a single one. When separating this group connection, the three items are gained as single items.[51] This is depicted by the rectangle around the items C, D, and H. To avoid that any module combination exists with item H in it, it has to be assured that no other item (i.e., A, B, E, F, and G) can be taken off prior to the group of item C, D, and H. This is achieved by adding precedence information. These are the three lines from the connection between H and G to the items A, B, and E. Lines to F and G are unnecessary, because the item B disables the separation of item F and G. The resulting graph is that of Fig. 4.18, top left. In addition, we assume that for any reason (e.g., handling) either both front wheels have to be taken off or none of the two. Thus, the two items are again grouped and the resulting graph is depicted on the bottom left.[52]

Based on the connection graph the corresponding disassembly state graph is the one shown in Fig. 4.19, top. Even though the complete core is not a feasible module, because of the consisting item H, it has to be included according to the above approach to calculate the disassembly cost and time. Compared to the initial disassembly state graph in Fig. 4.4, two levels are

[51] The separation of more than one item or module with one separation is also called ternary operation and appears in practice. Cf. KWAK / HONG / CHO (2009): *Eco-architecture analysis for end-of-life*, p. 6247. Thus, a ternary operation in practice would result in the same representation like the grouped items.

[52] If the latter aspect is considered also, the remaining modules reduces to $m \in \{28, 39, 41, 48, 49\}$.

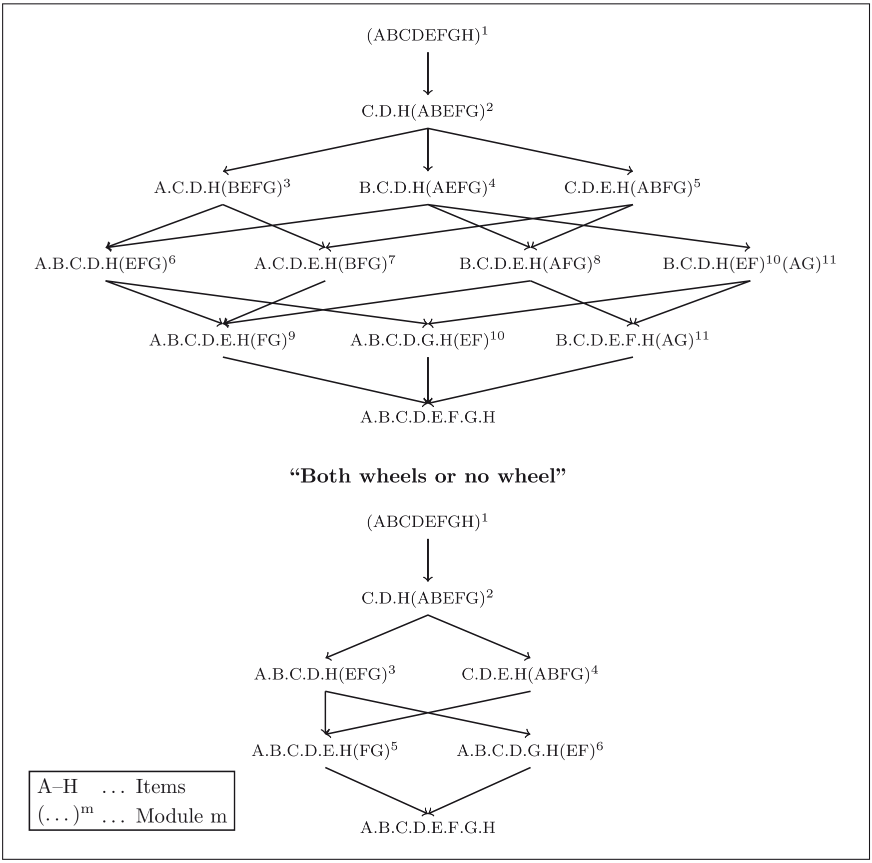

Fig. 4.19 Disassembly state graph of core 1 without hazardous modules

reduced, because three connections are grouped to one. Remember, the number of levels of the disassembly state graph equals the number of connections that hold the core together. The resulting graph contains 13 states. In total 11 modules remain. These are renumbered to form a sequence starting at one and ending with the number of modules and states. For completeness, the corresponding and/or graph is shown in Fig. 4.20, left. The notation of the arrows is as follows: module $m = 4$ can be disassembled to module 6 or 8 or (10 and 11). This significant decrease of states and modules is caused by the fact that item H is rather "deep" in the core. If item H would have been the first to remove anyways, no reduction of states would occur.

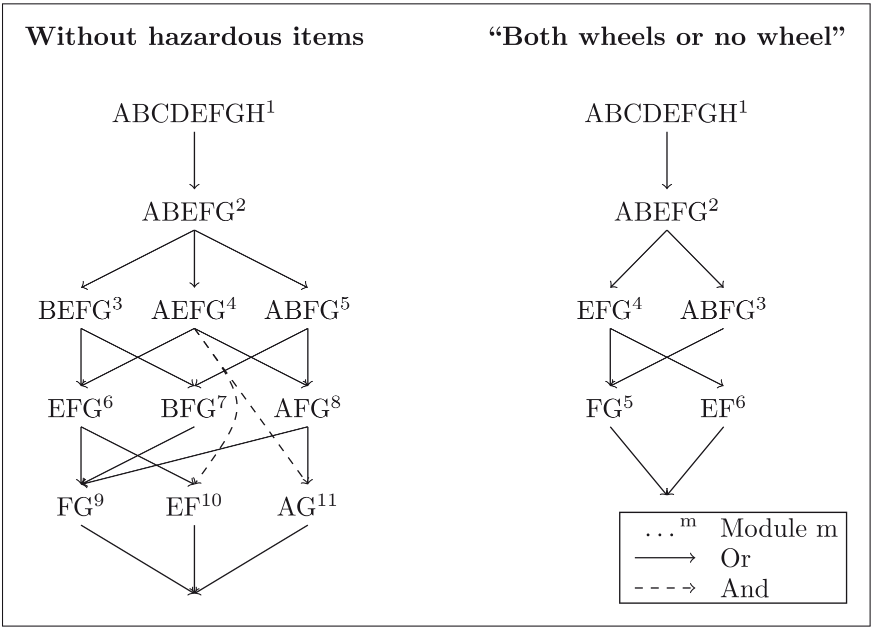

Fig. 4.20 And/Or graph of core 1 without hazardous modules

Adding the aspect of taking off either both wheels or no wheel reduces to disassembly state graph even more (see Fig. 4.19, bottom). Starting from the state with module ABEFG either item E is separated or items A and B are taken off. Hence, only two further states result. Of course, the and/or graph is also reduced, because all modules with item A and B in different modules are not allowed. Other than with not allowing hazardous modules, the grouping of A and B leads to modules with different size on the same level in the and/or graph (see Fig. 4.20, right).

The information of the graphs is coded in the module definition and additional item matrix. Whereas the module definitions of the corresponding rows of the original size are identical, the additional item matrix has to be recalculated. For example, module EF has only item B as additional item in the original version (see Table 4.2 on page 164, row $m = 49$). But with the reduced possibilities the number of additional items increases, because now module EF always appears with the single items B, C, D, and H (for the non-hazardous version) and A, B, C, D, G, and H (for the two-wheels version). The matrices are listed in Table 4.35.

Table 4.35 Reduced module definition and additional item matrix core 1

non-hazardous version																
	module definition matrix $\delta_{1,m,i}$								additional item matrix $\alpha_{1,m,i}$							
module	item								item							
m	A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H
1	1	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.
2	1	1	.	.	1	1	1	.	.	.	1	1	.	.	.	1
3	.	1	.	.	1	1	1	.	1	.	1	1	.	.	.	1
4	1	.	.	.	1	1	1	.	.	1	1	1	.	.	.	1
5	1	1	.	.	.	1	1	.	.	.	1	1	1	.	.	1
6	.	.	.	.	1	1	1	.	1	1	1	1	.	.	.	1
7	.	1	.	.	.	1	1	.	1	.	1	1	1	.	.	1
8	1	.	.	.	.	1	1	.	.	1	1	1	1	.	.	1
9	.	.	.	.	.	1	1	.	1	1	1	1	1	.	.	1
10	.	.	.	.	1	1	.	.	.	1	1	1	.	.	.	1
11	1	.	.	.	.	.	1	.	.	1	1	1	.	.	.	1
"both wheels or no wheel" version																
	module definition matrix $\delta_{1,m,i}$								additional item matrix $\alpha_{1,m,i}$							
module	item								item							
m	A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H
1	1	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.
2	1	1	.	.	1	1	1	.	.	.	1	1	.	.	.	1
3	.	.	.	.	1	1	1	.	1	1	1	1	.	.	.	1
4	1	1	.	.	.	1	1	.	.	.	1	1	1	.	.	1
5	.	.	.	.	.	1	1	.	1	1	1	1	1	.	.	1
6	.	.	.	.	1	1	.	.	1	1	1	1	.	.	1	1

A dot denotes a value of zero.

The update of the modules entails an update of the saved cost and time information for this core. The original values are listed in Table 4.14. The joint times and cost stay unchanged so that the saved cost and time is calculated straightforward (see Table 4.36). The variables for module $m = 1$ of core $c = 1$ must be set to zero, i.e., $Y^{\mathrm{M}}_{1,1} = Y^{\mathrm{R}}_{1,1,r} = Y^{\mathrm{D}}_{1,1,d} = 0$. The number of modules $\overline{M}_1$ is either set to 11 or six and the mapping between the nodes of the distribution, recycling, and disposal graph and the module and item index $L^{\mathrm{A}}_{1,w}$ needs to be updated, too.

With these modifications the model can be solved. A profit of 30,739 € and 30,645 € is gained with solution times of 2,698 s and 24,456 s for the non-hazardous and two-wheels version, respectively. We see, that the first equals the optimal solution found faster and the second is obviously a suboptimal

Table 4.36 Saved cost and time of modules of reduced core 1

	non-hazardous version										
	module m										
	1	2	3	4	5	6	7	8	9	10	11
$c^{\mathrm{J}}_{1,m}$	300	112.5	108.75	108.75	97.5	105	93.75	93.75	90	15	3.75
$t^{\mathrm{J}}_{1,m}$	10	3.75	3.625	3.625	3.25	3.5	3.125	3.125	3	0.5	0.125
	"both wheels or no wheel" version										
	module m										
	1	2	3	4	5	6					
$c^{\mathrm{J}}_{1,m}$	300	112.5	105	97.5	90	15					
$t^{\mathrm{J}}_{1,m}$	10	3.75	3	3.25	3	0.5					

solution. Comparing the solution times shows that reducing the number of decision variables not always leads to a reduction of the solution time. What we also see is that only the reduction of the number of modules does not really help on speeding up the solution time. On the other hand, this is expected, because the main driver of the model size is the core graph, which solely depends on the number of items.

4.5.4 Alternative condition constraints

In Sect. 4.2.2.2 the developing of the condition constraints started with a set of constraints considering superordinate modules to limit the available quantities for particular module and item usage. As shown there, this formulation does not prohibit all infeasible solutions. However, if the found solution is feasible—in terms of the modules and item usage according to the expected condition of the cores—this solution is an optimal one. This means that the solution of the flexible planning model delivers the same optimal profit. (Note that the solution besides the profit can be different.)

The mentioned approach does not include the core, distribution, recycling, and disposal graphs, which leads to a significant decrease of model size. Thus, a significant speed up of the solution time is expected so that it is worth trying to solve this model first. If the resulting solution is feasible, the optimal solution is found. Otherwise, further steps are necessary. These steps could include solving the correct flexible planning model or any of the above mentioned approaches.

The flexible planning model with the substituted condition constraints is listed in the sequel. The equations are not in detail explained, because they are discussed in the relevant Sect. 4.2.2.2 and 4.2.3. The model is labelled **(AC)** in the following.

Objective function:

$$\text{Maximise} \quad P = R - C \tag{4.208}$$

$$R = \sum_e r_e^{\mathrm{I}} Q_e^{\mathrm{I}} + \sum_f r_f^{\mathrm{M}} Q_f^{\mathrm{M}} + \sum_r r_r^{\mathrm{R}} Q_r^{\mathrm{R}} \tag{4.209}$$

$$C = \sum_c \left(c_c^{\mathrm{A}} + c_{c,1}^{\mathrm{J}}\right) Q_c^{\mathrm{C}} - \sum_c \sum_{m=1}^{\overline{M}_c} c_{cm}^{\mathrm{J}} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}}\right) + \sum_d c_d^{\mathrm{D}} Q_d^{\mathrm{D}} \tag{4.210}$$

Item flow constraints:

$$Q_c^{\mathrm{C}} = X_{ci}^{\mathrm{I}} + \sum_r X_{cir}^{\mathrm{R}} + \sum_d X_{cid}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}}\right) \quad \forall\ c, i \in \{1, \ldots, \bar{I}_c\} \tag{4.211}$$

$$Q_r^{\mathrm{R}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X_{cir}^{\mathrm{R}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmr}^{\mathrm{R}}\right) \quad \forall\ r \tag{4.212}$$

$$Q_d^{\mathrm{D}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X_{cid}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmd}^{\mathrm{D}}\right) \quad \forall\ d \tag{4.213}$$

$$X_{ci}^{\mathrm{I}} + \sum_r X_{cir}^{\mathrm{R}} + \sum_d X_{cid}^{\mathrm{D}} \geq \sum_{m=1}^{\overline{M}_c} \alpha_{cmi} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}}\right) \quad \forall\ c, i \in \{1, \ldots, \bar{I}_c\} \tag{4.214}$$

$$Q_e^{\mathrm{I}} = \sum_{(c,i) \in \mathcal{P}_e} X_{ci}^{\mathrm{I}} \quad \forall\ e \tag{4.215}$$

$$X_{ci}^{\mathrm{I}} = 0 \quad \forall\ (c,i) \notin \bigcup_e \mathcal{P}_e \tag{4.216}$$

$$Q_f^{\mathrm{M}} = \sum_{(c,m)\in\mathcal{R}_f} Y_{cm}^{\mathrm{M}} \quad \forall\ f \tag{4.217}$$

$$Y_{cm}^{\mathrm{M}} = 0 \quad \forall\ (c,m) \notin \bigcup_f \mathcal{R}_f \tag{4.218}$$

Condition & Damaging constraints:

$$\sum_{\tilde{m}\in\left\{\tilde{m}\left|\substack{\delta_{cmi}\le\delta_{c\tilde{m}i}\ \forall\, i,\\ (c,\tilde{m})\in\bigcup_f \mathcal{R}_f}\right.\right\}} Y_{c\tilde{m}}^{\mathrm{M}} \le \prod_{i\in\{i|\delta_{cmi}=1\}} (1-\zeta_{ci})(1-\eta_{ci})Q_c^{\mathrm{C}} \quad \forall\ (c,m)\in\bigcup_f \mathcal{R}_f \tag{4.219}$$

$$X_{ci}^{\mathrm{I}} + (1-\theta_{ci}) \sum_{m\in\left\{m\left|\substack{\delta_{cmi}=1,\\ (c,m)\in\bigcup_f \mathcal{R}_f}\right.\right\}} Y_{cm}^{\mathrm{M}} \le (1-\zeta_{ci})(1-\eta_{ci})(1-\theta_{ci})Q_c^{\mathrm{C}} \qquad \forall\ (c,i)\in\bigcup_e \mathcal{P}_e \tag{4.220}$$

$$\sum_{\tilde{m}\in\left\{\tilde{m}\left|\substack{\delta_{cmi}\le\delta_{c\tilde{m}i}\ \forall\, i,\\ (c,\tilde{m})\in\{1,\ldots,\overline{M}_c\}}\right.\right\}} \left(Y_{c\tilde{m}}^{\mathrm{M}} + \sum_r Y_{c\tilde{m}r}^{\mathrm{R}}\right) \le \prod_{i\in\{i|\delta_{cmi}=1\}} (1-\zeta_{ci}\iota_{ci})Q_c^{\mathrm{C}} \qquad \forall\ c, m\in\{1,\ldots,\overline{M}_c\} \tag{4.221}$$

$$\sum_r \left(\sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmr}^{\mathrm{R}} + X_{cir}^{\mathrm{R}}\right) + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cm}^{\mathrm{M}} + X_{ci}^{\mathrm{I}} \le (1-\zeta_{ci}\iota_{ci})Q_c^{\mathrm{C}} \qquad \forall\ c, i\in\{1,\ldots,\bar{I}_c\} \tag{4.222}$$

Purity constraints:

$$\omega_r Q_r^{\mathrm{R}} \le \sum_c \sum_{i=1}^{\bar{I}_c} \pi_{cir} w_{ci} \left(X_{cir}^{\mathrm{R}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmr}^{\mathrm{R}}\right) \quad \forall\ r \tag{4.223}$$

$$X^{\mathrm{R}}_{cir} = 0 \quad \forall\ (c,i) \in \mathcal{H}, r \tag{4.224}$$

$$X^{\mathrm{D}}_{cid} = 0 \quad \forall\ (c,i) \in \mathcal{H}, d \in \{1\} \tag{4.225}$$

$$Y^{\mathrm{R}}_{cmr} = 0 \quad \forall\ (c,m) \in \big\{(c,m)|\delta_{cmi} = 1, (c,i) \in \mathcal{H}, m \in \{1,\dots,\overline{M}_c\}\big\}, r \tag{4.226}$$

$$Y^{\mathrm{D}}_{cmd} = 0 \quad \forall\ (c,m) \in \big\{(c,m)|\delta_{cmi} = 1, (c,i) \in \mathcal{H}, m \in \{1,\dots,\overline{M}_c\}\big\}, d \in \{1\} \tag{4.227}$$

Limit constraints:

$$\underline{Q}^{\mathrm{C}}_c \le Q^{\mathrm{C}}_c \le \overline{Q}^{\mathrm{C}}_c \quad \forall\ c \tag{4.228}$$

$$\underline{Q}^{\mathrm{I}}_e \le Q^{\mathrm{I}}_e \le D^{\mathrm{I}}_e \quad \forall\ e \tag{4.229}$$

$$\underline{Q}^{\mathrm{M}}_f \le Q^{\mathrm{M}}_f \le D^{\mathrm{M}}_f \quad \forall\ f \tag{4.230}$$

$$\underline{Q}^{\mathrm{R}}_r \le Q^{\mathrm{R}}_r \le D^{\mathrm{R}}_r \quad \forall\ r \tag{4.231}$$

$$\underline{Q}^{\mathrm{D}}_d \le Q^{\mathrm{D}}_d \le \overline{Q}^{\mathrm{D}}_d \quad \forall\ d \tag{4.232}$$

$$\sum_c t^{\mathrm{J}}_{c,1} Q^{\mathrm{C}}_c - \sum_c \sum_{m=1}^{\overline{M}_c} t^{\mathrm{J}}_{cm} \left(Y^{\mathrm{M}}_{cm} + \sum_r Y^{\mathrm{R}}_{cmr} + \sum_d Y^{\mathrm{D}}_{cmd} \right) \le \bar{L} \tag{4.233}$$

Domain:

$$X^{\mathrm{I}}_{ci}, X^{\mathrm{R}}_{cir}, X^{\mathrm{D}}_{cid}, Y^{\mathrm{M}}_{cm}, Y^{\mathrm{R}}_{cmr}, Y^{\mathrm{D}}_{cmd} \in \mathbb{Z}^* \quad \forall\ c, i \in \{1,\dots,\bar{I}_c\}, m \in \{1,\dots,\overline{M}_c\}, r, d \tag{4.234}$$

Solving this model considering all three cores with the given data, results in an infeasible solution. The quantities to acquire are 30, 218, and 31 for the three cores. These equal the ones from the optimal solution, but the module and item selection, e.g., for core 1, is infeasible. Thus, for our numerical example this model cannot be used straight away. With a solution time of only 0.42 s it would be too bad to not use this model.

One way of using it is the integration in the approach discussed in Sect. 4.5.1 where a solution based on the continuous solution is sought. For this approach the following modification integrates the above mentioned model. Every time the integral solution for each individual core is determined, it is first tried with the above (AC) model and if that fails the model with the correct condition constraints is used, as described in the above section. To avoid infeasible solutions, the bounding of the decision variables is set according to the continuous solution. The procedure is illustrated in the following.

Solving the model (AC) with all three cores leads to an infeasible solution. The infeasibility is detected after determining the states (see Sect. 4.4.1) and the attempted assignment of the states to the corresponding units of a core (see Sect. 4.4.2, Eqs. (4.173) et seqq.). When this assignment—given the planned quantity of cores with correctly expected conditions—is not possible, the found solution is infeasible. In the case of infeasibility, the continuous solution with the flexible planning model is determined (see Sect. 4.5.1). This continuous solution is depicted in Table 4.32. For core 1, we find the following values: $X^{\mathrm{I}}_{1,\mathrm{A}} = 13.5$, $X^{\mathrm{I}}_{1,\mathrm{B}} = 13.5$, $X^{\mathrm{I}}_{1,\mathrm{E}} = 29.7$, $X^{\mathrm{R}}_{1,\mathrm{C},1} = 29.96$, $X^{\mathrm{R}}_{1,\mathrm{D},1} = 29.96$, $X^{\mathrm{D}}_{1,\mathrm{C},1} = 0.04$, $X^{\mathrm{D}}_{1,\mathrm{D},1} = 0.04$, $X^{\mathrm{D}}_{1,\mathrm{H},2} = 30$, $Y^{\mathrm{R}}_{1,28,1} = 0.09$, $Y^{\mathrm{R}}_{1,34,1} = 0.07$, etc.

The solution we look for should be somewhere in the neighbourhood of these values. Of course, the specification of what the neighbourhood is, i.e., which specific interval, needs to be researched further. At this point, only an illustration of an alternative solution method is sketched. Therefore, a bound of a by one increased value for each decision variable that is greater than zero is chosen. This means that the decision variable $X^{\mathrm{I}}_{1,\mathrm{A}}$ in model (AC) is bounded by $\lfloor 13.5 \rfloor + 1 = 14$ based on the continuous solution. For recycling and disposal the bound calculation is generally the same, but the sum over all recycling or disposal categories is relevant. (In terms of condition, it is irrelevant which particular recycling or disposal bin the module or item is allocated to.) Thus, e.g., $\sum_r X^{\mathrm{R}}_{1,\mathrm{C},r}$ has to be lower than $29.96 + 1$, which equals $\lfloor 29.96 \rfloor + 1 = 30$ for integer values. All decision variables that are zero in the continuous solution with exception of the item disposal X^{D} have to be zero in the integral solution, too.

For a specific core $\tilde{c}$ that the integral solution is sought, the model to be solved is the following. It equals the one in Sect. 4.5.1 only that the condition constraints are substituted and the variable value bounding is added. The **objective function** and **item flow constraints** are unchanged, i.e.,

$$\text{Maximise} \quad P = R - C \,, \tag{4.235}$$

$$R = \sum_e r^{\mathrm{I}}_e Q^{\mathrm{I}}_e + \sum_f r^{\mathrm{M}}_f Q^{\mathrm{M}}_f + \sum_r r^{\mathrm{R}}_r Q^{\mathrm{R}}_r \,, \tag{4.236}$$

$$C = \left(c^{\mathrm{A}}_{\tilde{c}} + c^{\mathrm{J}}_{\tilde{c},1}\right) Q^{\mathrm{C}}_{\tilde{c}} - \sum_{m=1}^{\overline{M}_{\tilde{c}}} c^{\mathrm{J}}_{\tilde{c}m} \left(Y^{\mathrm{M}}_{\tilde{c}m} + \sum_r Y^{\mathrm{R}}_{\tilde{c}mr} + \sum_d Y^{\mathrm{D}}_{\tilde{c}md} \right) + \sum_d c^{\mathrm{D}}_d Q^{\mathrm{D}}_d \,, \tag{4.237}$$

$$Q^{\mathrm{C}}_{\tilde{c}} = X^{\mathrm{I}}_{\tilde{c}i} + \sum_{r} X^{\mathrm{R}}_{\tilde{c}ir} + \sum_{d} X^{\mathrm{D}}_{\tilde{c}id} + \sum_{m=1}^{\overline{M}_{\tilde{c}}} \delta_{\tilde{c}mi} \left(Y^{\mathrm{M}}_{\tilde{c}m} + \sum_{r} Y^{\mathrm{R}}_{\tilde{c}mr} + \sum_{d} Y^{\mathrm{D}}_{\tilde{c}md} \right)$$
$$\forall\ i \in \{1, \ldots, \bar{I}_{\tilde{c}}\}, \quad (4.238)$$

$$Q^{\mathrm{R}}_{r} = \sum_{i=1}^{\bar{I}_{\tilde{c}}} w_{\tilde{c}i} \left(X^{\mathrm{R}}_{\tilde{c}ir} + \sum_{m=1}^{\overline{M}_{\tilde{c}}} \delta_{\tilde{c}mi} Y^{\mathrm{R}}_{\tilde{c}mr} \right)$$
$$+ \left[\sum_{c \neq \tilde{c}} \sum_{i=1}^{\bar{I}_{c}} w_{ci} \left(X^{\mathrm{R}}_{cir} + \sum_{m=1}^{\overline{M}_{c}} \delta_{cmi} Y^{\mathrm{R}}_{cmr} \right) \right] \quad \forall\ r\,, \qquad (4.239)$$

$$Q^{\mathrm{D}}_{d} = \sum_{i=1}^{\bar{I}_{\tilde{c}}} w_{\tilde{c}i} \left(X^{\mathrm{D}}_{\tilde{c}id} + \sum_{m=1}^{\overline{M}_{\tilde{c}}} \delta_{\tilde{c}mi} Y^{\mathrm{D}}_{\tilde{c}md} \right)$$
$$+ \left[\sum_{c \neq \tilde{c}} \sum_{i=1}^{\bar{I}_{c}} w_{ci} \left(X^{\mathrm{D}}_{cid} + \sum_{m=1}^{\overline{M}_{c}} \delta_{cmi} Y^{\mathrm{D}}_{cmd} \right) \right] \quad \forall\ d\,, \qquad (4.240)$$

$$X^{\mathrm{I}}_{\tilde{c}i} + \sum_{r} X^{\mathrm{R}}_{\tilde{c}ir} + \sum_{d} X^{\mathrm{D}}_{\tilde{c}id} \geq \sum_{m=1}^{\overline{M}_{\tilde{c}}} \alpha_{\tilde{c}mi} \left(Y^{\mathrm{M}}_{\tilde{c}m} + \sum_{r} Y^{\mathrm{R}}_{\tilde{c}mr} + \sum_{d} Y^{\mathrm{D}}_{\tilde{c}md} \right)$$
$$\forall\ i \in \{1, \ldots, \bar{I}_{\tilde{c}}\}, \quad (4.241)$$

$$Q^{\mathrm{I}}_{e} = \sum_{\substack{(c,i) \in \mathcal{P}_e \\ c = \tilde{c}}} X^{\mathrm{I}}_{ci} + \left[\sum_{\substack{(c,i) \in \mathcal{P}_e \\ c \neq \tilde{c}}} X^{\mathrm{I}}_{ci} \right] \quad \forall\ e\,, \qquad (4.242)$$

$$X^{\mathrm{I}}_{\tilde{c}i} = 0 \quad \forall\ (\tilde{c}, i) \notin \bigcup_{e} \mathcal{P}_e\,, \qquad (4.243)$$

$$Q^{\mathrm{M}}_{f} = \sum_{\substack{(c,m) \in \mathcal{R}_f \\ c = \tilde{c}}} Y^{\mathrm{M}}_{cm} + \left[\sum_{\substack{(c,m) \in \mathcal{R}_f \\ c = \tilde{c}}} Y^{\mathrm{M}}_{cm} \right] \quad \forall\ f\,, \qquad (4.244)$$

and

$$Y^{\mathrm{M}}_{\tilde{c}m} = 0 \quad \forall\, (\tilde{c}, m) \notin \bigcup_f \mathcal{R}_f \,. \tag{4.245}$$

The **condition constraints** are the ones from the (AC) model.

$$\sum_{\tilde{m} \in \left\{ \tilde{m} \,\middle|\, \substack{\delta_{\tilde{c}mi} \leq \delta_{\tilde{c}\tilde{m}i} \ \forall\, i, \\ (\tilde{c},\tilde{m}) \in \bigcup_f \mathcal{R}_f} \right\}} Y^{\mathrm{M}}_{\tilde{c}\tilde{m}} \leq \prod_{i \in \{ i | \delta_{\tilde{c}mi} = 1 \}} (1 - \zeta_{\tilde{c}i})(1 - \eta_{\tilde{c}i}) Q^{\mathrm{C}}_{\tilde{c}} \quad \forall\, (\tilde{c}, m) \in \bigcup_f \mathcal{R}_f \tag{4.246}$$

$$X^{\mathrm{I}}_{\tilde{c}i} + (1 - \theta_{\tilde{c}i}) \sum_{m \in \left\{ m \,\middle|\, \substack{\delta_{\tilde{c}mi} = 1, \\ (\tilde{c},m) \in \bigcup_f \mathcal{R}_f} \right\}} Y^{\mathrm{M}}_{\tilde{c}m} \leq (1 - \zeta_{\tilde{c}i})(1 - \eta_{\tilde{c}i})(1 - \theta_{\tilde{c}i}) Q^{\mathrm{C}}_{\tilde{c}} \quad \forall\, (\tilde{c}, i) \in \bigcup_e \mathcal{P}_e \tag{4.247}$$

$$\sum_{\tilde{m} \in \left\{ \tilde{m} \,\middle|\, \substack{\delta_{\tilde{c}mi} \leq \delta_{\tilde{c}\tilde{m}i} \ \forall\, i, \\ (\tilde{c},\tilde{m}) \in \{1, \ldots, \overline{M}_{\tilde{c}}\}} \right\}} \left(Y^{\mathrm{M}}_{\tilde{c}\tilde{m}} + \sum_r Y^{\mathrm{R}}_{\tilde{c}\tilde{m}r} \right) \leq \prod_{i \in \{ i | \delta_{\tilde{c}mi} = 1 \}} (1 - \zeta_{\tilde{c}i} \iota_{\tilde{c}i}) Q^{\mathrm{C}}_{\tilde{c}} \quad \forall\, m \in \{1, \ldots, \overline{M}_{\tilde{c}}\} \tag{4.248}$$

$$\sum_r \left(\sum_{m=1}^{\overline{M}_{\tilde{c}}} \delta_{\tilde{c}mi} Y^{\mathrm{R}}_{\tilde{c}mr} + X^{\mathrm{R}}_{\tilde{c}ir} \right) + \sum_{m=1}^{\overline{M}_{\tilde{c}}} \delta_{\tilde{c}mi} Y^{\mathrm{M}}_{\tilde{c}m} + X^{\mathrm{I}}_{\tilde{c}i} \leq (1 - \zeta_{\tilde{c}i} \iota_{\tilde{c}i}) Q^{\mathrm{C}}_{\tilde{c}} \quad \forall\, i \in \{1, \ldots, \bar{I}_{\tilde{c}}\} \tag{4.249}$$

The **purity constraints** and the **limit constraints** are taken unchanged.

$$\begin{aligned} \omega_r Q^{\mathrm{R}}_r \leq & \sum_{i=1}^{\bar{I}_{\tilde{c}}} \pi_{\tilde{c}ir} w_{\tilde{c}i} \left(X^{\mathrm{R}}_{\tilde{c}ir} + \sum_{m=1}^{\overline{M}_{\tilde{c}}} \delta_{\tilde{c}mi} Y^{\mathrm{R}}_{\tilde{c}mr} \right) \\ & + \left[\sum_{c \neq \tilde{c}} \sum_{i=1}^{\bar{I}_c} \pi_{cir} w_{ci} \left(X^{\mathrm{R}}_{cir} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y^{\mathrm{R}}_{cmr} \right) \right] \quad \forall\, r \end{aligned} \tag{4.250}$$

$$X^{\mathrm{R}}_{\tilde{c}ir} = 0 \quad \forall\ (\tilde{c}, i) \in \mathcal{H}, r \tag{4.251}$$

$$X^{\mathrm{D}}_{\tilde{c}id} = 0 \quad \forall\ (\tilde{c}, i) \in \mathcal{H}, d \in \{1\} \tag{4.252}$$

$$Y^{\mathrm{R}}_{cmr} = 0$$
$$\forall\ (c,m) \in \left\{(c,m)|c = \tilde{c}, \delta_{cmi} = 1, (c,i) \in \mathcal{H}, m \in \{1, \dots, \overline{M}_c\}\right\}, r \tag{4.253}$$

$$Y^{\mathrm{D}}_{cmd} = 0$$
$$\forall\ (c,m) \in \left\{(c,m)|c = \tilde{c}, \delta_{cmi} = 1, (c,i) \in \mathcal{H}, m \in \{1, \dots, \overline{M}_c\}\right\}, d \in \{1\} \tag{4.254}$$

$$\underline{Q}^{\mathrm{C}}_{\tilde{c}} \le Q^{\mathrm{C}}_{\tilde{c}} \le \overline{Q}^{\mathrm{C}}_{\tilde{c}} \tag{4.255}$$

$$\underline{Q}^{\mathrm{I}}_{e} \le Q^{\mathrm{I}}_{e} \le D^{\mathrm{I}}_{e} \quad \forall\ e \tag{4.256}$$

$$\underline{Q}^{\mathrm{M}}_{f} \le Q^{\mathrm{M}}_{f} \le D^{\mathrm{M}}_{f} \quad \forall\ f \tag{4.257}$$

$$\underline{Q}^{\mathrm{R}}_{r} \le Q^{\mathrm{R}}_{r} \le D^{\mathrm{R}}_{r} \quad \forall\ r \tag{4.258}$$

$$\underline{Q}^{\mathrm{D}}_{d} \le Q^{\mathrm{D}}_{d} \le \overline{Q}^{\mathrm{D}}_{d} \quad \forall\ d \tag{4.259}$$

$$t^{\mathrm{J}}_{\tilde{c},1} Q^{\mathrm{C}}_{\tilde{c}} - \sum_{m=1}^{\overline{M}_{\tilde{c}}} t^{\mathrm{J}}_{\tilde{c}m} \left(Y^{\mathrm{M}}_{\tilde{c}m} + \sum_r Y^{\mathrm{R}}_{\tilde{c}mr} + \sum_d Y^{\mathrm{D}}_{\tilde{c}md} \right) \le$$
$$\bar{L} - \left[\sum_{c \ne \tilde{c}} \left(t^{\mathrm{J}}_{c,1} Q^{\mathrm{C}}_{c} - \sum_{m=1}^{\overline{M}_c} t^{\mathrm{J}}_{cm} \left(Y^{\mathrm{M}}_{cm} + \sum_r Y^{\mathrm{R}}_{cmr} + \sum_d Y^{\mathrm{D}}_{cmd} \right) \right) \right] \tag{4.260}$$

In addition, the limit constraints are extended by

$$Y^{\mathrm{M}}_{\tilde{c}m} \le \begin{cases} \left\lfloor \dot{Y}^{\mathrm{M}}_{\tilde{c}m} \right\rfloor + 1 & \dot{Y}^{\mathrm{M}}_{\tilde{c}m} > 0 \\ 0 & \text{else} \end{cases} \quad \forall\ m \in \{1, \dots, \overline{M}_{\tilde{c}}\}, \tag{4.261}$$

$$\sum_r Y^{\mathrm{R}}_{\tilde{c}mr} \le \begin{cases} \left\lfloor \sum_r \dot{Y}^{\mathrm{R}}_{\tilde{c}mr} \right\rfloor + 1 & \sum_r \dot{Y}^{\mathrm{R}}_{\tilde{c}mr} > 0 \\ 0 & \text{else} \end{cases} \quad \forall\ m \in \{1, \dots, \overline{M}_{\tilde{c}}\}, \tag{4.262}$$

$$\sum_d Y^{\mathrm{D}}_{\tilde{c}md} \le \begin{cases} \left\lfloor \sum_d \dot{Y}^{\mathrm{D}}_{\tilde{c}md} \right\rfloor + 1 & \sum_d \dot{Y}^{\mathrm{D}}_{\tilde{c}md} > 0 \\ 0 & \text{else} \end{cases} \quad \forall\, m \in \{1, \ldots, \overline{M}_{\tilde{c}}\}\,, \tag{4.263}$$

$$X^{\mathrm{I}}_{\tilde{c}i} \le \begin{cases} \left\lfloor \dot{X}^{\mathrm{I}}_{\tilde{c}i} \right\rfloor + 1 & \dot{X}^{\mathrm{I}}_{\tilde{c}i} > 0 \\ 0 & \text{else} \end{cases} \quad \forall\, i \in \{1, \ldots, \bar{I}_{\tilde{c}}\}\,, \text{ and} \tag{4.264}$$

$$\sum_r X^{\mathrm{R}}_{\tilde{c}ir} \le \begin{cases} \left\lfloor \sum_r \dot{X}^{\mathrm{R}}_{\tilde{c}ir} \right\rfloor + 1 & \sum_r \dot{X}^{\mathrm{R}}_{\tilde{c}ir} > 0 \\ 0 & \text{else} \end{cases} \quad \forall\, i \in \{1, \ldots, \bar{I}_{\tilde{c}}\} \tag{4.265}$$

using the values from the continuous solution $\dot{Y}^{\mathrm{M}}_{\tilde{c}m}$, $\dot{Y}^{\mathrm{R}}_{\tilde{c}mr}$, $\dot{Y}^{\mathrm{D}}_{\tilde{c}md}$, $\dot{X}^{\mathrm{I}}_{\tilde{c}i}$, and $\dot{X}^{\mathrm{R}}_{\tilde{c}ir}$. A limitation of item disposal is not necessary in addition to the constraints above. Lastly, the domain of the variables is

$$X^{\mathrm{I}}_{\tilde{c}i}, X^{\mathrm{R}}_{\tilde{c}ir}, X^{\mathrm{D}}_{\tilde{c}id}, Y^{\mathrm{M}}_{\tilde{c}m}, Y^{\mathrm{R}}_{\tilde{c}mr}, Y^{\mathrm{D}}_{\tilde{c}md} \in \mathbb{Z}^* \qquad \forall\, i \in \{1, \ldots, \bar{I}_{\tilde{c}}\}, m \in \{1, \ldots, \overline{M}_{\tilde{c}}\}, r, d \tag{4.266}$$

for the focussed core $\tilde{c}$.

When we start with core 1 for fixing the integer variables, the above model with $\tilde{c} = 1$ is solved. The solution for core 1 is feasible. Thus, the integral values replace the continuous values of core 1. The overall profit (i.e., for all three cores together) with integer variables for the first core reduces to 33,003.67 €. Applying the model on core 2 with the updated quantities also results in a feasible solution. After integrating the integral values into the continuous values leads to an overall profit of 31,707.89 €. The third core is optimised in the same way. A feasible solution is gained and merged into the continuous solution.[53] Finally, the resulting profit for the integral solution is 30,564.1 €. This is less than what has been achieved with the proposal in Sect. 4.5.1, but the solution time is only 29.48 s, where 29.42 s thereof are necessary to solve the (AC) model first (delivering an infeasible solution) and afterwards the continuous flexible planning model.

With this relative fast solving of the model (AC), the cores do not have to be solved individually. It is also possible to solve model (AC) with bounds based on the solution of the continuous flexible planning. Doing it as described above, i.e., allowing the decision variables a value of maximal the next bigger integer value from the continuous solution, all three cores can be optimised simultaneously. The resulting solution is gained in 0.11 s and has a profit of 30,619 €. In total, the solution time is 29.53 s with a profit

[53] The core, distribution, recycling, and disposal graph variables of the continuous are of course not updated.

of 30,619 €, which is faster and better than the approach in Sect. 4.5.1. Again, the proposed approaches in this section are just ideas to start developing methods of solving the flexible planning problem faster and most likely suboptimal. From the presented approaches this last one is probably very promising for further research.

4.6 Concluding remarks

In this chapter the considerations from the complete disassembly planning of the first chapter were extended to the flexible disassembly planning. With complete disassembly planning there exists no option of disassembling one core into different modules and items. However, the flexible disassembly gives the decision maker the freedom to gain different sets of modules and items from different units of the same core. This is an extension to the incomplete disassembly, because incomplete disassembly indicates that groups of items can be kept together and are not fully disassembled. If the disassembly sequence is identical for each unit of a core, such an incomplete disassembly problem can be planned with complete disassembly approaches. Not until the flexible planning the modelling of the disassembly depth in combination with the quantity planning is necessary, to gain a solution with best profit achievable.

From these two planning problems, the disassembly depth planning is generally more complex than the quantity planning. Thereby, graphical approaches like the and/or graph are very often used. Hereby, the problem is visualised with such a graph and usually the solution finding is based on such a graph. Furthermore, the large number of sequences is reduced by geographical, technical, and topological constraints. But still, for cores consisting of many items the graphs become huge, in general. When sequence dependent disassembly cost is accounted for, each feasible sequence has to be considered. For sequence independent disassembly cost many sequences could lead to the same disassembly state with the same cost. Thereby, a disassembly state represents the result of the disassembly process, i.e., which items and modules exist in the end.

In this work sequence independent disassembly cost are assumed. The reason is that they are truly sequence independent or that the differences are negligible, because the disassembling is done mostly manually and for the planning the approximation is sufficient. Because of this assumption the disassembly state graph is the basis for the flexible planning. It contains the necessary item and module information for the planning.

In addition, the expected condition of incoming cores is included in the planning, too. This is also considered in the complete disassembly planning, but for the flexible planning this causes a major increase of complexity.[54] A mathematical model to determine the optimal solution is developed. Thereby, an optimal solution is characterised by the maximum profit that can be achieved with quantities to acquire of several cores. The considered constraints are demand of reusable items and modules as well as recyclable items and modules, other acquisition and distribution limitations, purity requirements for recycling quantities, hazardous items, the expected condition of items of cores, damaging of items, and the available labour time. Depending on the condition (i.e., is it genuine, functioning, and/or of wrong material) the usage of items and modules is limited (distribution for reuse, recycling, and/or disposal).

The optimal planning is demonstrated with a numerical example comprising three cores, each consisting of eight items and 50 modules. The optimal solution of the planning model is gained and evaluated in comparison to a solution with no extra module demand, the best two-stage approach, and with limited number of disassembly states. With the resulting solution (i.e., quantities of cores, items, and modules) the corresponding disassembly states and assignments of the incoming units of cores need to be determined. For specifying the required disassembly states and how often a disassembly ending in this state is necessary (i.e., state quantity) an approach based on linear programming and an alternative one are developed. Given these state quantities, the condition of a unit of an incoming core, and the information about the usage specific planned quantities in combination with the expected quantities of items and modules, the assignment of the units to a particular disassembly state as well as the usage of the resulting items and modules is gained. The assignment can be done with mathematical programming for the batch of all units or based on priority values for a successive assignment, i.e., unit by unit. The latter has the benefit that not all units need to be tested prior the disassembly of the first unit.

The flexible disassembly planning is rather complex, which is easily observable by the relative long solution time of the model. Therefore, a few ideas on how to achieve faster solving with the aspects mentioned in this work are discussed. However, a faster solving usually comes along with a suboptimal solution. The trade-off the decision maker has to face is the solving time and the appearing gap to the optimal solution. Based on the

[54] The problem is $\mathcal{NP}$-complete especially since the integer planning is $\mathcal{NP}$-complete. Cf. Schrijver (2000): *Theory of LP and ILP*, p. 245. Besides this complexity definition the model size, i.e., the number of variables and constraints, is meant by the term complexity.

illustration of the ideas with the numerical example the approach that starts with the continuous solution of the flexible planning with a subsequent solving of a model with alternative condition constraints to find integer values is favoured. Admittedly, further research and evaluation is recommended. In addition, alternative approaches of finding good solutions fast (e.g., specific heuristics or meta-heuristics) can be developed and evaluated with the optimal flexible disassembly planning.

Chapter 5
Résumé

This work considers several aspects of the disassembly planning. The complete disassembly planning problem is the first aspect. It is the basis for all further extensions that are considered in this work. The planning aims at determining the optimal quantities of cores to acquire, items to distribute, material to recycle, and waste to dispose of in order to gain the maximal profit. Thereby, the profit calculation includes revenues of item distribution and material recycling. On the other hand, the cost covers acquisition, disassembly, and disposal cost. The planning problem comprises more than one core, which makes the inclusion of commonality and multiplicity necessary. Moreover, the planning aims at meeting a given demand for item distribution and material recycling. Thereby, it is assumed that only whole cores are acquired and supplied.

Furthermore, new aspects are incorporated like the special treatment for hazardous items, the material purity for recycling, the core condition, as well as further supply and distribution limits. The usage of hazardous items is restricted only to the options of distribution (if demanded) and hazardous waste disposal. In addition, the material purity requested by some recycling companies is explicitly considered in the planning. In general, the disassembling companies face uncertainties about the condition of the acquired cores. This includes defective and replaced items, even with the wrong material. Depending on the condition, the usage of the corresponding item is limited to certain usage options only. Thereby, the information about the conditions is given in form of probabilities per item and core, whereupon it is assumed that they are identically and independently distributed. The considered conditions are whether an item of a core is functioning or defective, genuine or non-genuine, and of the right or wrong material.

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In addition, the possible damaging of an item during the disassembly process is also integrated. These probabilities are used in a deterministic planning approach. The last added aspect is that of limits. One considered limit is that of the available labour time. The others refer to the acquisition and usage. This means that for all interfaces to external companies a lower and an upper limit can be set. Thereby, the given demand is equivalent to an upper limit for the corresponding item or material. If the lower limit equals that of the demand, the approach is equivalent to the disassembly-to-order planning. All the above mentioned is consistently integrated in every planning problem presented in this work, with the exception of the labour time in the multi-period planning.

The providing of limits can be used to mark intervals where the price or unit cost is identical for each unit, i.e., a fixed price or unit cost. But sometimes it might be necessary to find the optimal quantity where prices and unit cost change depending on the quantity without limiting the values to a certain interval. This way, the market behaviour can be explicitly integrated in the planning. In this work, two price-quantity dependencies are discussed.[1] The first is the case with linear dependencies between the quantity and the corresponding price or unit cost for item reuse, material recycling, core acquisition, and disposal. Hence, all interfaces to business partners are considered. Following the argumentation of the price and unit-cost development depending on the quantity, it is shown that the profit function is concave, which indicates a unique optimum. This can be determined with solution approaches for quadratic optimisation problems (e.g., the gradient projection method) and with standard solver software like GUROBI.

The second case is more general and the first case is a special one of the second. Here, piecewise linear price-quantity dependencies can be modelled. With this generalisation, more specific price-quantity dependencies and even better approximations of non-linear dependencies can be modelled. The drawback, to the best of our knowledge, is a missing solution method for the resulting problem. Therefore, a solution method is developed that can be used to solve the problem in the domain of real numbers. This method is in a further step enhanced to solve mixed integer problems with a concave objective function that also contains linear summands. The focus in this work is to achieve the solving with standard solver software for linear and quadratic problems, because they already have good performance and overcome the numerical issues when applying theoretical approaches to limited precision computers.

[1] The term price-quantity dependency always includes the unit-cost-quantity dependencies, too.

A further extension of the initial complete disassembly planning problem is the consideration of multiple periods. But this is not just a sequence of single periods planned together. A storage for cores and one for items is introduced. Thereby, the item storage contains a section for the storage of hazardous items. The storage spaces are limited and each storage has its particular cost. Furthermore, contractual aspects are integrated. This not only means the compliance with contracted quantities and purity, but also contractual penalties in the case that the quantities are not met. However, a core specific acquisition level is guaranteed. The contracting options with respect to the considered periods for the recycling material purity are discussed. We derive that the purity should be met in every period. Alternatively, past periods could be integrated, too. Moreover, the multi-period planning is realised with a rolling horizon planning, which incorporates the business practice in a way that an infinitely on-going business is assumed and that the inventory is not forced to a given value at the end of the planning horizon. The results of the planning are evaluated with a total planning of the setting of the numerical example. This evaluation shows a good performance of the approach for the given example.

After these considerations of the complete disassembly planning, the focus is shifted to the flexible disassembly planning. This is a generalisation from complete to incomplete disassembly. We chose the disassembly state graph as appropriate tool to identify paths to find possible modules and items. This allows the disassembly planning of cores with an arbitrary structure. If the decision maker picks only one of the possible states of the disassembly state graph a priori, the planning can afterwards be conducted with a complete disassembly planning approach. On the other hand, if for each core more than one different disassembly state can be selected, we denote the planning as flexible disassembly planning. With this given possibility, modules and module constituent items can be demanded separately. This is possible, because a fraction of the quantity of a core can be disassembled to one state where the module is gained and another fraction is disassembled to another state where the module is further separated into items.

In this approach the aspects of the basic model are incorporated, too. With respect to the modelling of the core condition, the model to solve becomes particularly large. The modelling is graph based with one graph representing the various item conditions of a core (we call it core graph) and three further graphs for the usage options (we call them distribution, recycling, and disposal graph). Using a numerical example, the benefit of the flexible planning is shown in comparison to the complete disassembly as well as the best two-stage approach. In addition, the effect of limiting the allowed number of disassembly states per core and for all cores is discussed.

The determined optimal quantities with regard to the profit cannot be used directly by the disassembly person or automated disassembly system. In a first step the necessary states are ascertained. Given these and the planned quantities for the usage options, for each single unit of a core the concrete disassembly state is assigned and the usage options for the resulting items and modules are specified. Hereby, a practical handling guidance is developed. Moreover, a comprehensive framework including the planning and disassembly guideline generation for the flexible disassembly planning problem is developed.

With the examinations in this work the initially formulated research questions Q1–Q4 in Sect. 1.2 should now be answered. Question one—regarding the incorporation of price-quantity dependencies—can be positively answered; with the limitation that only linear and piecewise linear dependencies for interfaces to all external partners were considered. Nonetheless, the piecewise linear dependencies can be used to approximate other non-linear dependencies to some extent. The second question focusses on the contracting support with dynamic planning. Firstly, contractual aspects like penalty cost and a guarantee level have been introduced. Afterwards, a framework has been developed that considers future period information in the planning of the period in focus. The framework also includes a decision support for the contracting in future periods, thus a possible contracting support is developed.

In order to answer question three about the usefulness of flexible planning, a corresponding planning model is generated. It could be shown that the flexible disassembly planning is beneficial. However, the planning leads to a large sized model. The resulting optimal solution can also be transferred to a concrete disassembling guideline, which makes the planning applicable for practice and answers the fourth question positively.

Even though the research questions could be sufficiently answered, some drawbacks exist and further research is necessary. To start with, we pick up on the last mentioned aspect: the flexible disassembly planning. The developed approach causes a rather large sized model, which makes the planning of real sized objects unmanageable. To overcome this problem either an alternative solution approach is to be developed or a heuristic solution approach should be found. First ideas for speeding up the solving with the methods developed in this work are already discussed. These include using the optimal real valued solution to find an integral solution for each core separately. This transforms the problem to some degree into single core problems. This approach delivers good results in a moderate time. On the contrary, fixing the solution time results in worse objective values than the aforementioned approach. The reason is the finding of a feasible solution is already time consuming.

The reduction of the disassembly state graph by undesirable states helps mainly in reducing the number of modules and states, but has marginal influence on the solution time if not negative influence. This nicely illustrates that the size of the model does not necessary correlates with the solution time. A last sketched idea is the use of the optimal real valued solution and determine the integral solution for each core separately or jointly, but with the help of different condition constraints. These constraints partly allow infeasible solutions, but the resulting model is solved much faster. If the gained solution is feasible the solution is used and otherwise the correct condition constraints must be used. This latter approach is the one to favour according to the analysis with the numerical example in this work. However, this result is not sound such that further research is highly advisable in this area.

Furthermore, the assignment of the incoming cores to the appropriate disassembly states and usage options is presented for a prior batch testing and a successive alternative. The latter is priority based and should be validated with further test sets. In addition, the usage option assignment might be extended to the specific material recycling boxes and disposal bins. This extension is straightforward, because the planned quantities of items and modules of the cores together with the assignment of the items and modules to recycling and disposal is given. The only problem that might occur is that of a different condition realisation compared to the expected one. However, this problem already arises in the assignment stage, even in the assignment based on the batch testing. Here, options for a preferably small deviation from the optimal solution are sought.

With regard to the price-quantity dependencies, a consideration of other often used dependencies formulated with isoelastic, exponential, and algebraic functions are of interest. In addition, the consistent integration of condition considerations, purity requirements, and hazardous items in other research areas, like disassembly sequencing, scheduling, and line balancing, should be forced, where applicable. Lastly, the integration of the disassembly states for the flexible disassembly planning is a step towards a combination of disassembly sequencing and disassembly (or disassembly-to-order) planning. This could be pushed even further and towards an automated disassembly, because of presumably growing quantities of products to disassembly to gain valuable materials and to recover the value added in products.

Appendix A
Appendix to Chapter 2

Infeasible solution of Kongar / Gupta DTO system

The optimal solution presented in the paper by Kongar / Gupta is not feasible.[1] This can be shown with two examples. Taking the obtained value of $\lambda = 0.417$ and the value of $g_3 = NRU + NRC = 47{,}087$ and using constraint (41) of the original paper, the inequality

$$0.20\lambda \leq 0.001(g_3 - 48{,}000) + 0.8 \tag{A.1}$$

is not fulfilled. Furthermore, the level of achievement of the membership function μ_3 (see constraint (40) of the original paper) results in $\mu_3(g_3 = 47{,}087) = 0.2175$ which does not equal the stated value of 0.1044.

Another unsatisfied constraint is that of inequality (23) of the original paper. A weaker formulation of the constraint is

$$\sum_j D_j \left(1 + \alpha_j + \beta_j + \gamma_j\right) \leq \sum_j \sum_i X_{ij}. \tag{A.2}$$

Using the given data for D_j, α_j, β_j, and γ_j the left side of the inequality equals 10,530. The right side equals NRU (see Eq. (29) of the original paper). But, the value of NRU is not explicitly given in the results. But NRU can be calculated by using $NRU + NRC = 47{,}087$ and $NRC = 37{,}337$, so that $NRU = 9{,}750$. This leads to an unmet inequality (A.2), because the left side is greater than the right side. Since this weaker constraint is not satisfied the more specific constraint (23) cannot be fulfilled either.

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[1] Cf. Kongar / Gupta (2006b): *Disassembly to order*, pp. 552–559.

C. Ullerich, *Advanced Disassembly Planning*,
DOI 10.1007/978-3-658-03118-3, © Springer Fachmedien Wiesbaden 2014

Appendix B
Appendix to Chapter 3

B.1 Basic model with destructive and non-destructive disassembly cost differentiation

The basic model in Sect. 3.1.2 does not include a differentiation in destructive and non-destructive disassembly cost. Following the work by KONGAR / GUPTA an inclusion of cost differentiation leads to the following model.[1] The model structure is depicted in Fig. B.1. As can be seen, the variables X^{N} and X^{F} are added for non-destructively and destructively, respectively, gained items. Furthermore, several additional constraints need to be added as well and more data for the planning must be collected. The calculation of the revenues and the profit is identical to the basic model.

$$\text{Maximise} \quad P = R - C \tag{B.1}$$

$$R = \sum_{e} r_e^{\mathrm{I}} Q_e^{\mathrm{I}} + \sum_{r} r_r^{\mathrm{R}} Q_r^{\mathrm{R}} \tag{B.2}$$

But the cost is expanded by the differentiated cost factors c_{ci}^{JN} and c_{ci}^{JF} that represent the disassembly cost for an item with non-destructive and destructive disassembly, respectively.

$$C = \sum_{c} c_c^{\mathrm{A}} Q_c^{\mathrm{C}} + \sum_{c} \sum_{i=1}^{\bar{I}_c} \left(c_{ci}^{\mathrm{JN}} X_{ci}^{\mathrm{N}} + c_{ci}^{\mathrm{JF}} X_{ci}^{\mathrm{F}} \right) + \sum_{d} c_d^{\mathrm{D}} Q_d^{\mathrm{D}} \tag{B.3}$$

[1] Cf. KONGAR / GUPTA (2006b): *Disassembly to order*.

C. Ullerich, *Advanced Disassembly Planning*,
DOI 10.1007/978-3-658-03118-3, © Springer Fachmedien Wiesbaden 2014

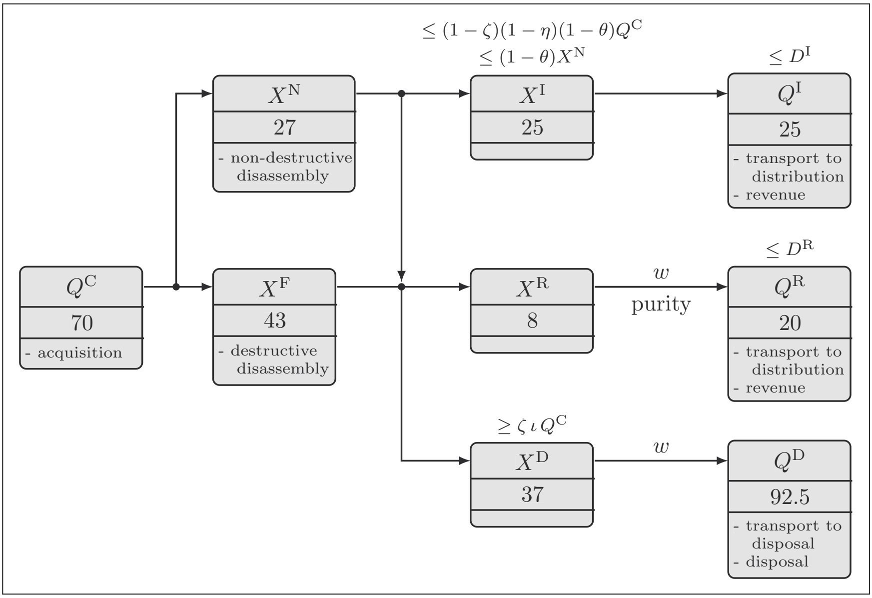

Fig. B.1 Basic model structure with disassembly differentiation

Item flow constraints The flow of items through the two disassembly options requires two new constraints. The first one assures that all cores are disassembled in one of the two ways.

$$Q_c^{\mathrm{C}} = X_{ci}^{\mathrm{N}} + X_{ci}^{\mathrm{F}} \quad \forall\, c, i \in \{1, \ldots, \bar{I}_c\} \tag{B.4}$$

The second constraint limits the items to reuse additionally to the ones that are non-destructively disassembled and undamaged during the process.

$$X_{ci}^{\mathrm{I}} \leq (1-\theta_{ci})\, X_{ci}^{\mathrm{N}} \quad \forall\, (c,i) \in \bigcup_e \mathcal{P}_e \tag{B.5}$$

The remaining item flow constraints are kept as are most of the constraints.

$$Q_c^{\mathrm{C}} = X_{ci}^{\mathrm{I}} + \sum_r X_{cir}^{\mathrm{R}} + \sum_d X_{cid}^{\mathrm{D}} \quad \forall\, c, i \in \{1, \ldots, \bar{I}_c\} \tag{B.6}$$

$$Q_r^{\mathrm{R}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{cir}^{\mathrm{R}} \quad \forall\, r \tag{B.7}$$

$$Q_d^{\mathrm{D}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{cid}^{\mathrm{D}} \quad \forall\ d \tag{B.8}$$

$$Q_e^{\mathrm{I}} = \sum_{(c,i)\in\mathcal{P}_e} w_{ci} X_{ci}^{\mathrm{I}} \quad \forall\ e \tag{B.9}$$

$$X_{ci}^{\mathrm{I}} = 0 \quad \forall\ (c,i) \notin \bigcup_e \mathcal{P}_e \tag{B.10}$$

Condition constraints

$$\sum_d X_{cid}^{\mathrm{D}} \geq \zeta_{ci} \iota_{ci} Q_c^{\mathrm{C}} \quad \forall\ c, i \in \{1,\ldots,\bar{I}_c\} \tag{B.11}$$

$$X_{ci}^{\mathrm{I}} \leq (1-\zeta_{ci})(1-\eta_{ci})(1-\theta_{ci}) Q_c^{\mathrm{C}} \quad \forall\ (c,i) \in \bigcup_e \mathcal{P}_e \tag{B.12}$$

The second of the optional constraints (i.e., Eq. (3.11)) is skipped here.

Purity constraints No changes are necessary for the purity considerations.

$$\omega_r Q_r^{\mathrm{R}} \leq \sum_c \sum_{i=1}^{\bar{I}_c} \pi_{cir} w_{ci} X_{cir}^{\mathrm{R}} \quad \forall\ r \tag{B.13}$$

$$X_{cir}^{\mathrm{R}} = 0 \quad \forall\ (c,i) \in \mathcal{H}, r \tag{B.14}$$

$$X_{cid}^{\mathrm{D}} = 0 \quad \forall\ (c,i) \in \mathcal{H}, d \in \{1\} \tag{B.15}$$

Limits constraints

$$\underline{Q}_c^{\mathrm{C}} \leq Q_c^{\mathrm{C}} \leq \overline{Q}_c^{\mathrm{C}} \quad \forall\ c \tag{B.16}$$

$$\underline{Q}_e^{\mathrm{I}} \leq Q_e^{\mathrm{I}} \leq D_e^{\mathrm{I}} \quad \forall\ e \tag{B.17}$$

$$\underline{Q}_r^{\mathrm{R}} \leq Q_r^{\mathrm{R}} \leq D_r^{\mathrm{R}} \quad \forall\ r \tag{B.18}$$

$$\underline{Q}_d^{\mathrm{D}} \leq Q_d^{\mathrm{D}} \leq \overline{Q}_d^{\mathrm{D}} \quad \forall\ d \tag{B.19}$$

If the disassembly cost is differentiated, it is obvious that the disassembly times differ, too. Especially, because the manual labour is the main driver for the disassembly cost. Thus, the aggregated times t_c^{J} are substituted by t_{ci}^{JN} and t_{ci}^{JF} as with the cost in Eq. (B.3).

$$\sum_c \sum_{i=1}^{\bar{I}_c} \left(t_{ci}^{\mathrm{JN}} X_{ci}^{\mathrm{N}} + t_{ci}^{\mathrm{JF}} X_{ci}^{\mathrm{F}} \right) \leq \bar{L} \tag{B.20}$$

Because of the (equality) Eqs. (B.4) and (B.6), one of the new variables is completely explained. Thus, only one of them (e.g., X^{N}_{ci}) is added to the list of variables with the integer domain.

$$X^{\mathrm{I}}_{ci}, X^{\mathrm{N}}_{ci}, X^{\mathrm{R}}_{cir}, X^{\mathrm{D}}_{cid} \in \mathbb{Z}^{*} \quad \forall\, c, i \in \{1, \ldots, \bar{I}_c\}, r, d\,. \tag{B.21}$$

The accessory properties for the disassembly differentiation cause that the number of decision variables and the number of constraints increase by $\sum_c \bar{I}_c$ and $\sum_c \bar{I}_c + |\bigcup_e \mathcal{P}_e|$, respectively, compared to the model in appendix B.2.

B.2 Compact basic model formulation

The model presented here is equivalent to the one in Sect. 3.1.2. The difference is that the variables, which are completely explained by some constraints, are substituted by the explaining term. This starts already with the objective function. The variables P, R, and C are the first ones to substitute. Within these equations the variables Q^{C}_c, Q^{I}_e, Q^{R}_r, and Q^{D}_d need to be substituted, too. The resulting objective is

$$\begin{aligned}\text{Maximise}\quad & \sum_e r^{\mathrm{I}}_e \sum_{(c,i)\in\mathcal{P}_e} w_{ci} X^{\mathrm{I}}_{ci} + \sum_r r^{\mathrm{R}}_r \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X^{\mathrm{R}}_{cir} \\ & - \sum_c \left(c^{\mathrm{A}}_c + c^{\mathrm{J}}_c\right)\left(X^{\mathrm{I}}_{c,1} + \sum_r X^{\mathrm{R}}_{c,1,r} + \sum_d X^{\mathrm{D}}_{c,1,d}\right) \\ & - \sum_d c^{\mathrm{D}}_d \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X^{\mathrm{D}}_{cid}\,. \end{aligned} \tag{B.22}$$

Analogously, the variables Q^{C}_c, Q^{I}_e, Q^{R}_r, and Q^{D}_d are substituted by the corresponding terms in all other constraints. The constraints are also arranged in groups for a better comparison.

Item flow constraints

$$X^{\mathrm{I}}_{c,1} + \sum_r X^{\mathrm{R}}_{c,1,r} + \sum_d X^{\mathrm{D}}_{c,1,d} = X^{\mathrm{I}}_{ci} + \sum_r X^{\mathrm{R}}_{cir} + \sum_d X^{\mathrm{D}}_{cid} \quad \forall\, c, i \in \{2,\ldots,\bar{I}_c\} \tag{B.23}$$

$$X^{\mathrm{I}}_{ci} = 0 \quad \forall\, (c,i) \notin \bigcup_e \mathcal{P}_e \tag{B.24}$$

Condition constraints

$$\sum_d X^{\mathrm{D}}_{cid} \geq \zeta_{ci}\iota_{ci}\left(X^{\mathrm{I}}_{c,1} + \sum_r X^{\mathrm{R}}_{c,1,r} + \sum_d X^{\mathrm{D}}_{c,1,d}\right) \quad \forall\, c, i \in \{1,\ldots,\bar{I}_c\} \tag{B.25}$$

$$X^{\mathrm{I}}_{ci} \leq (1-\zeta_{ci})(1-\eta_{ci})(1-\theta_{ci})\left(X^{\mathrm{I}}_{c,1} + \sum_r X^{\mathrm{R}}_{c,1,r} + \sum_d X^{\mathrm{D}}_{c,1,d}\right) \quad \forall\, (c,i) \in \bigcup_e \mathcal{P}_e \tag{B.26}$$

The redundant Eq. (3.11) is neglected here.

Purity constraints

$$\omega_r \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{cir}^{\mathrm{R}} \leq \sum_c \sum_{i=1}^{\bar{I}_c} \pi_{cir} w_{ci} X_{cir}^{\mathrm{R}} \quad \forall\ r \tag{B.27}$$

$$X_{cir}^{\mathrm{R}} = 0 \quad \forall\ (c,i) \in \mathcal{H}, r \tag{B.28}$$

$$X_{cid}^{\mathrm{D}} = 0 \quad \forall\ (c,i) \in \mathcal{H}, d \in \{1\} \tag{B.29}$$

Limits constraints

$$\underline{Q}_c^{\mathrm{C}} \leq X_{c,1}^{\mathrm{I}} + \sum_r X_{c,1,r}^{\mathrm{R}} + \sum_d X_{c,1,d}^{\mathrm{D}} \leq \overline{Q}_c^{\mathrm{C}} \quad \forall\ c \tag{B.30}$$

$$\underline{Q}_e^{\mathrm{I}} \leq \sum_{(c,i)\in\mathcal{P}_e} w_{ci} X_{ci}^{\mathrm{I}} \leq D_e^{\mathrm{I}} \quad \forall\ e \tag{B.31}$$

$$\underline{Q}_r^{\mathrm{R}} \leq \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{cir}^{\mathrm{R}} \leq D_r^{\mathrm{R}} \quad \forall\ r \tag{B.32}$$

$$\underline{Q}_d^{\mathrm{D}} \leq \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{cid}^{\mathrm{D}} \leq \overline{Q}_d^{\mathrm{D}} \quad \forall\ d \tag{B.33}$$

$$\sum_c t_c^{\mathrm{J}} \left(X_{c,1}^{\mathrm{I}} + \sum_r X_{c,1,r}^{\mathrm{R}} + \sum_d X_{c,1,d}^{\mathrm{D}} \right) \leq \bar{L} \tag{B.34}$$

$$X_{ci}^{\mathrm{I}}, X_{cir}^{\mathrm{R}}, X_{cid}^{\mathrm{D}} \in \mathbb{Z}^* \quad \forall\ c, i \in \{1, \ldots, \bar{I}_c\}, r, d \tag{B.35}$$

Once the (optimal) solution is available, the values of the substituted variables can be determined by the following equations.

$$Q_c^{\mathrm{C}} = X_{c,1}^{\mathrm{I}} + \sum_r X_{c,1,r}^{\mathrm{R}} + \sum_d X_{c,1,d}^{\mathrm{D}} \quad \forall\ c \tag{B.36}$$

$$Q_r^{\mathrm{R}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{cir}^{\mathrm{R}} \quad \forall\ r \tag{B.37}$$

$$Q_d^{\mathrm{D}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{cid}^{\mathrm{D}} \quad \forall\ d \tag{B.38}$$

$$Q_e^{\mathrm{I}} = \sum_{(c,i)\in\mathcal{P}_e} w_{ci} X_{ci}^{\mathrm{I}} \quad \forall\ e \tag{B.39}$$

$$R = \sum_{e} r_e^{\mathrm{I}} Q_e^{\mathrm{I}} + \sum_{r} r_r^{\mathrm{R}} Q_r^{\mathrm{R}} \tag{B.40}$$

$$C = \sum_{c} \left(c_c^{\mathrm{A}} + c_c^{\mathrm{J}} \right) Q_c^{\mathrm{C}} + \sum_{d} c_d^{\mathrm{D}} Q_d^{\mathrm{D}} \tag{B.41}$$

$$P = R - C \tag{B.42}$$

B.3 Order of section optima

The first order derivative of the objective function is

$$\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s (\check{Q}_s - \check{Q}_{s-1}) + \hat{r}_{\tilde{s}} \left(2\,Q - \check{Q}_{\tilde{s}-1}\right) \tag{B.43}$$

for the section $\tilde{s}$, i.e., in the interval $\check{Q}_{\tilde{s}-1} \leq Q \leq \check{Q}_{\tilde{s}}$ (see Eq. (3.31)). To find the optimal Q the first order derivative must equal zero, i.e.,

$$\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s (\check{Q}_s - \check{Q}_{s-1}) + \hat{r}_{\tilde{s}} \left(2\,Q - \check{Q}_{\tilde{s}-1}\right) = 0\,. \tag{B.44}$$

Transforming the equation leads to the optimal solution

$$Q_{\tilde{s}}^{\mathrm{opt}} = \frac{1}{2}\,\check{Q}_{\tilde{s}-1} - \frac{1}{2\,\hat{r}_{\tilde{s}}} \left(\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right) \right) \tag{B.45}$$

for the section $\tilde{s}$. The optimal solution for the succeeding section $\tilde{s}+1$ is

$$Q_{\tilde{s}+1}^{\mathrm{opt}} = \frac{1}{2}\,\check{Q}_{\tilde{s}} - \frac{1}{2\,\hat{r}_{\tilde{s}+1}} \left(\bar{r} + \sum_{s=1}^{\tilde{s}} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right) \right)\,. \tag{B.46}$$

Comparing the two optimal solutions in Eqs. (B.45) and (B.46) of the succeeding sections $\tilde{s}$ and $\tilde{s}+1$, respectively, results in

$$Q_{\tilde{s}}^{\mathrm{opt}} \overset{?}{\lessgtr} Q_{\tilde{s}+1}^{\mathrm{opt}}\,. \tag{B.47}$$

The expression $\overset{?}{\lessgtr}$ shall indicate that we want to know which of the two relations holds. Transforming the expression leads to

$$0 \overset{?}{\lessgtr} Q_{\tilde{s}+1}^{\mathrm{opt}} - Q_{\tilde{s}}^{\mathrm{opt}} \tag{B.48}$$

$$\begin{aligned} 0 \overset{?}{\lessgtr} &\frac{1}{2}\,\check{Q}_{\tilde{s}} - \frac{1}{2\,\hat{r}_{\tilde{s}+1}} \left(\bar{r} + \sum_{s=1}^{\tilde{s}} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right) \right) \\ &- \frac{1}{2}\,\check{Q}_{\tilde{s}-1} - \frac{1}{2\,\hat{r}_{\tilde{s}}} \left(\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s (\check{Q}_s - \check{Q}_{s-1}) \right) \end{aligned} \tag{B.49}$$

$$0 \overset{?}{\lessgtr} \frac{1}{2}\left(\check{Q}_{\tilde{s}} - \check{Q}_{\tilde{s}-1}\right) + \frac{1}{2\,\hat{r}_{\tilde{s}}}\left(\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right)\right)$$
$$- \frac{1}{2\,\hat{r}_{\tilde{s}+1}}\left(\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right)\right) - \frac{\hat{r}_{\tilde{s}}}{2\,\hat{r}_{\tilde{s}+1}}\left(\check{Q}_{\tilde{s}} - \check{Q}_{\tilde{s}-1}\right) \tag{B.50}$$

$$0 \overset{?}{\lessgtr} \underbrace{\left(1 - \frac{\hat{r}_{\tilde{s}}}{\hat{r}_{\tilde{s}+1}}\right)}_{<\,0} \underbrace{\left(\check{Q}_{\tilde{s}} - \check{Q}_{\tilde{s}-1}\right)}_{>\,0} + \underbrace{\left(\frac{1}{\hat{r}_{\tilde{s}}} - \frac{1}{\hat{r}_{\tilde{s}+1}}\right)}_{<\,0} \underbrace{\left(\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right)\right)}_{\geq\,0} . \tag{B.51}$$

Since $\hat{r}_{\tilde{s}}$ is greater than $\hat{r}_{\tilde{s}+1}$, the first and third term in brackets are negative. In addition, the section border $\check{Q}_{\tilde{s}}$ is also greater than $\check{Q}_{\tilde{s}-1}$, which lets the second term in brackets become positive. The fourth term represents the price on the section border $\check{Q}_{\tilde{s}-1}$, i.e., the section border where section $\tilde{s}$ begins. We assume that the prices are non-negative and therefore the overall expression on the right hand side is negative, i.e.,

$$0 > \left(1 - \frac{\hat{r}_{\tilde{s}}}{\hat{r}_{\tilde{s}+1}}\right)\left(\check{Q}_{\tilde{s}} - \check{Q}_{\tilde{s}-1}\right) + \left(\frac{1}{\hat{r}_{\tilde{s}}} - \frac{1}{\hat{r}_{\tilde{s}+1}}\right)\left(\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right)\right) \tag{B.52}$$

which means that

$$Q_{\tilde{s}}^{\mathrm{opt}} > Q_{\tilde{s}+1}^{\mathrm{opt}} \tag{B.53}$$

holds. Hence, the optimal solution of the section $\tilde{s}$ is greater than the optimal solution of the section $\tilde{s}+1$. This is true for all neighbouring section and because of transitivity the optimal solution of the first section is the greatest of all section solutions and the solution of the last section is the smallest.

B.4 Proof of objective dominance

The objective functions of the individual sections have got another property which is very useful for determining the optimal value. As we will see in the sequel, the value of the corresponding individual objective function of the actual section is always less (or equal on the section borders) than the individual objective functions of all other sections in the actual section. This also means that the resulting objective function is the minimum of all objective functions of the individual sections.

Equation (3.25) can be rewritten to

$$r(Q) = \min_{\tilde{s}} \{r_{\tilde{s}}(Q)\} = \min_{\tilde{s}} \left\{ \bar{r} + \sum_{s=0}^{\tilde{s}-1} (\hat{r}_s - \hat{r}_{s+1})\, \check{Q}_s + \hat{r}_{\tilde{s}}\, Q \right\} \tag{B.54}$$

with $\hat{r}_0 = 0$ and $\check{Q}_0 = 0$ and the first section is indexed with $\tilde{s} = 1$. This is equivalent to

$$r(Q) = \min_{\tilde{s}} \left\{ \bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right) + \hat{r}_{\tilde{s}} \left(Q - \check{Q}_{\tilde{s}-1}\right) \right\} . \tag{B.55}$$

Because of the monotone transformation by multiplying with a positive number Q

$$\min_{\tilde{s}} \{r_{\tilde{s}}(Q)\}\, Q = \min_{\tilde{s}} \{r_{\tilde{s}}(Q)\, Q\} \tag{B.56}$$

holds. Hence, multiplying the equations of $r(Q)$ with Q leads to

$$r(Q)\, Q = \min_{\tilde{s}} \left\{ \left(\bar{r} + \sum_{s=0}^{\tilde{s}-1} (\hat{r}_s - \hat{r}_{s+1})\, \check{Q}_s + \hat{r}_{\tilde{s}} Q \right) Q \right\} \tag{B.57}$$

$$= \min_{\tilde{s}} \left\{ \left(\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right) + \hat{r}_{\tilde{s}} \left(Q - \check{Q}_{\tilde{s}-1}\right) \right) Q \right\} . \tag{B.58}$$

This property can also be shown using the objective function directly. The objective value of an arbitrary section $\tilde{s}$ is given by

$$r_{\tilde{s}}(Q)\, Q = \left(\bar{r} + \sum_{s=1}^{\tilde{s}-1} \hat{r}_s \left(\check{Q}_s - \check{Q}_{s-1}\right) + \hat{r}_{\tilde{s}} \left(Q - \check{Q}_{\tilde{s}-1}\right) \right) Q \tag{B.59}$$

(see Eq. (3.30)) or

$$r_{\tilde{s}}(Q)\,Q = \left(\bar{r} + \sum_{s=0}^{\tilde{s}-1} (\hat{r}_s - \hat{r}_{s+1})\,\check{Q}_s + \hat{r}_{\tilde{s}}\,Q\right) Q\,. \tag{B.60}$$

Let us compare the objective values of two succeeding sections $\tilde{s}$ and $\tilde{s}+1$ for the same value Q.

$$\left(\bar{r} + \sum_{s=0}^{\tilde{s}-1} (\hat{r}_s - \hat{r}_{s+1})\,\check{Q}_s + \hat{r}_{\tilde{s}}\,Q\right) Q \overset{?}{\lesseqgtr} \left(\bar{r} + \sum_{s=0}^{\tilde{s}} (\hat{r}_s - \hat{r}_{s+1})\,\check{Q}_s + \hat{r}_{\tilde{s}+1}\,Q\right) Q \tag{B.61}$$

Transforming the expression (with $Q > 0$) leads to

$$\bar{r} + \sum_{s=0}^{\tilde{s}-1} (\hat{r}_s - \hat{r}_{s+1})\,\check{Q}_s + \hat{r}_{\tilde{s}}\,Q \overset{?}{\lesseqgtr} \bar{r} + \sum_{s=0}^{\tilde{s}-1} (\hat{r}_s - \hat{r}_{s+1})\,\check{Q}_s + (\hat{r}_{\tilde{s}} - \hat{r}_{\tilde{s}+1})\,\check{Q}_{\tilde{s}} + \hat{r}_{\tilde{s}+1}\,Q \tag{B.62}$$

$$0 \overset{?}{\lesseqgtr} (\hat{r}_{\tilde{s}} - \hat{r}_{\tilde{s}+1})\,\check{Q}_{\tilde{s}} + \hat{r}_{\tilde{s}+1}\,Q - \hat{r}_{\tilde{s}}\,Q \tag{B.63}$$

$$0 \overset{?}{\lesseqgtr} \underbrace{(\hat{r}_{\tilde{s}} - \hat{r}_{\tilde{s}+1})}_{>0}\left(\check{Q}_{\tilde{s}} - Q\right)\,. \tag{B.64}$$

The term in the first pair of brackets is positive, because $\hat{r}_s > \hat{r}_{s+1}$. For $Q = 0$ the objective value of all sections are equal with a value of zero. For positive Q the value of the objective function belonging to section $\tilde{s}$ is lower than the one of section $\tilde{s}+1$ for Q less than $\check{Q}_{\tilde{s}}$, i.e., the section border between section $\tilde{s}$ and $\tilde{s}+1$. This is even true for the complete interval of $0 < Q < \check{Q}_{\tilde{s}}$. For all Q greater than the section border $Q > \check{Q}_{\tilde{s}}$ the value of the objective function of section $\tilde{s}$ is greater than the one of section $\tilde{s}+1$. This is illustrated in Fig. B.2. Derived from this, all section functions have a common intersection at $Q = 0$. Furthermore, each pair of neighbouring section functions has got a further intersection at the section border. (This is necessary to have a continuous objective function.) Since no more intersections exist we can further derive that the value of the objective functions of all other sections (greater or less) are greater than the value of objective function of the corresponding section to a given Q. In the figure we can identify that always three functions are greater than the actual objective function of the section. What we also see is that the function of, e.g., section four is always greater than the one of lower sections in the interval of zero and the corresponding upper section border. For example, function four is greater than function one from zero to the first border, greater than function two from zero to the second border, and greater than function three from

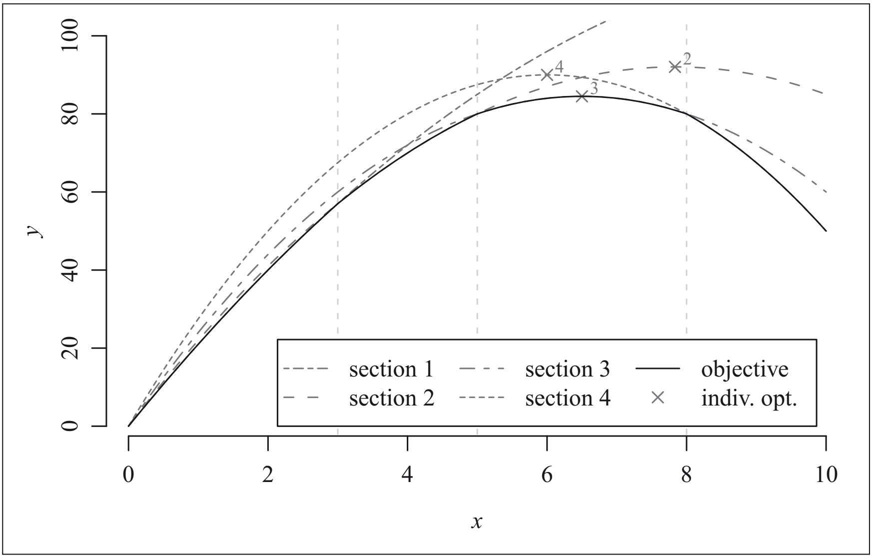

Fig. B.2 Course of section objective functions

zero to the third section border. But function four is not necessarily greater than function two in section three. The markings of the section number on the section individual optima illustrates the ordering of the optima from right to left (i.e., big to small) as shown in Sect. B.3.

B.5 Algorithm for solving QLP with partially defined objective functions

In Fig. B.3 a general approach to find the optimum of the partially defined concave quadratic objective function is depicted. Firstly, we try to find the optimal solution based on the gradient solely with LP solving. If this fails, a QLP solver should be used that considers the jumping gradient at the section borders of the partially defined objective function. One such consideration could be the procedure depicted in Fig. 3.21 for the gradient projection method by Rosen.[2] The drawback of the gradient projection method by Rosen is the enormous matrix calculations and, thereby, especially the inversion of matrices.

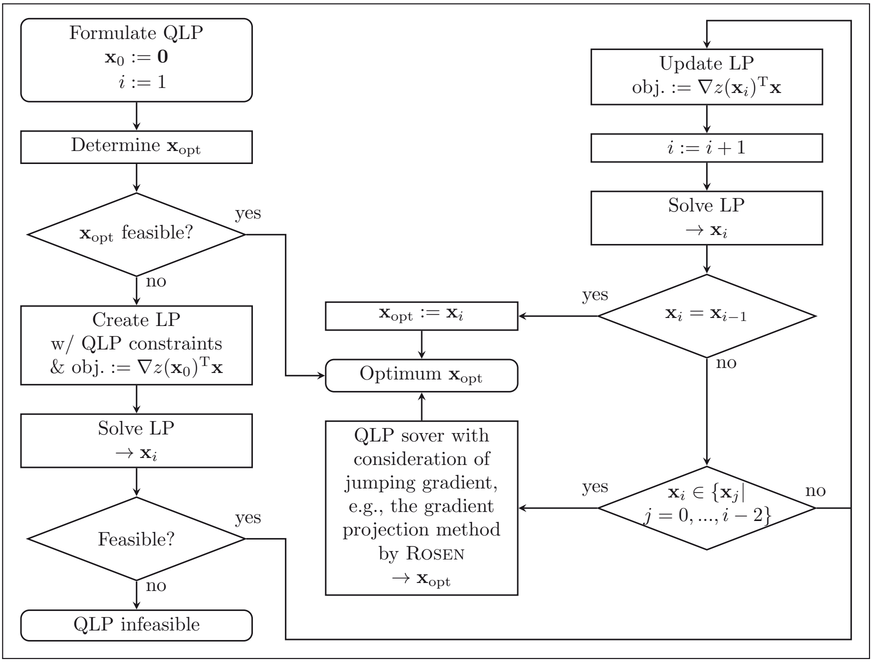

Fig. B.3 Flow diagram LP and QLP solver

[2] Cf. Rosen (1960): *Gradient projection method.*

B.6 Proof of symmetry of quadratic function

To prove that a quadratic function of the form $f(x) = a\,x^2 + b\,x + c$ is symmetric, we start with the equation $0 = 0$. Each step we extend both sides until we reach the desired term. The variable ϵ denotes the distance from the axis of symmetry, which is the turning point $x_{\mathrm{opt}} = -\frac{b}{2\,a}$. Starting with

$$0 = 0\,, \tag{B.65}$$

substituting 0 by $b\,\epsilon - b\,\epsilon$

$$b\,\epsilon - b\,\epsilon = -b\,\epsilon + b\,\epsilon\,, \tag{B.66}$$

substituting b by $-2\,a\,x_{\mathrm{opt}}$

$$-2\,a\,\epsilon\,x_{\mathrm{opt}} - b\,\epsilon = 2\,a\,\epsilon\,x_{\mathrm{opt}} + b\,\epsilon\,, \tag{B.67}$$

adding on both sides $a\,x_{\mathrm{opt}}^2 + a\,\epsilon^2 + b\,x_{\mathrm{opt}} + c$

$$\begin{aligned} a\,x_{\mathrm{opt}}^2 - 2\,a\,\epsilon\,x_{\mathrm{opt}} + a\,\epsilon^2 + b\,x_{\mathrm{opt}} - b\,\epsilon + c = \\ a\,x_{\mathrm{opt}}^2 + 2\,a\,\epsilon\,x + a\,\epsilon^2 + b\,x_{\mathrm{opt}} + b\,\epsilon + c\,, \end{aligned} \tag{B.68}$$

and factoring both sides

$$a\,\left(x_{\mathrm{opt}} - \epsilon\right)^2 + b\,(x_{\mathrm{opt}} - \epsilon) + c = a\,\left(x_{\mathrm{opt}} + \epsilon\right)^2 + b\,(x_{\mathrm{opt}} + \epsilon) + c \tag{B.69}$$

shows that moving away from x_{opt} by ϵ in both directions leads to the identical function values. Hence, the function is symmetric around x_{opt}.

B.7 Proof of maximal underrun of purity level

The formula for determining the maximal underrun of the required purity level ω depending on the study horizon length s and the number of periods of the past p included in the planning is

$$\epsilon(s,p) = \frac{s-1}{p+1}(1-\omega) \tag{B.70}$$

(see Eq. (3.135)). As we see in Table 3.21 on 113 a reduction of the study horizon by one period leads to a decrease of the minimal purity level possible in the planning period by the allowed impurity (i.e., $1-\omega$). This is caused by the requirement of an average of ω over $p+s$ periods. If s is reduced by one, a grey entry with 100 % is taken away. This leaves one value less to balance the low value of for example 60 % and thus an increase to 65 % is necessary. Conversely, this means that an increase of the study horizon by one leads to a decrease of the overall purity average, which means that the maximal underrun increases. This increase is not the allowed impurity but a fraction of it. And the fraction depends on the number of periods of the past included in the planning.

As we see in the afore mentioned table, when p periods of the past are included, $p+1$ periods form a group that is repeated over and over to result in the overall achieved purity for an infinite repeating of the planning. Hence, every additional period in the study horizon results in an increase of the maximal underrun of the purity by $1-\omega$ in relation to the group length $p+1$, i.e., $\frac{1-\omega}{p+1}$.

To prove the dependency on s we use the mathematical induction and start to show that the equation is true for a given $s=2$.[3] Note that $s=1$ is our minimal study horizon. Using the values $p=3$ and $\omega=0.95$ the maximal underrun is $\frac{2-1}{3+1}(1-0.95)=0.0125$. This equals an overall average purity of $\omega - 0.0125 = 93.75\,\%$. Thus, we showed that the formula is true for an arbitrary s. In the next step we show that the formula is true for the next value of s. This means we assume that the formula is correct for s and when we calculate the value for $s+1$, we need to show that the change from s to $s+1$ is correct. Calculating ϵ for $s+1$ leads to

$$\epsilon(s+1,p) = \frac{(s+1)-1}{p+1}(1-\omega) = \frac{s-1}{p+1}(1-\omega) + \frac{1-\omega}{p+1} = \epsilon(s,p) + \frac{1-\omega}{p+1}\,. \tag{B.71}$$

[3] Usually the lowest value is chosen, but to compare the result with the one in Table 3.21 we choose $s=2$. The proof can easily be repeated for $s=1$.

We see that the increase of s by one leads to an increase of ϵ by $\frac{1-\omega}{p+1}$ compared to $\epsilon(s,p)$. This is exactly what we developed above. Therefore, the equation is correct with respect to s.

Taking a look at the differences between the example for three and two periods of the past together with a study horizon of length three, we notice that the maximal underrun increases with less periods of the past. With every period we include from the past the periods in the planning are increased by one and the repeating group for the determination of the overall purity level is extended by one (e.g., (75, 100, 100) for $p = 2$ compared to (70, 100, 100, 100) for $p = 3$). The first effect equals that of extending the study horizon, which is the decrease of the minimal purity by $1 - \omega$. The second effect is that the average is based on one more value of 100 %. Let us develop the change of ϵ starting with the change of the average purity. The initial average purity (e.g., 91 2/3) is denoted by $\bar{\omega}_0$. When a further period of the past is included, another value of 100 % is added, which causes an extension of the number of periods used for the averaging. Thus, the minimal purity value can be reduced by $1 - \omega$ and the group for determining the average is extended by one. Hence, the average purity value $\bar{\omega}_1$ with one more period of the past included is

$$\bar{\omega}_1 = \frac{\bar{\omega}_0(p+1) + 1 - (1-\omega)}{p+2} = \frac{\bar{\omega}_0(p+1) - \omega}{p+2}\,. \tag{B.72}$$

To achieve the change of number of periods the average is calculated on, we first need to multiply the “old” average with the “old” number of periods $p + 1$, do the absolute changes (plus 1 and minus $1 - \omega$), and divide by the “new” number of periods $p + 2$. But, what we want to calculate is the maximal underrun and not the average purity. Thus, the $\bar{\omega}$ are substituted by $\omega - \epsilon$.

$$\omega - \epsilon_1 = \frac{(\omega - \epsilon_0)(p+1) - \omega}{p+2} \tag{B.73}$$

$$\epsilon_1 = \omega - \frac{(\omega - \epsilon_0)(p+1) - \omega}{p+2} = \epsilon_0 \frac{p+1}{p+2} \tag{B.74}$$

This is the change of ϵ, when we increase the number of periods of the past by one.

Again using the mathematical induction we showed already that the equation is correct for an arbitrary p.[4] Now we need to demonstrate that the increase from an arbitrary value is correct. Therefore, we calculate

[4] We showed it for $p = 3$, which could easily repeated with $p = 0$ as minimal value of p.

$$\epsilon(s, p+1) = \frac{s-1}{(p+1)+1}(1-\omega) = \frac{s-1}{p+1}(1-\omega)\frac{p+1}{p+2} = \epsilon(s,p)\frac{p+1}{p+2} \quad \text{(B.75)}$$

and see that the change is exactly the term $\frac{p+1}{p+2}$ multiplicatively linked to $\epsilon(s, p)$. Hence, we also proved the correctness with respect to p and subsume that the equation is correct for determining the maximal underrun of the purity requirement for a long-term consideration depending on study horizon s and number of past periods p. We do not consider ω separately in the equation, because it is obvious that the allowed impurity $1 - \omega$ has linear influence on the maximal underrun.

B.8 Compact dynamic model

The transformation applied are based on the following equations under the assumption that the t are greater than or equal τ.

$$Q^{\mathrm{C}}_{tc} = X^{\mathrm{I}}_{t,c,1} + \sum_r X^{\mathrm{R}}_{t,c,1,r} + \sum_d X^{\mathrm{D}}_{t,c,1,d} \tag{B.76}$$

$$Q^{\mathrm{R}}_{tr} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X^{\mathrm{R}}_{tcir} \tag{B.77}$$

$$Q^{\mathrm{D}}_{td} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X^{\mathrm{D}}_{tcid} \tag{B.78}$$

$$Q^{\mathrm{I}}_{te} = \sum_{(c,i)\in\mathcal{P}_e} X^{\mathrm{I}}_{tci} \tag{B.79}$$

$$V^{\mathrm{C}}_{tc} = V^{\mathrm{C}}_{\tau c} + \sum_{l=\tau}^{t-1} \left(\widetilde{Q}^{\mathrm{C}}_{lc} - X^{\mathrm{I}}_{l,c,1} - \sum_r X^{\mathrm{R}}_{l,c,1,r} - \sum_d X^{\mathrm{D}}_{l,c,1,d} \right) \tag{B.80}$$

$$V^{\mathrm{I}}_{te} = V^{\mathrm{I}}_{\tau e} + \sum_{l=\tau}^{t-1} \left(\sum_{(c,i)\in\mathcal{P}_e} X^{\mathrm{I}}_{lci} - \widetilde{Q}^{\mathrm{I}}_{le} \right) \tag{B.81}$$

$$V^{\mathrm{R}}_{tr} = V^{\mathrm{R}}_{\tau r} + \sum_{l=\tau}^{t-1} \left(\sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X^{\mathrm{R}}_{lcir} - \widetilde{Q}^{\mathrm{R}}_{lr} \right) \tag{B.82}$$

$$V^{\mathrm{D}}_{td} = V^{\mathrm{D}}_{\tau d} + \sum_{l=\tau}^{t-1} \left(\sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X^{\mathrm{D}}_{lcid} - \widetilde{Q}^{\mathrm{D}}_{ld} \right) \tag{B.83}$$

Note that $V^{\mathrm{C}}_{\tau c}$, $V^{\mathrm{I}}_{\tau e}$, $V^{\mathrm{R}}_{\tau r}$, and $V^{\mathrm{D}}_{\tau d}$ are given values and no decision variables. The objective function could still be more compact, but for a better reading the P_τ, R_τ, C_τ, C^{V}_τ, and C^{S}_τ are kept.

$$\text{Max } P_\tau = R_\tau - C_\tau - C^{\mathrm{V}}_\tau - C^{\mathrm{S}}_\tau \tag{B.84}$$

$$R_\tau = \sum_{t=\tau}^{\tau+\bar{\tau}-1} \left(\sum_e r^{\mathrm{I}}_{te} \widetilde{Q}^{\mathrm{I}}_{te} + \sum_r r^{\mathrm{R}}_{tr} \widetilde{Q}^{\mathrm{R}}_{tr} \right) \left(z^{t-\tau} + \frac{z^{\bar{\tau}}}{\bar{\tau}(1-z)} \right) \tag{B.85}$$

$$C_\tau = \sum_{t=\tau}^{\tau+\bar{\tau}-1} \left(\sum_c \left(c^{\mathrm{A}}_{tc} \widetilde{Q}^{\mathrm{C}}_{tc} + c^{\mathrm{J}}_c \left(X^{\mathrm{I}}_{t,c,1} + \sum_r X^{\mathrm{R}}_{t,c,1,r} + \sum_d X^{\mathrm{D}}_{t,c,1,d} \right) \right) \right)$$

$$+\sum_d c^{\mathrm{D}}_{td}\widetilde{Q}^{\mathrm{D}}_{td}\Bigg)\cdot\left(z^{t-\tau}+\frac{z^{\bar{\tau}}}{\bar{\tau}(1-z)}\right) \tag{B.86}$$

$$\begin{aligned}
C^{\mathrm{V}}_{\tau} = \sum_{t=\tau}^{\tau+\bar{\tau}-1}\Bigg[&\sum_c h^{\mathrm{C}}_c\Bigg(V^{\mathrm{C}}_{\tau c}+\sum_{l=\tau}^{t-1}\Bigg(\widetilde{Q}^{\mathrm{C}}_{lc}-X^{\mathrm{I}}_{l,c,1}-\sum_r X^{\mathrm{R}}_{l,c,1,r}-\sum_d X^{\mathrm{D}}_{l,c,1,d}\Bigg)\\
&+\widetilde{Q}^{\mathrm{C}}_{tc}-\frac{1}{2}\Bigg(X^{\mathrm{I}}_{t,c,1}+\sum_r X^{\mathrm{R}}_{t,c,1,r}+\sum_d X^{\mathrm{D}}_{t,c,1,d}\Bigg)\Bigg)\\
&+\sum_e h^{\mathrm{I}}_e\Bigg(V^{\mathrm{I}}_{\tau e}+\sum_{l=\tau}^{t-1}\Bigg(\sum_{(c,i)\in\mathcal{P}_e}X^{\mathrm{I}}_{lci}-\widetilde{Q}^{\mathrm{I}}_{le}\Bigg)+\frac{1}{2}\sum_{(c,i)\in\mathcal{P}_e}X^{\mathrm{I}}_{tci}\Bigg)\\
&+\sum_r h^{\mathrm{R}}_r\Bigg(V^{\mathrm{R}}_{\tau r}+\sum_{l=\tau}^{t-1}\Bigg(\sum_c\sum_{i=1}^{\bar{I}_c}w_{ci}X^{\mathrm{R}}_{lcir}-\widetilde{Q}^{\mathrm{R}}_{lr}\Bigg)\\
&\qquad+\frac{1}{2}\sum_c\sum_{i=1}^{\bar{I}_c}w_{ci}X^{\mathrm{R}}_{tcir}\Bigg)\\
&+\sum_d h^{\mathrm{D}}_d\Bigg(V^{\mathrm{D}}_{\tau d}+\sum_{l=\tau}^{t-1}\Bigg(\sum_c\sum_{i=1}^{\bar{I}_c}w_{ci}X^{\mathrm{D}}_{lcid}-\widetilde{Q}^{\mathrm{D}}_{ld}\Bigg)\\
&\qquad+\frac{1}{2}\sum_c\sum_{i=1}^{\bar{I}_c}w_{ci}X^{\mathrm{D}}_{tcid}\Bigg)\Bigg]z^{t-\tau}\\
+\Bigg[&\sum_c h^{\mathrm{C}}_c\Bigg(V^{\mathrm{C}}_{\tau c}+\sum_{t=\tau}^{\tau+\bar{\tau}-1}\Bigg(\widetilde{Q}^{\mathrm{C}}_{tc}-X^{\mathrm{I}}_{t,c,1}-\sum_r X^{\mathrm{R}}_{t,c,1,r}-\sum_d X^{\mathrm{D}}_{t,c,1,d}\Bigg)\Bigg)\\
&+\sum_e h^{\mathrm{I}}_e\Bigg(V^{\mathrm{I}}_{\tau e}+\sum_{t=\tau}^{\tau+\bar{\tau}-1}\Bigg(\sum_{(c,i)\in\mathcal{P}_e}X^{\mathrm{I}}_{tci}-\widetilde{Q}^{\mathrm{I}}_{te}\Bigg)\Bigg)\\
&+\sum_r h^{\mathrm{R}}_r\Bigg(V^{\mathrm{R}}_{\tau r}+\sum_{t=\tau}^{\tau+\bar{\tau}-1}\Bigg(\sum_c\sum_{i=1}^{\bar{I}_c}w_{ci}X^{\mathrm{R}}_{tcir}-\widetilde{Q}^{\mathrm{R}}_{tr}\Bigg)\Bigg)\\
&+\sum_d h^{\mathrm{D}}_d\Bigg(V^{\mathrm{D}}_{\tau d}+\sum_{t=\tau}^{\tau+\bar{\tau}-1}\Bigg(\sum_c\sum_{i=1}^{\bar{I}_c}w_{ci}X^{\mathrm{D}}_{tcid}-\widetilde{Q}^{\mathrm{D}}_{td}\Bigg)\Bigg)\Bigg]\frac{z^{\bar{\tau}}}{1-z}
\end{aligned} \tag{B.87}$$

$$C^{\mathrm{S}}_{\tau} = \sum_{t=\tau}^{\tau+\bar{\tau}-1} \left(\sigma^{\mathrm{C}} \sum_{c} c^{\mathrm{A}}_{tc} \left(\overline{Q}^{\mathrm{C}}_{tc} - \widetilde{Q}^{\mathrm{C}}_{tc} \right) + \sigma^{\mathrm{I}} \sum_{e} r^{\mathrm{I}}_{te} \left(D^{\mathrm{I}}_{te} - \widetilde{Q}^{\mathrm{I}}_{te} \right) \right.$$
$$\left. + \sigma^{\mathrm{R}} \sum_{r} r^{\mathrm{R}}_{tr} \left(D^{\mathrm{R}}_{tr} - \widetilde{Q}^{\mathrm{R}}_{tr} \right) + \sigma^{\mathrm{D}} \sum_{d} c^{\mathrm{D}}_{td} \left(\overline{Q}^{\mathrm{D}}_{td} - \widetilde{Q}^{\mathrm{D}}_{td} \right) \right) z^{t-\tau} \tag{B.88}$$

$$\left\lceil \beta_c \overline{Q}^{\mathrm{C}}_{tc} \right\rceil \leq \widetilde{Q}^{\mathrm{C}}_{tc} \leq \overline{Q}^{\mathrm{C}}_{tc} \quad \forall\ t \in \widetilde{T}, c \tag{B.89}$$

$$\widetilde{Q}^{\mathrm{I}}_{te} \leq D^{\mathrm{I}}_{te} \quad \forall\ t \in \widetilde{T}, e \tag{B.90}$$
$$\widetilde{Q}^{\mathrm{R}}_{tr} \leq D^{\mathrm{R}}_{tr} \quad \forall\ t \in \widetilde{T}, r \tag{B.91}$$
$$\widetilde{Q}^{\mathrm{D}}_{td} \leq \overline{Q}^{\mathrm{D}}_{td} \quad \forall\ t \in \widetilde{T}, d \tag{B.92}$$

$$X^{\mathrm{I}}_{t,c,1} + \sum_{r} X^{\mathrm{R}}_{t,c,1,r} + \sum_{d} X^{\mathrm{D}}_{t,c,1,d} = X^{\mathrm{I}}_{tci} + \sum_{r} X^{\mathrm{R}}_{tcir} + \sum_{d} X^{\mathrm{D}}_{tcid}$$
$$\forall\ t \in \widetilde{T}, c, i \in \{2, \ldots, \bar{I}_c\} \tag{B.93}$$

$$\sum_{d} X^{\mathrm{D}}_{tcid} \geq \zeta_{ci} \iota_{ci} \left(X^{\mathrm{I}}_{t,c,1} + \sum_{r} X^{\mathrm{R}}_{t,c,1,r} + \sum_{d} X^{\mathrm{D}}_{t,c,1,d} \right)$$
$$\forall\ t \in \widetilde{T}, c, i \in \{1, \ldots, \bar{I}_c\} \tag{B.94}$$

$$X^{\mathrm{I}}_{tci} \leq (1 - \zeta_{ci})(1 - \eta_{ci})(1 - \theta_{ci}) \left(X^{\mathrm{I}}_{t,c,1} + \sum_{r} X^{\mathrm{R}}_{t,c,1,r} + \sum_{d} X^{\mathrm{D}}_{t,c,1,d} \right)$$
$$\forall\ t \in \widetilde{T}, (c, i) \in \bigcup_{e} \mathcal{P}_e \tag{B.95}$$

$$\omega_r \sum_{l=t-\mathcal{I}_r}^{t} \sum_{c} \sum_{i=1}^{\bar{I}_c} w_{ci} X^{\mathrm{R}}_{lcir} \leq \sum_{c} \sum_{i=1}^{\bar{I}_c} w_{ci} \pi_{cir} \sum_{l=t-\mathcal{I}_r}^{t} X^{\mathrm{R}}_{lcir} \quad \forall\ t \in \widetilde{T}, r \tag{B.96}$$

$$\bar{V}^1 \geq \sum_{c} \nu^{\mathrm{C}}_{c} \left(V^{\mathrm{C}}_{\tau c} + \sum_{l=\tau}^{t-1} \left(\widetilde{Q}^{\mathrm{C}}_{lc} - X^{\mathrm{I}}_{l,c,1} - \sum_{r} X^{\mathrm{R}}_{l,c,1,r} - \sum_{d} X^{\mathrm{D}}_{l,c,1,d} \right) + \widetilde{Q}^{\mathrm{C}}_{\tau c} \right)$$
$$\forall\ t \in \widetilde{T} \tag{B.97}$$

$$\begin{aligned}\bar{V}^2 \geq & \sum_e \nu_e^{\mathrm{I}} \left(V_{\tau e}^{\mathrm{I}} + \sum_{l=\tau}^{t-1} \left(\sum_{(c,i)\in\mathcal{P}_e} X_{lci}^{\mathrm{I}} - \widetilde{Q}_{le}^{\mathrm{I}} \right) + \sum_{(c,i)\in\mathcal{P}_e} X_{tci}^{\mathrm{I}} \right) \\ & + \sum_r \nu_r^{\mathrm{R}} \left(V_{\tau r}^{\mathrm{R}} + \sum_{l=\tau}^{t-1} \left(\sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{lcir}^{\mathrm{R}} - \widetilde{Q}_{lr}^{\mathrm{R}} \right) + \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{tcir}^{\mathrm{R}} \right) \\ & + \sum_d \nu_d^{\mathrm{D}} \left(V_{\tau d}^{\mathrm{D}} + \sum_{l=\tau}^{t-1} \left(\sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{lcid}^{\mathrm{D}} - \widetilde{Q}_{ld}^{\mathrm{D}} \right) + \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{tcid}^{\mathrm{D}} \right) \\ & \forall\ t \in \widetilde{T} \quad \text{(B.98)}\end{aligned}$$

$$\begin{aligned}\bar{V}^3 \geq & \sum_{e\in\{e|\mathcal{P}_e\subseteq\mathcal{H}\}} \nu_e^{\mathrm{I}} \left(V_{\tau e}^{\mathrm{I}} + \sum_{l=\tau}^{t-1} \left(\sum_{(c,i)\in\mathcal{P}_e} X_{lci}^{\mathrm{I}} - \widetilde{Q}_{le}^{\mathrm{I}} \right) + \sum_{(c,i)\in\mathcal{P}_e} X_{tci}^{\mathrm{I}} \right) \\ & + \nu_2^{\mathrm{D}} \left(V_{\tau,2}^{\mathrm{D}} + \sum_{l=\tau}^{t-1} \left(\sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{l,c,i,2}^{\mathrm{D}} - \widetilde{Q}_{l,2}^{\mathrm{D}} \right) + \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} X_{t,c,i,2}^{\mathrm{D}} \right) \\ & \forall\ t \in \widetilde{T} \quad \text{(B.99)}\end{aligned}$$

$$X_{tci}^{\mathrm{I}} = 0 \quad \forall\ t, (c,i) \notin \bigcup_e \mathcal{P}_e \qquad \text{(B.100)}$$

$$X_{tcir}^{\mathrm{R}} = 0 \quad \forall\ t, (c,i) \in \mathcal{H}, r \qquad \text{(B.101)}$$

$$X_{tcid}^{\mathrm{D}} = 0 \quad \forall\ t, (c,i) \in \mathcal{H}, d \in \{1\} \qquad \text{(B.102)}$$

All decision variables are non-negative.

$$\widetilde{Q}_{\tau c}^{\mathrm{C}}, X_{\tau ci}^{\mathrm{I}}, X_{\tau cir}^{\mathrm{R}}, X_{\tau cid}^{\mathrm{D}}, \widetilde{Q}_{\tau e}^{\mathrm{I}} \in \mathbb{Z}^* \quad \forall\ c, i \in \{1, \ldots, \bar{I}_c\}, r, d, e \qquad \text{(B.103)}$$

B.9 Optimal values X^{I}_{tci}, X^{R}_{tcir}, and X^{D}_{tcid}

Table B.1 Optimal solution of first iteration of X^{I}_{tci}, X^{R}_{tcir}, and X^{D}_{tcid}

		X^{I}_{tci}					X^{R}_{tcir}									
							$r=1$ t					$r=2$ t				
c	i	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	A	10	8.65	10.31	.	13.43	14	10.57	12.6	.	16.42	.	.	.	.	.
	B	10	8.65	10.31	.	13.43	14	10.57	12.6	.	16.42	.	.	.	.	.
	C	.	.	.	.	.	23	19.2	22.87	.	29.8	.	.	.	.	.
	D	.	.	.	.	.	23	19.2	22.87	.	29.8	.	.	.	.	.
	E	23	19.03	22.68	.	29.55	1	0.19	0.23	.	0.3	.	.	.	.	.
	F	.	.	.	.	.	24	19.23	22.91	.	29.85	.	.	.	.	.
	G	.	.	.	.	.	.	.	.	.	0.27	24	19.23	22.91	.	29.58
	H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
2	A	67	86.11	63.23	90.83	77.79	125	105.25	77.28	.	95.08	.	.	.	118.42	.
	B	85	70.72	61.12	94.17	77.79	107	120.64	79.39	.	95.08	.	.	.	115.09	.
	C	.	.	.	.	.	192	191.36	140.51	.	.	.	.	.	209.26	172.87
	D	.	.	.	.	.	192	191.36	140.51	.	25.08	.	.	.	209.26	147.79
	E	190	189.44	139.11	207.17	171.14	2	1.91	1.41	2.09	.	.	.	.	.	1.73
	F	.	.	.	.	.	192	191.36	140.51	209.26	172.87	.	.	.	.	.
	G	190	189.44	139.11	207.17	171.14	.	.	.	2.09	1.73	2	1.91	1.41	.	.
	H	.	.	.	.	.	10	11.55	12.92	10.78	.	182	179.81	127.59	198.48	172.87
3	A	.	8.43	28.02	13.5	13.05	.	10.31	34.24	.	15.95	.	.	.	16.5	.
	B	.	8.43	28.02	13.5	13.05	.	10.31	34.24	.	15.95	.	.	.	16.5	.
	C	.	.	.	.	.	.	18.71	62.16	.	28.96	.	.	.	29.95	.
	D	.	.	.	.	.	.	18.71	62.16	.	28.96	.	.	.	29.95	.
	E	.	.	.	.	.	.	18.74	62.26	30	29	.	.	.	.	.
	F	.	.	.	.	.	.	18.74	62.26	30	2.58	.	.	.	.	26.42
	G	.	18.56	61.63	29.7	28.71	.	.	.	0.3	0.29	.	0.19	0.62	.	.
	H	.	.	.	.	.	.	.	.	.	.	.	18.74	62.26	30	29

		X^{R}_{tcir}										X^{D}_{tcid}									
		$r=3$ t					$r=4$ t					$d=1$ t					$d=2$ t				
c	i	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	B	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	C	.	.	.	.	.	.	.	.	.	.	1	0.03	0.03	.	0.04	.	.	.	.	.
	D	.	.	.	.	.	.	.	.	.	.	1	0.03	0.03	.	0.04	.	.	.	.	.
	E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	G	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	24	19.23	22.91	.	29.85
2	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	B	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	C	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	D	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	G	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
3	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	B	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	C	.	.	.	.	.	.	.	.	.	.	.	0.03	0.09	0.04	0.04	.	.	.	.	.
	D	.	.	.	.	.	.	.	.	.	.	.	0.03	0.09	0.04	0.04	.	.	.	.	.
	E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	G	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Dots denote zero values. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

B.10 Optimal values X^{I}_{tci}, X^{R}_{tcir}, and X^{D}_{tcid} (20 periods)

Table B.2 Optimal solutions of X^{I}_{tci}

		X^{I}_{tci}																			
		period t																			
c	i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	18	17	18	19	20
1	A	10	8	12	3	13	36	11	11	11	10	6	12	12	11	11	13	12	12	11	12
	B	10	3	12	3	13	36	11	11	11	10	6	12	12	11	11	13	12	12	11	12
	C	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	D	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	E	23	18	26	6	28	80	24	24	24	22	13	26	27	24	25	28	26	26	24	26
	F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	G	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
2	A	67	86	60	90	82	51	62	72	70	72	67	73	65	61	67	79	61	66	63	66
	B	85	86	60	90	82	51	62	72	70	72	67	73	65	61	67	79	61	66	63	66
	C	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	D	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	E	190	190	133	198	181	112	136	160	155	158	147	162	144	135	148	174	135	145	140	145
	F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	G	190	190	133	198	181	112	136	160	155	157	147	162	144	135	148	174	135	145	140	145
	H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
3	A	.	.	21	13	9	10	7	7	7	6	7	.	13	8	.	.	17	10	6	6
	B	.	8	28	13	9	10	7	7	7	6	7	.	13	8	.	.	17	10	6	6
	C	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	D	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	G	.	17	62	29	19	23	15	15	15	9	11	.	29	17	.	.	37	22	14	14
	H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Dots denote zero values.

Table B.3 Optimal solutions of X^{D}_{tcid}

			X^{D}_{tcid}																			
			period t																			
d	c	i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	18	17	18	19	20
regular waste ($d = 1$)	1	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		B	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		C	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		D	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		G	.	.	.	.	.	21	.	.	.	.	.	.	.	1	.	.	.	.	.	1
		H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	2	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		B	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		C	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		D	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		G	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	2
		H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	3	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		B	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		C	.	1	1	1	1	1	1	1	1	1	1	.	1	1	.	.	1	1	1	1
		D	.	1	1	1	1	1	1	1	1	1	1	.	1	1	.	.	1	1	1	1
		E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		G	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1
		H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
hazardous waste ($d = 2$)	1	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		B	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		C	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		D	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		G	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		H	24	19	27	7	29	81	25	25	25	23	14	27	28	25	26	29	27	27	25	27
	2	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		B	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		C	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		D	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		G	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	3	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		B	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		C	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		D	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		G	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Dots denote zero values.

Table B.4 Optimal solutions of X^{R}_{tcir}

			X^{R}_{tcir}																			
			period t																			
r	c	i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	18	17	18	19	20
steel ($r=1$)	1	A	14	11	8	3	16	.	.	.	.	.	6	.	.	.	.	.	.	15	14	15
		B	14	16	3	4	14	.	.	.	.	.	8	15	.	.	.	.	.	15	11	15
		C	23	18	26	6	.	.	24	2	.	.	.	.	1	.	.	.	4	26	24	26
		D	23	18	26	6	1	.	24	.	.	.	11	.	2	.	.	.	.	26	24	26
		E	1	1	1	.	.	.	.	1	1	1	1	1	1	.	.	.	.	.	.	.
		F	24	19	27	6	16	.	17	25	25	23	14	27	28	25	23	29	27	27	25	27
		G	.	.	2	.	1	.	1	.	1	.	.	3	2	3	1	3	2	1	1	1
		H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	2	A	125	106	75	16	.	.	5	90	.	.	82	91	.	.	2	.	.	81	79	81
		B	107	106	75	16	.	.	5	90	.	.	.	17	.	43	2	.	11	81	79	81
		C	192	192	135	7	105	.	138	.	.	.	149	.	3	33	.	176	.	147	142	147
		D	192	190	135	52	181	5	138	.	.	.	149	.	.	22	89	94	.	147	142	147
		E	2	2	2	.	2	.	1	2	2	.	1	2	2	.	1	.	.	2	.	.
		F	192	192	135	198	183	1	138	162	157	160	149	164	145	137	146	176	137	141	129	147
		G	.	.	.	.	.	.	1	2	2	3	.	.	2	.	2	.	2	.	.	.
		H	10	11	.	1	.	.	.	8	7	.	8	.	.	.	.	.	.	.	.	.
	3	A	.	18	42	16	.	.	.	.	.	.	.	.	.	.	.	.	.	13	9	9
		B	.	10	5	16	10	.	9	9	.	.	10	.	.	.	.	.	.	13	9	9
		C	.	17	44	.	.	4	7	.	.	.	16	.	1	17	.	.	.	22	14	14
		D	.	17	61	.	.	.	15	.	.	.	14	.	1	.	.	.	.	22	6	14
		E	.	18	63	30	.	3	1	16	16	14	.	.	15	13	.	.	33	.	4	10
		F	.	18	63	27	20	.	14	16	16	14	17	.	30	18	.	.	38	16	15	15
		G	.	.	.	1	1	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.
		H	.	1	.	30	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1
metal ($r=2$)	1	A	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		B	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		C	.	.	.	.	28	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.
		D	.	.	.	.	3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		E	.	.	.	1	1	1	1	.	.	.	.	.	.	1	1	1	1	1	1	1
		F	.	.	.	1	13	81	8	.	.	.	.	.	.	.	3	.	.	.	.	.
		G	24	19	25	7	28	60	24	25	24	23	14	24	26	21	25	26	25	26	24	25
		H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	2	A	.	.	.	94	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		B	.	.	.	94	32	.	.	.	.	.	82	.	.	.	.	.	.	.	.	.
		C	.	.	.	193	75	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		D	.	2	.	148	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		E	.	.	.	2	.	2	1	.	.	2	1	.	.	2	1	2	2	.	2	2
		F	.	.	.	2	.	113	.	.	.	.	.	.	1	.	4	.	.	6	13	.
		G	2	2	2	2	2	.	1	.	.	.	2	2	.	2	.	2	.	2	2	.
		H	182	181	135	199	183	114	138	154	150	160	141	164	146	137	150	176	137	147	142	147
	3	A	.	.	.	1	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		B	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		C	.	.	.	29	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		D	.	.	.	29	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		E	.	.	.	.	20	21	15	.	.	.	17	.	15	5	.	.	5	23	11	5
		F	.	.	.	3	.	24	2	.	.	.	.	.	.	.	.	.	.	7	.	.
		G	.	1	1	.	.	.	1	1	1	4	6	.	1	1	.	.	1	1	1	.
		H	.	17	63	.	18	24	16	16	16	14	17	.	30	18	.	.	38	23	15	14

Table B.5 Optimal solutions of X^{R}_{tcir} (cont.)

			X^{R}_{tcir}																			
			period t																			
r	c	i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	18	17	18	19	20
rubber ($r = 3$)	1	A	.	.	7	.	.	45	14	14	14	13	2	15	16	14	15	16	15	.	.	.
		B	.	.	12	.	.	45	14	14	14	13	.	.	16	14	15	16	15	.	3	.
		C	.	.	.	.	.	80	.	22	24	22	11	26	26	24	25	28	22	.	.	.
		D	.	.	.	.	24	80	.	24	24	22	2	26	25	24	25	28	26	.	.	.
		E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		G	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	2	A	.	.	.	.	100	63	71	.	87	88	.	.	81	76	81	97	76	.	.	.
		B	.	.	.	.	69	63	71	.	87	88	.	74	81	33	81	97	65	.	.	.
		C	.	.	.	.	3	114	.	162	157	160	.	164	143	104	150	.	137	.	.	.
		D	.	.	.	.	2	109	.	162	157	160	.	164	146	115	61	82	137	.	.	.
		E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		G	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		H	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	3	A	.	.	.	.	9	14	9	9	9	8	10	.	17	10	.	.	21	.	.	.
		B	.	.	30	.	1	14	.	.	9	8	.	.	17	10	.	.	21	.	.	.
		C	.	.	18	.	19	19	8	15	15	13	.	.	28	.	.	.	37	.	.	.
		D	.	.	1	.	19	23	.	15	15	13	2	.	28	17	.	.	37	.	8	.
		E	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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B.11 Potential for profit increasing

Two further ideas to evaluate the gained solution with the rolling planning are discussed in the sequel. Firstly, we build a model for the optimisation of all 24 periods together. Thereby, for the periods one through 20 the integrality constraints must apply. Furthermore, only the given contracts for the first five periods are relevant while for the remaining periods no contract exists and we assume that we can acquire and distribute arbitrary numbers of cores. The disposal constraints stay identical to the rolling horizon planning model. In addition, the inventory at the end of period 24 is limited by the inventory of the result of the rolling horizon planning and the lower acquisition limits of 25, 20, and 15 units of core 1, 2, and 3, respectively, are included, to keep it comparable. Solving this model leads to a profit of 380,475.06 € for the periods one through 20. (The profit for all 24 periods is 703,204.93 €.) The development of the revenues, cost, and profits is displayed in Fig. B.4. We see that the revenues are relatively constant compared to the cost. The cost has a high impact on the profit, such that it is very volatile, too.

In the rolling horizon planning we added a random influence of plus minus 10 % to simulate a suboptimal contracting and time varying data.

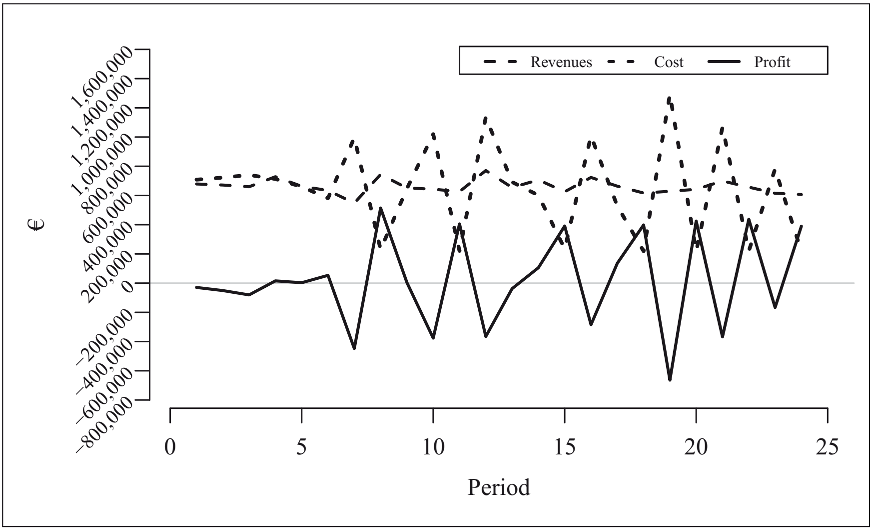


Fig. B.4 Profit development without further contracts

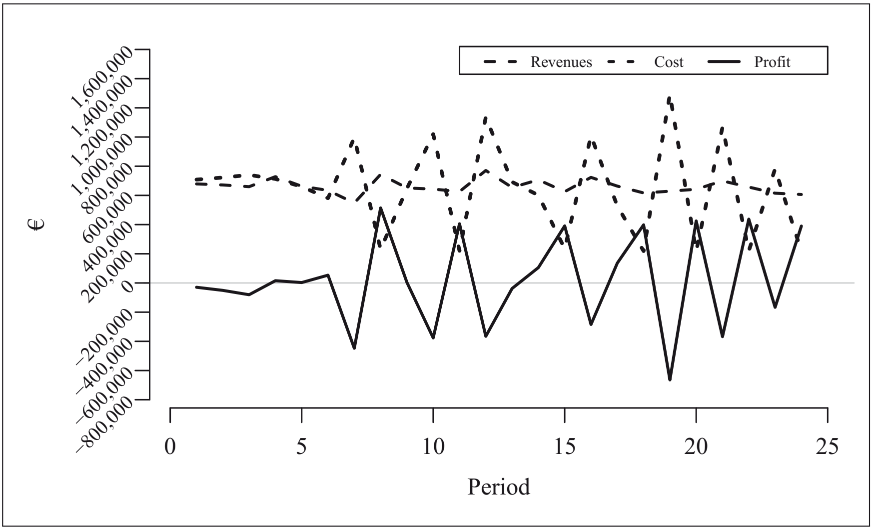


Fig. B.5 Profit development with further contracts

This should be considered here, too. Therefore, we take the solution from above and the values for $\widetilde{Q}^{\mathrm{C}}_{6..24,c}$, $\widetilde{Q}^{\mathrm{I}}_{6..24,e}$, and $\widetilde{Q}^{\mathrm{R}}_{6..24,r}$ are used to create contracts as with the pre-planning. So we extend the above model by the contracts. Solving this model leads to a profit of 114,801.58 € for periods one through 20 and 407,466.62 € for all 24 periods. Interestingly, the 20 period profit is rather low. But a glance at Fig. B.5 shows the reason. The cost and profit are highly volatile, which is a similar development compared to the solution above. The volatile cost is mainly driven by the acquisition cost and here especially of core 2. This illustrates Fig. B.6.

To compare this result with the rolling horizon planning (the one with the increased penalty factor) and its ex-post solution we take the profit over 20 and all 24 periods as well. The profit of the ex-post solution is 318,885.34 € and 329,828.34 € for 20 and 24 periods, respectively. For the rolling horizon planning a profit of 280,246.93 € as well as 288,318.56 € results.

Comparing the profit of the contracting using total planning with the ex-post solution and the rolling horizon planning shows a gap of $\frac{407466.62}{329828.34} - 1 = 23.54\,\%$ and $\frac{407466.62}{288318.56} - 1 = 41.33\,\%$, respectively. Especially, the ex-post solution shows a relative moderate gap. However, the rolling horizon planning shows a rather steady development of revenues, cost, and profit (see Fig. B.7). This development is usually preferred (this is one aim of the rolling horizon planning we have in this work) especially when a steady

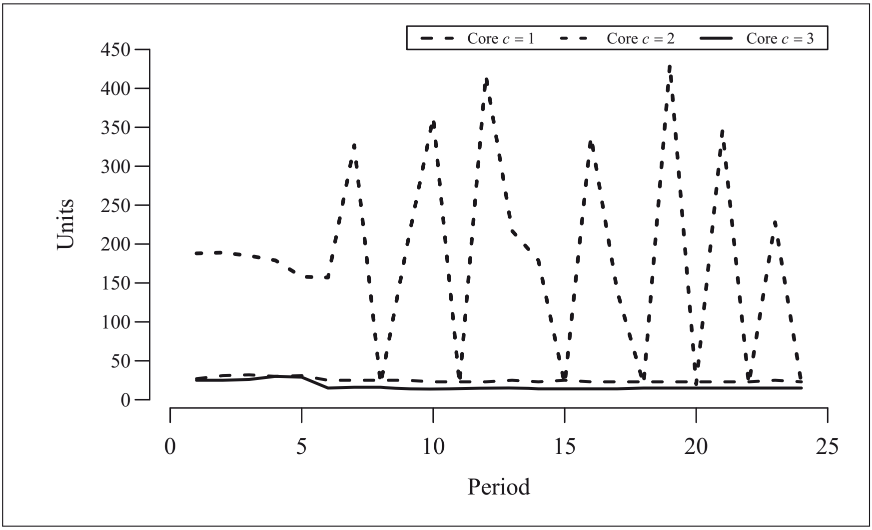


Fig. B.6 Quantities of acquired cores

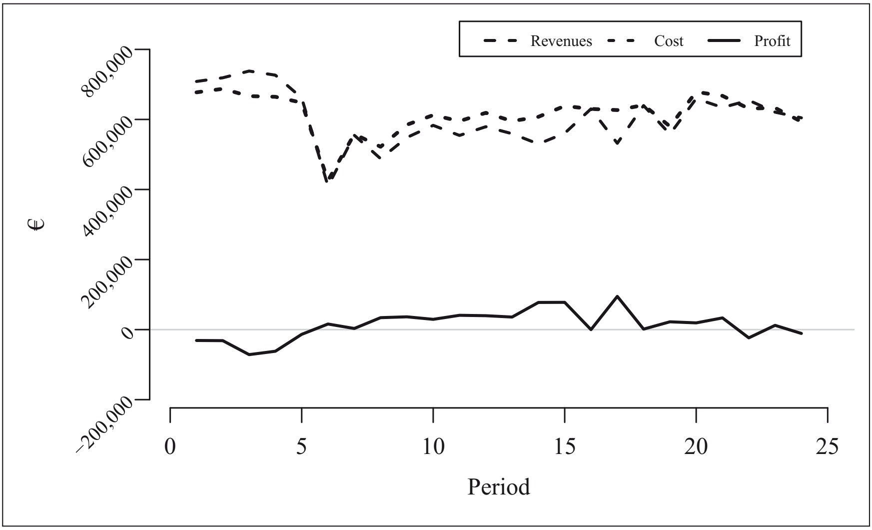


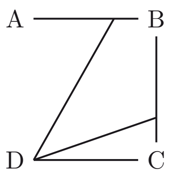
Fig. B.7 Profit development based on rolling horizon planning availability on cores and a steady demand of items and material exists. From this example set we can derive that a further (significant) increase of profit is possible, but maybe only with a volatile solution.

Appendix C
Appendix to Chapter 4

C.1 Linear and star core configuration

The two extreme cases of core configurations in terms of complexity are compared. The first is the linear structure, which can be seen as the lower bound, and the second is the star structure, which represents the upper bound with regard to the number of modules. A linear core structure is depicted in Fig. C.1. The core is limited to four items to keep the illustration straightforward. The corresponding connectivity matrix is also included in the figure. The geometric and technical constraints (as illustrated in the figure) are "AB not D" and "BC not D". These prevent any other disassembly sequence than taking off A, then B, and lastly C (or D, which is the same). The corresponding disassembly state graph and the and/or graph are depicted in Fig. C.2. The disassembly state graph consists of four nodes, i.e.,

Extended connection graph



Connectivity matrix

	A	B	C	D
A	0	1	0	0
B	1	0	2	0
C	0	2	0	3
D	0	0	3	0

Fig. C.1 Extended connection graph of linear core structure

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Disassembly state graph

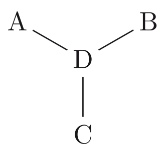
$$\begin{array}{c} (ABCD) \\ \downarrow \\ A(BCD) \\ \downarrow \\ A.B(CD) \\ \downarrow \\ A.B.C.D \end{array}$$

And/Or graph

$$\begin{array}{c} ABCD \\ \downarrow \\ BCD \\ \downarrow \\ CD \\ \downarrow \end{array}$$

Fig. C.2 Disassembly state and and/or graph of linear core structure

Extended connection graph



Connectivity matrix

	A	B	C	D
A	0	0	0	1
B	0	0	0	2
C	0	0	0	3
D	1	2	3	0

Fig. C.3 Extended connection graph of star core structure

only one node per level. In total three modules exist, which form the and/or graph.

On the contrary, the star structured core without any geometric or technical constraints—connection graph and connectivity matrix can be found in Fig. C.3—results in a larger disassembly state graph. This graph is depicted in Fig. C.4. The graph consists of $2^3 = 8$ nodes, which is the upper limit for three connections. The corresponding and/or graph has the same size (neglecting the node representing the completely disassembled core), because there exists no connection in the core which joins two modules. Hence, as soon as more than one module per connection cutting results, an and-relationship is included in the and/or graph and the number of nodes becomes smaller than the ones of the disassembly state graph. With these two extreme examples regarding the number of nodes we see that in practice the complexity is somewhere in between these two.

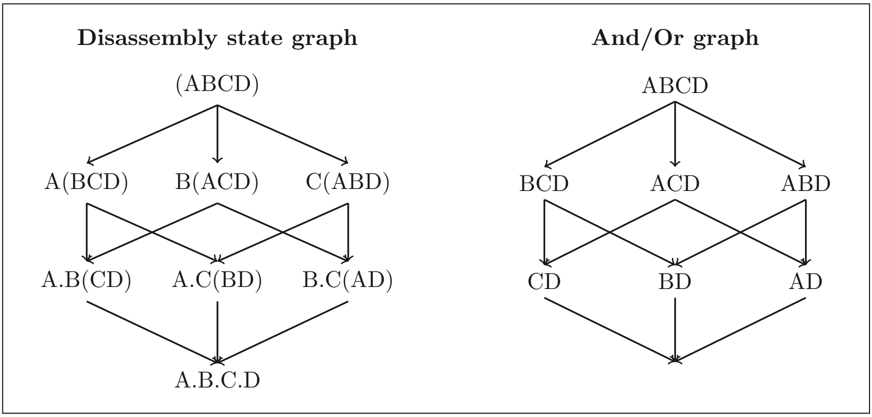


Fig. C.4 Disassembly state and and/or graph of star core structure

C.2 Number of edges of core graph

The basis for determining the number of edges of a core graph are the number of items n, the usage categories u, and the property that higher usage categories can always be used for all lower usage categories. This means that a functioning item can always be recycled and disposed of, whereas a recyclable item can always be disposed of. Obviously, the lowest category (i.e., the disposal) marks the end, because these items can only be disposed of. There exists no lower category. We assume, that no matter how many usage categories u exist, $u-1$ categories have lower categories. For the illustration we focus on the node $\underline{\underline{A}}\underline{B}\underline{\underline{C}}D$ of the core graph in Fig. 4.10 on page 197. This node contains two entries with the lowest category marked by the grey underlined letter $\underline{\underline{A}}$ and $\underline{\underline{C}}$. Since two entries have the lowest category only arrows for the remaining entries emerge from this node, i.e., two. This means that there exist nodes with n, $n-1$, $n-2$, $\ldots$, and $n-n$ lowest category entries, i.e., $n-k$ for $k \in \{0, \ldots, n\}$, which corresponds to k emerging edges at the same time.

The number of nodes for each number of edges is determined as follows. A node with k outgoing edges contains $n-k$ entries with a grey underlined letter. These $n-k$ entries can be placed arbitrarily among the n entries as long as always $n-k$ of the n exist. This results in $\binom{n}{n-k}$ possibilities. For each of these possibilities the remaining k entries can be filled with an arbitrary mix of the remaining categories, i.e., $u-1$. This equals a permutation with repetition of $u-1$ elements on k positions. Hence, $(u-1)^k$ possibilities per node of the $\binom{n}{n-k}$ exist. This makes $\binom{n}{n-k}(u-1)^k$ nodes with k emerging edges. Thus, the number of edges from these nodes is $\binom{n}{n-k}(u-1)^k k$. Lastly, the sum over all edge numbers k delivers the number of edges of the core graph with

$$\sum_{k=0}^{n} \binom{n}{n-k} (u-1)^k \, k \, . \tag{C.1}$$

The transformation into a more compact term is as follows.

$$\begin{aligned} & \sum_{k=0}^{n} \binom{n}{n-k} (u-1)^k \, k \\ = & \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} (u-1)^k \, k \end{aligned}$$

$$
\begin{aligned}
&= \sum_{k=1}^{n} \frac{n!}{(n-k)!k!} (u-1)^k \, k \\
&= \sum_{k=1}^{n} \frac{n!}{(n-k)!(k-1)!} (u-1)^k \\
&= \sum_{k=1}^{n} \frac{n(n-1)!}{(n-k)!(k-1)!} (u-1)^{k-1} (u-1) \\
&= (u-1) n \sum_{k=1}^{n} \frac{(n-1)!}{(n-k)!(k-1)!} (u-1)^{k-1} \\
&= (u-1) n \sum_{k=1}^{n} \frac{(n-1)!}{((n-1)-(k-1))!(k-1)!} (u-1)^{k-1} \\
&= (u-1) n \sum_{k=0}^{n-1} \frac{(n-1)!}{((n-1)-k)!k!} (u-1)^k \\
&= (u-1) n \sum_{k=0}^{n-1} \binom{n-1}{k} (u-1)^k \, 1^{n-1-k}
\end{aligned}
$$

The binomial theorem $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} \, b^k$ helps to transform the sum when substituting a with 1 and b with $u-1$ and using $n-1$ instead of n.[1]

$$
\begin{aligned}
&= (u-1) n (u-1+1)^{n-1} \\
&= (u-1) n \, u^{n-1} \qquad \text{(C.2)}
\end{aligned}
$$

The final step is to replace n by $\bar{I}_c$, to use the notation of the model, i.e., the number of edges for a core c is

$$
(u-1) \bar{I}_c \, u^{\bar{I}_c - 1} \, . \qquad \text{(C.3)}
$$

[1] Cf. Gellert et al. (1965): *Kleine Enzyklopdie – Mathematik*, p. 45.

C.3 Compact flexible model formulation

The transformation of the model in Sect. 4.2.3 is analogue to the one of the basic model in Sect. 3.1.2 to the compact basic model in appendix B.2. All variables completely explained by other variables are substituted, i.e., P, R, C, Q^{C}_c, Q^{I}_e, Q^{R}_r, and Q^{D}_d, as in the basic model. In addition, Q^{M}_f for modules is substituted, too.

$$
\begin{aligned}
\text{Maximise}\quad & \sum_e r^{\mathrm{I}}_e \left(\sum_{(c,i)\in\mathcal{P}_e} X^{\mathrm{I}}_{ci} \right) + \sum_f r^{\mathrm{M}}_f \left(\sum_{(c,m)\in\mathcal{R}_f} Y^{\mathrm{M}}_{cm} \right) \\
& + \sum_r r^{\mathrm{R}}_r \left(\sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X^{\mathrm{R}}_{cir} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y^{\mathrm{R}}_{cmr} \right) \right) \\
& - \sum_c \left(c^{\mathrm{A}}_c + c^{\mathrm{J}}_{c,1} \right) \Bigg(X^{\mathrm{I}}_{c,1} + \sum_r X^{\mathrm{R}}_{c,1,r} + \sum_d X^{\mathrm{D}}_{c,1,d} \\
& \qquad + \sum_{m=1}^{\overline{M}_c} \delta_{c,m,1} \left(Y^{\mathrm{M}}_{cm} + \sum_r Y^{\mathrm{R}}_{cmr} + \sum_d Y^{\mathrm{D}}_{cmd} \right) \Bigg) \\
& + \sum_c \sum_{m=1}^{\overline{M}_c} c^{\mathrm{J}}_{cm} \left(Y^{\mathrm{M}}_{cm} + \sum_r Y^{\mathrm{R}}_{cmr} + \sum_d Y^{\mathrm{D}}_{cmd} \right) \\
& - \sum_d c^{\mathrm{D}}_d \left(\sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X^{\mathrm{D}}_{cid} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y^{\mathrm{D}}_{cmd} \right) \right) \\
= & \sum_e \sum_{(c,i)\in\mathcal{P}_e} r^{\mathrm{I}}_e X^{\mathrm{I}}_{ci} + \sum_f \sum_{(c,m)\in\mathcal{R}_f} r^{\mathrm{M}}_f Y^{\mathrm{M}}_{cm} \\
& + \sum_c \Bigg[\sum_r \sum_{i=1}^{\bar{I}_c} r^{\mathrm{R}}_r w_{ci} \left(X^{\mathrm{R}}_{cir} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y^{\mathrm{R}}_{cmr} \right) \\
& \quad - \left(c^{\mathrm{A}}_c + c^{\mathrm{J}}_{c,1} \right) \Bigg(X^{\mathrm{I}}_{c,1} + \sum_r X^{\mathrm{R}}_{c,1,r} + \sum_d X^{\mathrm{D}}_{c,1,d} \\
& \qquad + \sum_{m=1}^{\overline{M}_c} \delta_{c,m,1} \left(Y^{\mathrm{M}}_{cm} + \sum_r Y^{\mathrm{R}}_{cmr} + \sum_d Y^{\mathrm{D}}_{cmd} \right) \Bigg)
\end{aligned}
$$

$$+\sum_{m=1}^{\overline{M}_c} c^{\mathrm{J}}_{cm}\left(Y^{\mathrm{M}}_{cm}+\sum_r Y^{\mathrm{R}}_{cmr}+\sum_d Y^{\mathrm{D}}_{cmd}\right)$$
$$\left.-\sum_d\sum_{i=1}^{\bar{I}_c} c^{\mathrm{D}}_d\, w_{ci}\left(X^{\mathrm{D}}_{cid}+\sum_{m=1}^{\overline{M}_c}\delta_{cmi}Y^{\mathrm{D}}_{cmd}\right)\right] \tag{C.4}$$

Item and module flow constraints

$$X^{\mathrm{I}}_{c,1}+\sum_r X^{\mathrm{R}}_{c,1,r}+\sum_d X^{\mathrm{D}}_{c,1,d}+\sum_{m=1}^{\overline{M}_c}\delta_{c,m,1}\left(Y^{\mathrm{M}}_{cm}+\sum_r Y^{\mathrm{R}}_{cmr}+\sum_d Y^{\mathrm{D}}_{cmd}\right)$$
$$=X^{\mathrm{I}}_{ci}+\sum_r X^{\mathrm{R}}_{cir}+\sum_d X^{\mathrm{D}}_{cid}+\sum_{m=1}^{\overline{M}_c}\delta_{cmi}\left(Y^{\mathrm{M}}_{cm}+\sum_r Y^{\mathrm{R}}_{cmr}+\sum_d Y^{\mathrm{D}}_{cmd}\right)$$
$$\forall\ c, i\in\{2,\ldots,\bar{I}_c\} \tag{C.5}$$

$$X^{\mathrm{I}}_{ci}+\sum_r X^{\mathrm{R}}_{cir}+\sum_d X^{\mathrm{D}}_{cid}\geq\sum_{m=1}^{\overline{M}_c}\alpha_{cmi}\left(Y^{\mathrm{M}}_{cm}+\sum_r Y^{\mathrm{R}}_{cmr}+\sum_d Y^{\mathrm{D}}_{cmd}\right)$$
$$\forall\ c, i\in\{1,\ldots,\bar{I}_c\} \tag{C.6}$$

$$X^{\mathrm{I}}_{ci}=0 \quad \forall\ (c,i)\notin\bigcup_e \mathcal{P}_e \tag{C.7}$$

$$Y^{\mathrm{M}}_{cm}=0 \quad \forall\ (c,m)\notin\bigcup_f \mathcal{R}_f \tag{C.8}$$

Condition constraints

Core graph:

$$V^{\mathrm{C}}_{c,1}\leq\tilde{\rho}_{c,1}\left(X^{\mathrm{I}}_{c,1}+\sum_r X^{\mathrm{R}}_{c,1,r}+\sum_d X^{\mathrm{D}}_{c,1,d}\right.$$
$$\left.+\sum_{m=1}^{\overline{M}_c}\delta_{c,m,1}\left(Y^{\mathrm{M}}_{cm}+\sum_r Y^{\mathrm{R}}_{cmr}+\sum_d Y^{\mathrm{D}}_{cmd}\right)\right)$$
$$+\sum_{\tilde{v}\in\{\tilde{v}\mid\tilde{v}>1,E^{\mathrm{C}}_{c,\tilde{v},1}=1\}} Z^{\mathrm{C}}_{c,\tilde{v},1} \quad \forall\ c \tag{C.9}$$

$$\sum_{\tilde{v}\in\{\tilde{v}|\tilde{v}<v,E^{\mathrm{C}}_{cv\tilde{v}}=1\}} Z^{\mathrm{C}}_{cv\tilde{v}} + V^{\mathrm{C}}_{cv} = \tilde{\rho}_{cv}\left(X^{\mathrm{I}}_{c,1} + \sum_r X^{\mathrm{R}}_{c,1,r} + \sum_d X^{\mathrm{D}}_{c,1,d}\right.$$
$$\left. + \sum_{m=1}^{\overline{M}_c} \delta_{c,m,1}\left(Y^{\mathrm{M}}_{cm} + \sum_r Y^{\mathrm{R}}_{cmr} + \sum_d Y^{\mathrm{D}}_{cmd}\right)\right)$$
$$+ \sum_{\tilde{v}\in\{\tilde{v}|\tilde{v}>v,E^{\mathrm{C}}_{c\tilde{v}v}=1\}} Z^{\mathrm{C}}_{c\tilde{v}v} \quad \forall\, c, v \in \{2,\ldots,3^{\bar{I}_c}\} \tag{C.10}$$

$$\sum_v V^{\mathrm{C}}_{cv} = \left(X^{\mathrm{I}}_{c,1} + \sum_r X^{\mathrm{R}}_{c,1,r} + \sum_d X^{\mathrm{D}}_{c,1,d}\right.$$
$$\left. + \sum_{m=1}^{\overline{M}_c} \delta_{c,m,1}\left(Y^{\mathrm{M}}_{cm} + \sum_r Y^{\mathrm{R}}_{cmr} + \sum_d Y^{\mathrm{D}}_{cmd}\right)\right) \quad \forall\, c \tag{C.11}$$

$$Z^{\mathrm{C}}_{cv\tilde{v}} = 0 \quad \forall\, c, v, \tilde{v} \in \left\{\tilde{v}\,\middle|\, E^{\mathrm{C}}_{cv\tilde{v}} = 0\right\} \tag{C.12}$$

Distribution graph:

$$\sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{cw\tilde{w}}=1\right\}} Z^{\mathrm{I}}_{cw\tilde{w}} + Y^{\mathrm{M}}_{c,L^{\mathrm{A}}_{cw}} = \sum_{v\in L^{\mathrm{I}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{I}}_{c\tilde{w}w}$$
$$+ \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{I}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\, c, w \in \left\{w\,\middle|\, w \in \left\{1,\ldots,2^{\bar{I}_c} - \bar{I}_c - 1\right\}, L^{\mathrm{A}}_{cw} > 0\right\} \tag{C.13}$$

$$\sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{cw\tilde{w}}=1\right\}} Z^{\mathrm{I}}_{cw\tilde{w}} = \sum_{v\in L^{\mathrm{I}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{I}}_{c\tilde{w}w}$$
$$+ \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{I}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\, c, w \in \left\{w\,\middle|\, w \in \left\{1,\ldots,2^{\bar{I}_c} - \bar{I}_c - 1\right\}, L^{\mathrm{A}}_{cw} = 0\right\} \tag{C.14}$$

$$X^{\mathrm{I}}_{c,L^{\mathrm{A}}_{cw}} + X^{\mathrm{A}}_{c,L^{\mathrm{A}}_{cw}} = \sum_{v\in L^{\mathrm{I}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{I}}_{c\tilde{w}w} + \sum_{\tilde{w}\in\left\{\tilde{w}\middle|E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{I}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\, c, w \in \left\{2^{\bar{I}_c} - \bar{I}_c,\ldots,2^{\bar{I}_c} - 1\right\} \tag{C.15}$$

$$X^{\mathrm{A}}_{ci} = 0 \quad \forall\ (c,i) \notin \bigcup_{e} \mathcal{P}_e \tag{C.16}$$

Recycling graph:

$$\begin{aligned}\sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{cw\tilde{w}}=1\right\}} Z^{\mathrm{R}}_{cw\tilde{w}} + \sum_{r} Y^{\mathrm{R}}_{c,L^{\mathrm{A}}_{cw},r} = \sum_{v\in L^{\mathrm{R}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{R}}_{c\tilde{w}w}\\ + \sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{R}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}\\ \forall\ c, w \in \left\{w \,\middle|\, w\in\left\{1,\ldots,2^{\bar{I}_c}-\bar{I}_c-1\right\}, L^{\mathrm{A}}_{cw}>0\right\}\end{aligned} \tag{C.17}$$

$$\begin{aligned}\sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{cw\tilde{w}}=1\right\}} Z^{\mathrm{R}}_{cw\tilde{w}} = \sum_{v\in L^{\mathrm{R}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{R}}_{c\tilde{w}w}\\ + \sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{R}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}\\ \forall\ c, w \in \left\{w \,\middle|\, w\in\left\{1,\ldots,2^{\bar{I}_c}-\bar{I}_c-1\right\}, L^{\mathrm{A}}_{cw}=0\right\}\end{aligned} \tag{C.18}$$

$$\begin{aligned}\sum_{r} X^{\mathrm{R}}_{c,L^{\mathrm{A}}_{cw},r} + \sum_{d} X^{\mathrm{D}}_{c,L^{\mathrm{A}}_{cw},d} - \widetilde{X}^{\mathrm{D}}_{c,L^{\mathrm{A}}_{cw}} - X^{\mathrm{A}}_{c,L^{\mathrm{A}}_{cw}}\\ = \sum_{v\in L^{\mathrm{R}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{R}}_{c\tilde{w}w} + \sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{R}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}\\ \forall\ c, w \in \left\{2^{\bar{I}_c}-\bar{I}_c,\ldots,2^{\bar{I}_c}-1\right\}\end{aligned} \tag{C.19}$$

Disposal graph:

$$\begin{aligned}\sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{cw\tilde{w}}=1\right\}} Z^{\mathrm{D}}_{cw\tilde{w}} + \sum_{d} Y^{\mathrm{D}}_{c,L^{\mathrm{A}}_{cw},d} = \sum_{v\in L^{\mathrm{D}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{D}}_{c\tilde{w}w}\\ + \sum_{\tilde{w}\in\left\{\tilde{w}\,\middle|\,E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{D}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}\\ \forall\ c, w \in \left\{w \,\middle|\, w\in\left\{1,\ldots,2^{\bar{I}_c}-\bar{I}_c-1\right\}, L^{\mathrm{A}}_{cw}>0\right\}\end{aligned} \tag{C.20}$$

$$\sum_{\tilde{w} \in \left\{\tilde{w} \middle| E^{\mathrm{D}}_{cw\tilde{w}}=1\right\}} Z^{\mathrm{D}}_{cw\tilde{w}} = \sum_{v \in L^{\mathrm{D}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w} \in \left\{\tilde{w} \middle| E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{D}}_{c\tilde{w}w}$$
$$+ \sum_{\tilde{w} \in \left\{\tilde{w} \middle| E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{D}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\ c, w \in \left\{ w \middle| w \in \left\{1, \ldots, 2^{\bar{I}_c} - \bar{I}_c - 1\right\}, L^{\mathrm{A}}_{cw} = 0 \right\} \quad \text{(C.21)}$$

$$\widetilde{X}^{\mathrm{D}}_{c,L^{\mathrm{A}}_{cw}} = \sum_{v \in L^{\mathrm{D}}_{cw}} V^{\mathrm{C}}_{cv} + \sum_{\tilde{w} \in \left\{\tilde{w} \middle| E^{\mathrm{D}}_{c\tilde{w}w}=1\right\}} Z^{\mathrm{D}}_{c\tilde{w}w} + \sum_{\tilde{w} \in \left\{\tilde{w} \middle| E^{\mathrm{D}}_{c\tilde{w}w}>1\right\}} Z^{\mathrm{D}}_{c,\tilde{w},E^{\mathrm{D}}_{c\tilde{w}w}}$$
$$\forall\ c, w \in \left\{2^{\bar{I}_c} - \bar{I}_c, \ldots, 2^{\bar{I}_c} - 1\right\} \quad \text{(C.22)}$$

$$Z^{\mathrm{I}}_{cw\tilde{w}} = Z^{\mathrm{R}}_{cw\tilde{w}} = Z^{\mathrm{D}}_{cw\tilde{w}} = 0 \quad \forall\ c, w, \tilde{w} \in \left\{\tilde{w} \middle| E^{\mathrm{D}}_{cw\tilde{w}} \neq 1\right\} \quad \text{(C.23)}$$

Damaging:

$$(1 - \theta_{ci})\, X^{\mathrm{A}}_{ci} \geq \theta_{ci}\, X^{\mathrm{I}}_{ci} \quad \forall\ (c,i) \in \bigcup_e \mathcal{P}_e \quad \text{(C.24)}$$

$$\sum_d X^{\mathrm{D}}_{cid} \geq \widetilde{X}^{\mathrm{D}}_{ci} \quad \forall\ c, i \quad \text{(C.25)}$$

Purity constraints

$$\omega_r \left(\sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X^{\mathrm{R}}_{cir} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y^{\mathrm{R}}_{cmr} \right)\right)$$
$$\leq \sum_c \sum_{i=1}^{\bar{I}_c} \pi_{cir} w_{ci} \left(X^{\mathrm{R}}_{cir} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y^{\mathrm{R}}_{cmr} \right) \quad \forall\ r \quad \text{(C.26)}$$

$$X^{\mathrm{R}}_{cir} = 0 \quad \forall\ (c,i) \in \mathcal{H}, r \quad \text{(C.27)}$$
$$X^{\mathrm{D}}_{cid} = 0 \quad \forall\ (c,i) \in \mathcal{H}, d \in \{1\} \quad \text{(C.28)}$$

$$Y^{\mathrm{R}}_{cmr} = 0 \quad \forall\ (c,m) \in \left\{(c,m) | \delta_{cmi} = 1, (c,i) \in \mathcal{H}, m \in \{1, \ldots, \overline{M}_c\}\right\}, r \quad \text{(C.29)}$$

$$Y^{\mathrm{D}}_{cmd} = 0 \quad \forall\ (c,m) \in \left\{(c,m) | \delta_{cmi} = 1, (c,i) \in \mathcal{H}, m \in \{1, \ldots, \overline{M}_c\}\right\}, d \in \{1\} \quad \text{(C.30)}$$

Limits constraints

$$\underline{Q}_c^{\mathrm{C}} \leq X_{c,1}^{\mathrm{I}} + \sum_r X_{c,1,r}^{\mathrm{R}} + \sum_d X_{c,1,d}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_c} \delta_{c,m,1} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}} \right) \leq \overline{Q}_c^{\mathrm{C}} \quad \forall\ c \quad \text{(C.31)}$$

$$\underline{Q}_e^{\mathrm{I}} \leq \sum_{(c,i)\in\mathcal{P}_e} X_{ci}^{\mathrm{I}} \leq D_e^{\mathrm{I}} \quad \forall\ e \quad \text{(C.32)}$$

$$\underline{Q}_f^{\mathrm{M}} \leq \sum_{(c,m)\in\mathcal{R}_f} Y_{cm}^{\mathrm{M}} \leq D_f^{\mathrm{M}} \quad \forall\ f \quad \text{(C.33)}$$

$$\underline{Q}_r^{\mathrm{R}} \leq \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X_{cir}^{\mathrm{R}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmr}^{\mathrm{R}} \right) \leq D_r^{\mathrm{R}} \quad \forall\ r \quad \text{(C.34)}$$

$$\underline{Q}_d^{\mathrm{D}} \leq \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X_{cid}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmd}^{\mathrm{D}} \right) \leq \overline{Q}_d^{\mathrm{D}} \quad \forall\ d \quad \text{(C.35)}$$

$$\sum_c t_{c,1}^{\mathrm{J}} \left(X_{c,1}^{\mathrm{I}} + \sum_r X_{c,1,r}^{\mathrm{R}} + \sum_d X_{c,1,d}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_c} \delta_{c,m,1} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}} \right) \right) - \sum_c \sum_{m=1}^{\overline{M}_c} t_{cm}^{\mathrm{J}} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}} \right) \leq \bar{L} \quad \text{(C.36)}$$

$$X_{ci}^{\mathrm{I}}, X_{ci}^{\mathrm{A}}, \widetilde{X}_{ci}^{\mathrm{D}}, X_{cir}^{\mathrm{R}}, X_{cid}^{\mathrm{D}}, Y_{cm}^{\mathrm{M}}, Y_{cmr}^{\mathrm{R}}, Y_{cmd}^{\mathrm{D}} \in \mathbb{Z}^* \quad \forall\ c, i \in \{1, \ldots, \bar{I}_c\}, m \in \{1, \ldots, \overline{M}_c\}, r, d \quad \text{(C.37)}$$

$$V_{cv}^{\mathrm{C}}, Z_{cv\tilde{v}}^{\mathrm{C}}, Z_{cw\tilde{w}}^{\mathrm{I}}, Z_{cw\tilde{w}}^{\mathrm{R}}, Z_{cw\tilde{w}}^{\mathrm{D}} \geq 0 \quad \forall\ c, v \,\&\, \tilde{v} \in \{1, \ldots, 3^{\bar{I}_c}\}, w \,\&\, \tilde{w} \in \{1, \ldots, 2^{\bar{I}_c} - 1\} \quad \text{(C.38)}$$

Once the solution with the remaining decision variables is gained, the values of the other variables can be easily determined with the following equations.

$$Q_c^{\mathrm{C}} = X_{c,1}^{\mathrm{I}} + \sum_r X_{c,1,r}^{\mathrm{R}} + \sum_d X_{c,1,d}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_c} \delta_{c,m,1} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}} \right) \quad \forall\ c \qquad \text{(C.39)}$$

$$Q_r^{\mathrm{R}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X_{cir}^{\mathrm{R}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmr}^{\mathrm{R}} \right) \quad \forall\ r \qquad \text{(C.40)}$$

$$Q_d^{\mathrm{D}} = \sum_c \sum_{i=1}^{\bar{I}_c} w_{ci} \left(X_{cid}^{\mathrm{D}} + \sum_{m=1}^{\overline{M}_c} \delta_{cmi} Y_{cmd}^{\mathrm{D}} \right) \quad \forall\ d \qquad \text{(C.41)}$$

$$Q_e^{\mathrm{I}} = \sum_{(c,i)\in\mathcal{P}_e} X_{ci}^{\mathrm{I}} \quad \forall\ e \qquad \text{(C.42)}$$

$$Q_f^{\mathrm{M}} = \sum_{(c,m)\in\mathcal{R}_f} Y_{cm}^{\mathrm{M}} \quad \forall\ f \qquad \text{(C.43)}$$

$$R = \sum_e r_e^{\mathrm{I}} Q_e^{\mathrm{I}} + \sum_f r_f^{\mathrm{M}} Q_f^{\mathrm{M}} + \sum_r r_r^{\mathrm{R}} Q_r^{\mathrm{R}} \qquad \text{(C.44)}$$

$$C = \sum_c \left(c_c^{\mathrm{A}} + c_{c,1}^{\mathrm{J}} \right) Q_c^{\mathrm{C}} - \sum_c \sum_{m=1}^{\overline{M}_c} c_{cm}^{\mathrm{J}} \left(Y_{cm}^{\mathrm{M}} + \sum_r Y_{cmr}^{\mathrm{R}} + \sum_d Y_{cmd}^{\mathrm{D}} \right) + \sum_d c_d^{\mathrm{D}} Q_d^{\mathrm{D}} \qquad \text{(C.45)}$$

$$P = R - C \qquad \text{(C.46)}$$

In this model we not only have integer variables. The variables in Eq. (C.38) are real valued variables. For every node of the core graph of each core c one variable V_{cv}^{C} exists, i.e., $\sum_c 3^{\bar{I}_c}$. In addition, all edges of the core graph are represented by a variable $Z_{cv\tilde{v}}^{\mathrm{C}}$, too. Depending on the number of items per core $\bar{I}_c$ a core graph has $2 \cdot 3^{\bar{I}_c - 1} \bar{I}_c$ edges (see page 193). For all cores we get $\frac{2}{3} \sum_c 3^{\bar{I}_c} \bar{I}_c$ edges. The distribution, recycling, and disposal graph have each $\frac{1}{2}\left(3^{\bar{I}_c} - 2^{\bar{I}_c+1} + 1\right)$ edges per core (see Eq. (4.36) on page 186). Note that always two edges are treated as one so that the number of decision variables is half the amount of the edges drawn in Fig. 4.8 or 4.9. For all three graphs and the number of cores we get $\frac{3}{2} \sum_c \left(3^{\bar{I}_c} - 2^{\bar{I}_c+1} + 1\right)$. Hence, all together the model has

$$\sum_c 3^{\bar{I}_c} + \frac{2}{3} \sum_c 3^{\bar{I}_c} \bar{I}_c + \frac{3}{2} \sum_c \left(3^{\bar{I}_c} - 2^{\bar{I}_c+1} + 1 \right)$$

$$= \sum_c \left(3^{\bar{I}_c} + \frac{2}{3} 3^{\bar{I}_c} \bar{I}_c + \frac{3}{2} 3^{\bar{I}_c} - \frac{3}{2} 2^{\bar{I}_c + 1} + \frac{3}{2} \right)$$

$$= \sum_c \left(\left(\frac{2}{3} \bar{I}_c + \frac{5}{2} \right) 3^{\bar{I}_c} - 3 \cdot 2^{\bar{I}_c} \right) + \frac{3}{2} c \tag{C.47}$$

real valued variables. Note that the term $\alpha \cdot c$ is the short form for $\alpha \sum_c 1$.

The number of integer variables is determined in a similar way. The relevant variables are listed in Eq. (C.37). These are X^{I}_{ci} and X^{A}_{ci}. Because of Eqs. (C.7) and (C.16), they can have a value other than zero only for the number of elements of the demand position sets. This makes $2 \left|\bigcup_e \mathcal{P}_e\right|$ number of variables. The same applies to the variable Y^{M}_{cm}, only that the demand for modules is relevant (see Eq. (C.8)), i.e., $\left|\bigcup_f \mathcal{R}_f\right|$. The variable $\widetilde{X}^{\mathrm{D}}_{ci}$ appears in the model for each core $\bar{I}_c$ times so that in total $\sum_c \bar{I}_c$ variables exist. In general, we find $\sum_r \sum_c \bar{I}_c$ (in short: $r \sum_c \bar{I}_c$) variables X^{R}_{cir} in the model. But, Eq. (C.27) states that recycling for hazardous items is not an option. Thus, the number of hazardous items for each recycling box can be subtracted. The same applies to the disposal with the exception that one disposal bin $d = 2$ is for hazardous items (see Eq. (C.28)). This makes $(r+d) \sum_c \bar{I}_c - r|\mathcal{H}| - (d-1)|\mathcal{H}|$ variables X^{R}_{cir} and X^{D}_{cid}. The variables Y^{R}_{cmr} and Y^{D}_{cmd} have the indices c, m, and r or d which implies that $(r+d) \sum_c \overline{M}_c$ variables exist in the model. Looking at the Eqs. (C.29) and (C.30) indicates that in the case of hazardous items the modules containing such items must not be recycled or disposed of as non-hazardous waste. But, depending on the given data even if an item is hazardous it does not necessarily mean that more than just one module is affected. At least the module $m = 1$ (complete core) is always affected, because this has to exist always in order to determine the disassembly cost and times. But this is just one variable of many. Therefore, we neglect the possible reducing by hazardous items and use the number of variables as upper bound. Summarising the integer variables leads to at most

$$2 \left| \bigcup_e \mathcal{P}_e \right| + \left| \bigcup_f \mathcal{R}_f \right| + \sum_c \bar{I}_c + (r+d) \sum_c \bar{I}_c - (r+d)|\mathcal{H}| + |\mathcal{H}|$$
$$+ (r+d) \sum_c \overline{M}_c$$
$$= 2 \left| \bigcup_e \mathcal{P}_e \right| + \left| \bigcup_f \mathcal{R}_f \right| + (r+d+1) \sum_c \bar{I}_c + (r+d) \sum_c \overline{M}_c - (r+d-1)|\mathcal{H}|$$

Table C.1 Upper bound of number of decision variables and constraints

real variables	$\sum_c \left(\left(\frac{2}{3}\bar{I}_c + \frac{5}{2}\right) 3^{\bar{I}_c} - 3 \cdot 2^{\bar{I}_c}\right) + \frac{3}{2}c$
integer variables	$2\left\lvert\bigcup_e \mathcal{P}_e\right\rvert + \left\lvert\bigcup_f \mathcal{R}_f\right\rvert + (r+d)\left(\sum_c \left(\bar{I}_c + \overline{M}_c\right) - \lvert\mathcal{H}\rvert\right) + \sum_c \bar{I}_c + \lvert\mathcal{H}\rvert$
constraints	$\sum_c \left(3^{\bar{I}_c} + 3 \cdot 2^{\bar{I}_c} + 3\,\bar{I}_c\right) + \left\lvert\bigcup_e \mathcal{P}_e\right\rvert - c + 3\,r + 2\,(e+f+d) + 1$

$$= 2\left\lvert\bigcup_e \mathcal{P}_e\right\rvert + \left\lvert\bigcup_f \mathcal{R}_f\right\rvert + (r+d)\left(\sum_c \left(\bar{I}_c + \overline{M}_c\right) - \lvert\mathcal{H}\rvert\right) + \sum_c \bar{I}_c + \lvert\mathcal{H}\rvert \tag{C.48}$$

integer variables in the model.

The number of constraints are $\sum_c \left(\bar{I}_c - 1\right)$ (C.5), $\sum_c \bar{I}_c$ (C.6), $\sum_c 3^{\bar{I}_c}$ (C.9) and (C.10), c (C.11), $3\sum_c \left(2^{\bar{I}_c} - 1\right)$ (C.13)–(C.22), $\lvert\bigcup_e \mathcal{P}_e\rvert$ (C.24), $\sum_c \bar{I}_c$ (C.25), r (C.26), $2(c+e+f+r+d)$ (C.31)–(C.35), and one (C.36). Put together to one formula, we get

$$\begin{aligned}
&\sum_c \left(\bar{I}_c - 1\right) + \sum_c \bar{I}_c + \sum_c 3^{\bar{I}_c} + c + 3\sum_c \left(2^{\bar{I}_c} - 1\right) + \left\lvert\bigcup_e \mathcal{P}_e\right\rvert + \sum_c \bar{I}_c \\
&+ r + 2\,(c+e+f+r+d) + 1 \\
&= \sum_c 3^{\bar{I}_c} + 3\sum_c 2^{\bar{I}_c} + 3\sum_c \bar{I}_c + \left\lvert\bigcup_e \mathcal{P}_e\right\rvert - c + 3\,r + 2\,(e+f+d) + 1 \\
&= \sum_c \left(3^{\bar{I}_c} + 3 \cdot 2^{\bar{I}_c} + 3\,\bar{I}_c\right) + \left\lvert\bigcup_e \mathcal{P}_e\right\rvert - c + 3\,r + 2\,(e+f+d) + 1 \qquad \text{(C.49)}
\end{aligned}$$

constraints. The results are summarised in Table C.1.

C.4 Data excerpt of core, distribution, recycling, and disposal graph

Table C.2 Excerpt of ρ_{cv}

core $c = 1$						core $c = 2$						core $c = 3$					
v	ρ_{cv}	v	ρ_{cv}	v	ρ_{cv}	v	ρ_{cv}	v	ρ_{cv}	v	ρ_{cv}	v	ρ_{cv}	v	ρ_{cv}	v	ρ_{cv}
1	0	4334	0.00463187	6530	0.06521768	1	0	4334	0.00423685	6530	0.06604381	1	0	4334	0.00092637	6530	0.06796369
2	0	4335	0.00378971	6531	0.05335992	2	0	4335	0.00346651	6531	0.05403585	2	0	4335	0.00075794	6531	0.05560665
3	0	4336	0	6532	0	3	0	4336	0	6532	0	3	0	4336	0	6532	0
4	0	4337	0.00378971	6533	0.05335992	4	0	4337	0.00346651	6533	0.05403585	4	0	4337	0.00075794	6533	0.05560665
5	0	4338	0.00310067	⋮	⋮	5	0	4338	0.00283624	⋮	⋮	5	0	4338	0.00062013	⋮	⋮
6	0	4339	0	6543	0.00011419	6	0	4339	0	6543	0.00012091	6	0	4339	0	6543	0.00011900
7	0	4340	0	6544	0	7	0	4340	0	6544	0	7	0	4340	0	6544	0
8	0	4341	0	6545	0	8	0	4341	0	6545	0	8	0	4341	0	6545	0
9	0	4342	0	6546	0	9	0	4342	0	6546	0	9	0	4342	0	6546	0
10	0	4343	0.00343251	6547	0	10	0	4343	0.00347599	6547	0	10	0	4343	0.00068650	6547	0
11	0	4344	0.00280842	6548	0.06521768	11	0	4344	0.00285177	6548	0.06604381	11	0	4344	0.00056168	6548	0.06796369
12	0	⋮	⋮	6549	0.05335992	12	0	⋮	⋮	6549	0.05403585	12	0	⋮	⋮	6549	0.05560665
13	0	6518	0	6550	0	13	0	6518	0	6550	0	13	0	6518	0	6550	0
14	0	6519	0	6551	0.05335992	14	0	6519	0	6551	0.05403585	14	0	6519	0	6551	0.05560665
15	0	6520	0	6552	0.04365811	15	0	6520	0	6552	0.04421115	15	0	6520	0	6552	0.04549635
16	0	6521	0.08800550	6553	0	16	0	6521	0.08050007	6553	0	16	0	6521	0.09171100	6553	0
⋮	⋮	6522	0.07200450	6554	0	⋮	⋮	6522	0.06586369	6554	0	⋮	⋮	6522	0.07503627	6554	0
4327	0	6523	0	6555	0	4327	0	6523	0	6555	0	4327	0	6523	0	6555	0
4328	0.00000991	6524	0.07200450	6556	0	4328	0.00000948	6524	0.06586369	6556	0	4328	0.00000198	6524	0.07503627	6556	0
4329	0.00000811	6525	0.05891277	6557	0.04833045	4329	0.00000776	6525	0.05388847	6557	0.05418362	4329	0.00000162	6525	0.06139331	6557	0.05036542
4330	0	6526	0	6558	0.03954310	4330	0	6526	0	6558	0.04433205	4330	0	6526	0	6558	0.04120807
4331	0	6527	0	6559	0	4331	0	6527	0	6559	0	4331	0	6527	0	6559	0
4332	0	6528	0	6560	0.03954310	4332	0	6528	0	6560	0.04433205	4332	0	6528	0	6560	0.04120807
4333	0	6529	0	6561	0.03235344	4333	0	6529	0	6561	0.03627168	4333	0	6529	0	6561	0.03371569

Values are rounded to eight digits.

Table C.3 Excerpt of $E^{\mathrm{C}}_{cv\tilde{v}} = 1$

core $c=1$, $c=2$, and $c=3$																	
from	to	from	to	from	to	from	to	from	to	from	to	from	to	from	to	from	to
v	$\tilde{v}$	v	$\tilde{v}$	v	$\tilde{v}$	v	$\tilde{v}$	v	$\tilde{v}$	v	$\tilde{v}$	v	$\tilde{v}$	v	$\tilde{v}$	v	$\tilde{v}$
6561	6560	6556	6529	6550	6547	6545	6302	6539	5810	66	65	51	48	39	30	23	14
6561	6558	6556	6475	6550	6541	6545	5816	6539	4352	66	57	51	42	39	12	22	19
6561	6552	6556	6313	6550	6523	6545	4358	6538	6535	66	39	51	24	38	37	22	13
6561	6534	6556	5827	6550	6469	6544	6535	6538	6511	65	64	50	49	38	29	21	20
6561	6480	6556	4369	6550	6307	6544	6517	6538	6457	65	56	50	47	38	11	21	12
6561	6318	6555	6554	6550	5821	6544	6463	6538	6295	65	38	50	41	37	28	20	19
6561	5832	6555	6546	6550	4363	6544	6301	6538	5809	64	55	50	23	37	10	20	11
6561	4374	6555	6528	6549	6548	6544	5815	6538	4351	64	37	49	46	36	35	19	10
6560	6559	6555	6474	6549	6546	6544	4357	6537	6536	63	62	49	40	36	33	18	17
6560	6557	6555	6312	6549	6540	6543	6542	6537	6510	63	60	49	22	36	9	18	15
6560	6551	6555	5826	6549	6522	6543	6540	6537	6456	63	36	48	47	35	34	18	9
6560	6533	6555	4368	6549	6468	6543	6516	6537	6294	62	61	48	39	35	32	17	16
6560	6479	6554	6553	6549	6306	6543	6462	6537	5808	62	59	48	21	35	8	17	14
6560	6317	6554	6545	6549	5820	6543	6300	6537	4350	62	35	47	46	34	31	17	8
6560	5831	6554	6527	6549	4362	6543	5814	6536	6535	61	58	47	38	34	7	16	13
6560	4373	6554	6473	6548	6547	6543	4356	6536	6509	61	34	47	20	33	32	16	7
6559	6556	6554	6311	6548	6545	6542	6541	6536	6455	60	59	46	37	33	30	15	14
6559	6550	6554	5825	6548	6539	6542	6539	6536	6293	60	57	46	19	33	6	15	12
6559	6532	6554	4367	6548	6521	6542	6515	6536	5807	60	33	45	44	32	31	15	6
6559	6478	6553	6544	6548	6467	6542	6461	6536	4349	59	58	45	42	32	29	14	13
6559	6316	6553	6526	6548	6305	6542	6299	⋮	⋮	59	56	45	36	32	5	14	11
6559	5830	6553	6472	6548	5819	6542	5813	72	63	59	32	45	18	31	28	14	5
6559	4372	6553	6310	6548	4361	6542	4355	72	45	58	55	44	43	31	4	13	10
6558	6557	6553	5824	6547	6544	6541	6538	71	70	58	31	44	41	30	29	13	4
6558	6555	6553	4366	6547	6538	6541	6514	71	68	57	56	44	35	30	3	12	11
6558	6549	6552	6551	6547	6520	6541	6460	71	62	57	30	44	17	29	28	12	3
6558	6531	6552	6549	6547	6466	6541	6298	71	44	56	55	43	40	29	2	11	10
6558	6477	6552	6543	6547	6304	6541	5812	70	67	56	29	43	34	28	1	11	2
6558	6315	6552	6525	6547	5818	6541	4354	70	61	55	28	43	16	27	26	10	1
6558	5829	6552	6471	6547	4360	6540	6539	70	43	54	53	42	41	27	24	9	8
6558	4371	6552	6309	6546	6545	6540	6537	69	68	54	51	42	39	27	18	9	6
6557	6556	6552	5823	6546	6537	6540	6513	69	66	54	45	42	33	26	25	8	7
6557	6554	6552	4365	6546	6519	6540	6459	69	60	54	27	42	15	26	23	8	5
6557	6548	6551	6550	6546	6465	6540	6297	69	42	53	52	41	40	26	17	7	4
6557	6530	6551	6548	6546	6303	6540	5811	68	67	53	50	41	38	25	22	6	5
6557	6476	6551	6542	6546	5817	6540	4353	68	65	53	44	41	32	25	16	6	3
6557	6314	6551	6524	6546	4359	6539	6538	68	59	53	26	41	14	24	23	5	4
6557	5828	6551	6470	6545	6544	6539	6536	68	41	52	49	40	37	24	21	5	2
6557	4370	6551	6308	6545	6536	6539	6512	67	64	52	43	40	31	24	15	4	1
6556	6553	6551	5822	6545	6518	6539	6458	67	58	52	25	40	13	23	22	3	2
6556	6547	6551	4364	6545	6464	6539	6296	67	40	51	50	39	38	23	20	2	1

Table C.4 Excerpt of $E^{\mathrm{D}}_{cw\tilde{w}} \geq 1$

core $c=1$, $c=2$, and $c=3$																	
from	to	value	from	to	value	from	to	value	from	to	value	from	to	value	from	to	value
w	$\tilde{w}$	$E^{\mathrm{D}}_{cw\tilde{w}}$	w	$\tilde{w}$	$E^{\mathrm{D}}_{cw\tilde{w}}$	w	$\tilde{w}$	$E^{\mathrm{D}}_{cw\tilde{w}}$	w	$\tilde{w}$	$E^{\mathrm{D}}_{cw\tilde{w}}$	w	$\tilde{w}$	$E^{\mathrm{D}}_{cw\tilde{w}}$	w	$\tilde{w}$	$E^{\mathrm{D}}_{cw\tilde{w}}$
1	2	1	1	43	1	1	84	1	1	125	1	215	249	241	227	254	253
1	3	1	1	44	1	1	85	1	1	126	1	215	252	247	228	252	1
1	4	1	1	45	1	1	86	1	1	127	1	216	242	1	228	254	252
1	5	1	1	46	1	1	87	1	1	128	1	216	243	1	229	251	1
1	6	1	1	47	1	1	88	1	1	129	128	216	245	1	229	254	251
1	7	1	1	48	1	1	89	1	1	130	127	216	249	242	230	250	1
1	8	1	1	49	1	1	90	1	1	131	126	216	250	243	230	254	250
1	9	1	1	50	1	1	91	1	1	132	125	216	251	245	231	249	1
1	10	1	1	51	1	1	92	1	1	133	124	217	242	1	231	254	249
1	11	1	1	52	1	1	93	1	1	134	123	217	244	1	232	248	1
1	12	1	1	53	1	1	94	1	1	135	122	217	246	1	232	254	248
1	13	1	1	54	1	1	95	1	1	136	121	217	248	242	233	252	1
1	14	1	1	55	1	1	96	1	1	137	120	217	250	244	233	253	252
1	15	1	1	56	1	1	97	1	1	138	119	217	251	246	234	251	1
1	16	1	1	57	1	1	98	1	1	139	118	218	243	1	234	253	251
1	17	1	1	58	1	1	99	1	1	140	117	218	244	1	235	250	1
1	18	1	1	59	1	1	100	1	1	141	116	218	247	1	235	253	250
1	19	1	1	60	1	1	101	1	1	142	115	218	248	243	236	249	1
1	20	1	1	61	1	1	102	1	1	143	114	218	249	244	236	253	249
1	21	1	1	62	1	1	103	1	1	144	113	218	251	247	237	248	1
1	22	1	1	63	1	1	104	1	1	145	112	219	245	1	237	253	248
1	23	1	1	64	1	1	105	1	1	146	111	219	246	1	238	251	1
1	24	1	1	65	1	1	106	1	1	147	110	219	247	1	238	252	251
1	25	1	1	66	1	1	107	1	1	148	109	219	248	245	239	250	1
1	26	1	1	67	1	1	108	1	1	149	108	219	249	246	239	252	250
1	27	1	1	68	1	1	109	1	1	150	107	219	250	247	240	249	1
1	28	1	1	69	1	1	110	1	1	151	106	220	254	1	240	252	249
1	29	1	1	70	1	1	111	1	1	152	105	220	255	254	241	248	1
1	30	1	1	71	1	1	112	1	1	153	104	221	253	1	241	252	248
1	31	1	1	72	1	1	113	1	1	154	103	221	255	253	242	250	1
1	32	1	1	73	1	1	114	1	1	155	102	222	252	1	242	251	250
1	33	1	1	74	1	1	115	1	1	156	101	222	255	252	243	249	1
1	34	1	1	75	1	1	116	1	1	157	100	223	251	1	243	251	249
1	35	1	1	76	1	1	117	1	1	158	99	223	255	251	244	248	1
1	36	1	1	77	1	1	118	1	1	159	98	224	250	1	244	251	248
1	37	1	1	78	1	1	119	1	1	160	97	224	255	250	245	249	1
1	38	1	1	79	1	1	120	1	1	161	96	225	249	1	245	250	249
1	39	1	1	80	1	1	121	1	⋮	⋮	⋮	225	255	249	246	248	1
1	40	1	1	81	1	1	122	1	215	241	1	226	248	1	246	250	248
1	41	1	1	82	1	1	123	1	215	247	1	226	255	248	247	248	1
1	42	1	1	83	1	1	124	1	215	248	240	227	253	1	247	249	248

Table C.5 Excerpt of L^{I}_{cw}

core $c=1$, $c=2$, and $c=3$					
w	L^{I}_{cw}	w	L^{I}_{cw}	w	L^{I}_{cw}
1	$\{6561\}$	29	$\{4941, 5022, 5670, 5751\}$	247	$\{9, 18, 36, 45, 90, 99, 117, 126, 252, 261, 279,$
2	$\{6559, 6560\}$	30	$\{4617, 4860, 5346, 5589\}$		$288, 333, 342, 360, 369, 738, 747, 765, 774,$
3	$\{6555, 6558\}$	31	$\{2185, 2186, 4372, 4373\}$		$819, 828, 846, 855, 981, 990, 1008, 1017, 1062$
4	$\{6543, 6552\}$	32	$\{2181, 2184, 4368, 4371\}$		$1071, 1089, 1098, 2196, 2205, 2223, 2232, 2277,$
5	$\{6507, 6534\}$	33	$\{2169, 2178, 4356, 4365\}$		$2286, 2304, 2313, 2439, 2448, 2466, 2475, 2520,$
6	$\{6399, 6480\}$	34	$\{2133, 2160, 4320, 4347\}$		$2529, 2547, 2556, 2925, 2934, 2952, 2961, 3006,$
7	$\{6075, 6318\}$	35	$\{2025, 2106, 4212, 4293\}$		$3015, 3033, 3042, 3168, 3177, 3195, 3204, 3249,$
8	$\{5103, 5832\}$	36	$\{1701, 1944, 3888, 4131\}$		$3258, 3276, 3285\}$
9	$\{2187, 4374\}$	37	$\{729, 1458, 2916, 3645\}$	$\vdots$	$\vdots$
10	$\{6553, 6554, 6556, 6557\}$	38	$\{6535, 6536, 6538, 6539, 6544, 6545, 6547, 6548\}$	255	$\{4375, 4376, 4378, 4379, 4384, 4385, 4387, 4388,$
11	$\{6541, 6542, 6550, 6551\}$	39	$\{6499, 6500, 6502, 6503, 6526, 6527, 6529, 6530\}$		$4402, 4403, 4405, 4406, 4411, 4412, 4414, 4415,$
12	$\{6537, 6540, 6546, 6549\}$	40	$\{6487, 6488, 6496, 6497, 6514, 6515, 6523, 6524\}$		$4456, 4457, 4459, 4460, 4465, 4466, 4468, 4469,$
13	$\{6505, 6506, 6532, 6533\}$	41	$\{6483, 6486, 6492, 6495, 6510, 6513, 6519, 6522\}$		$4483, 4484, 4486, 4487, 4492, 4493, 4495, 4496,$
14	$\{6501, 6504, 6528, 6531\}$	$\vdots$	$\vdots$		$4618, 4619, 4621, 4622, 4627, 4628, 4630, 4631,$
15	$\{6489, 6498, 6516, 6525\}$	160	$\{237, 240, 480, 483, 966, 969, 1209, 1212, 2424,$		$4645, 4646, 4648, 4649, 4654, 4655, 4657, 4658,$
16	$\{6397, 6398, 6478, 6479\}$		$2427, 2667, 2670, 3153, 3156, 3396, 3399\}$		$4699, 4700, 4702, 4703, 4708, 4709, 4711, 4712,$
17	$\{6393, 6396, 6474, 6477\}$	161	$\{225, 234, 468, 477, 954, 963, 1197, 1206, 2412,$		$4726, 4727, 4729, 4730, 4735, 4736, 4738, 4739,$
18	$\{6381, 6390, 6462, 6471\}$		$2421, 2655, 2664, 3141, 3150, 3384, 3393\}$		$5104, 5105, 5107, 5108, 5113, 5114, 5116, 5117,$
19	$\{6345, 6372, 6426, 6453\}$	162	$\{189, 216, 432, 459, 918, 945, 1161, 1188, 2376,$		$5131, 5132, 5134, 5135, 5140, 5141, 5143, 5144,$
20	$\{6073, 6074, 6316, 6317\}$		$2403, 2619, 2646, 3105, 3132, 3348, 3375\}$		$5185, 5186, 5188, 5189, 5194, 5195, 5197, 5198,$
21	$\{6069, 6072, 6312, 6315\}$	163	$\{81, 162, 324, 405, 810, 891, 1053, 1134, 2268,$		$5212, 5213, 5215, 5216, 5221, 5222, 5224, 5225,$
22	$\{6057, 6066, 6300, 6309\}$		$2349, 2511, 2592, 2997, 3078, 3240, 3321\}$		$5347, 5348, 5350, 5351, 5356, 5357, 5359, 5360,$
23	$\{6021, 6048, 6264, 6291\}$	$\vdots$	$\vdots$		$5374, 5375, 5377, 5378, 5383, 5384, 5386, 5387,$
24	$\{5913, 5994, 6156, 6237\}$	219	$\{27, 54, 108, 135, 270, 297, 351, 378, 756, 783,$		$5428, 5429, 5431, 5432, 5437, 5438, 5440, 5441,$
25	$\{5101, 5102, 5830, 5831\}$		$837, 864, 999, 1026, 1080, 1107, 2214, 2241,$		$5455, 5456, 5458, 5459, 5464, 5465, 5467, 5468\}$
26	$\{5097, 5100, 5826, 5829\}$		$2295, 2322, 2457, 2484, 2538, 2565, 2943, 2970,$	256	$\emptyset$
27	$\{5085, 5094, 5814, 5823\}$		$3024, 3051, 3186, 3213, 3267, 3294\}$		
28	$\{5049, 5076, 5778, 5805\}$	$\vdots$	$\vdots$		

Table C.6 Excerpt of L^{R}_{cw}

core $c=1$, $c=2$, and $c=3$					
w	L^{R}_{cw}	w	L^{R}_{cw}	w	L^{R}_{cw}
1	{3281}	29	{2471, 2633, 3929, 4091}	247	{5, 23, 59, 77, 167, 185, 221, 239, 491, 509,
2	{3280, 3282}	30	{2309, 2795, 3767, 4253}		545, 563, 653, 671, 707, 725, 1463, 1481, 1517,
3	{3278, 3284}	31	{1093, 1095, 5467, 5469}		1535, 1625, 1643, 1679, 1697, 1949, 1967, 2003,
4	{3272, 3290}	32	{1091, 1097, 5465, 5471}		2021, 2111, 2129, 2165, 2183, 4379, 4397, 4433,
5	{3254, 3308}	33	{1085, 1103, 5459, 5477}		4451, 4541, 4559, 4595, 4613, 4865, 4883, 4919,
6	{3200, 3362}	34	{1067, 1121, 5441, 5495}		4937, 5027, 5045, 5081, 5099, 5837, 5855, 5891,
7	{3038, 3524}	35	{1013, 1175, 5387, 5549}		5909, 5999, 6017, 6053, 6071, 6323, 6341, 6377,
8	{2552, 4010}	36	{851, 1337, 5225, 5711}		6395, 6485, 6503, 6539, 6557}
9	{1094, 5468}	37	{365, 1823, 4739, 6197}	⋮	⋮
10	{3277, 3279, 3283, 3285}	38	{3268, 3270, 3274, 3276, 3286, 3288, 3292, 3294}	255	{2188, 2190, 2194, 2196, 2206, 2208, 2212, 2214,
11	{3271, 3273, 3289, 3291}	39	{3250, 3252, 3256, 3258, 3304, 3306, 3310, 3312}		2242, 2244, 2248, 2250, 2260, 2262, 2266, 2268,
12	{3269, 3275, 3287, 3293}	40	{3244, 3246, 3262, 3264, 3298, 3300, 3316, 3318}		2350, 2352, 2356, 2358, 2368, 2370, 2374, 2376,
13	{3253, 3255, 3307, 3309}	41	{3242, 3248, 3260, 3266, 3296, 3302, 3314, 3320}		2404, 2406, 2410, 2412, 2422, 2424, 2428, 2430,
14	{3251, 3257, 3305, 3311}	⋮	⋮		2674, 2676, 2680, 2682, 2692, 2694, 2698, 2700,
15	{3245, 3263, 3299, 3317}	160	{119, 125, 605, 611, 1577, 1583, 2063, 2069, 4493,		2728, 2730, 2734, 2736, 2746, 2748, 2752, 2754,
16	{3199, 3201, 3361, 3363}		4499, 4979, 4985, 5951, 5957, 6437, 6443}		2836, 2838, 2842, 2844, 2854, 2856, 2860, 2862,
17	{3197, 3203, 3359, 3365}	161	{113, 131, 599, 617, 1571, 1589, 2057, 2075, 4487,		2890, 2892, 2896, 2898, 2908, 2910, 2914, 2916,
18	{3191, 3209, 3353, 3371}		4505, 4973, 4991, 5945, 5963, 6431, 6449}		3646, 3648, 3652, 3654, 3664, 3666, 3670, 3672,
19	{3173, 3227, 3335, 3389}	162	{95, 149, 581, 635, 1553, 1607, 2039, 2093, 4469,		3700, 3702, 3706, 3708, 3718, 3720, 3724, 3726,
20	{3037, 3039, 3523, 3525}		4523, 4955, 5009, 5927, 5981, 6413, 6467}		3808, 3810, 3814, 3816, 3826, 3828, 3832, 3834,
21	{3035, 3041, 3521, 3527}	163	{41, 203, 527, 689, 1499, 1661, 1985, 2147, 4415,		3862, 3864, 3868, 3870, 3880, 3882, 3886, 3888,
22	{3029, 3047, 3515, 3533}		4577, 4901, 5063, 5873, 6035, 6359, 6521}		4132, 4134, 4138, 4140, 4150, 4152, 4156, 4158,
23	{3011, 3065, 3497, 3551}	⋮	⋮		4186, 4188, 4192, 4194, 4204, 4206, 4210, 4212,
24	{2957, 3119, 3443, 3605}	219	{14, 68, 176, 230, 500, 554, 662, 716, 1472, 1526,		4294, 4296, 4300, 4302, 4312, 4314, 4318, 4320,
25	{2551, 2553, 4009, 4011}		1634, 1688, 1958, 2012, 2120, 2174, 4388, 4442,		4348, 4350, 4354, 4356, 4366, 4368, 4372, 4374}
26	{2549, 2555, 4007, 4013}		4550, 4604, 4874, 4928, 5036, 5090, 5846, 5900,	256	∅
27	{2543, 2561, 4001, 4019}		6008, 6062, 6332, 6386, 6494, 6548}		
28	{2525, 2579, 3983, 4037}	⋮	⋮		

Table C.7 Excerpt of L_{cw}^{D}

core $c = 1$, $c = 2$, and $c = 3$					
w	L_{cw}^{D}	w	L_{cw}^{D}	w	L_{cw}^{D}
1	$\{1\}$	29	$\{811, 892, 1540, 1621\}$	247	$\{3277, 3286, 3304, 3313, 3358, 3367, 3385, 3394,$
2	$\{2, 3\}$	30	$\{973, 1216, 1702, 1945\}$		$3520, 3529, 3547, 3556, 3601, 3610, 3628, 3637,$
3	$\{4, 7\}$	31	$\{2189, 2190, 4376, 4377\}$		$4006, 4015, 4033, 4042, 4087, 4096, 4114, 4123,$
4	$\{10, 19\}$	32	$\{2191, 2194, 4378, 4381\}$		$4249, 4258, 4276, 4285, 4330, 4339, 4357, 4366,$
5	$\{28, 55\}$	33	$\{2197, 2206, 4384, 4393\}$		$5464, 5473, 5491, 5500, 5545, 5554, 5572, 5581,$
6	$\{82, 163\}$	34	$\{2215, 2242, 4402, 4429\}$		$5707, 5716, 5734, 5743, 5788, 5797, 5815, 5824,$
7	$\{244, 487\}$	35	$\{2269, 2350, 4456, 4537\}$		$6193, 6202, 6220, 6229, 6274, 6283, 6301, 6310,$
8	$\{730, 1459\}$	36	$\{2431, 2674, 4618, 4861\}$		$6436, 6445, 6463, 6472, 6517, 6526, 6544, 6553\}$
9	$\{2188, 4375\}$	37	$\{2917, 3646, 5104, 5833\}$	$\vdots$	$\vdots$
10	$\{5, 6, 8, 9\}$	38	$\{14, 15, 17, 18, 23, 24, 26, 27\}$	255	$\{1094, 1095, 1097, 1098, 1103, 1104, 1106, 1107,$
11	$\{11, 12, 20, 21\}$	39	$\{32, 33, 35, 36, 59, 60, 62, 63\}$		$1121, 1122, 1124, 1125, 1130, 1131, 1133, 1134,$
12	$\{13, 16, 22, 25\}$	40	$\{38, 39, 47, 48, 65, 66, 74, 75\}$		$1175, 1176, 1178, 1179, 1184, 1185, 1187, 1188,$
13	$\{29, 30, 56, 57\}$	41	$\{40, 43, 49, 52, 67, 70, 76, 79\}$		$1202, 1203, 1205, 1206, 1211, 1212, 1214, 1215,$
14	$\{31, 34, 58, 61\}$	$\vdots$	$\vdots$		$1337, 1338, 1340, 1341, 1346, 1347, 1349, 1350,$
15	$\{37, 46, 64, 73\}$	160	$\{3163, 3166, 3406, 3409, 3892, 3895, 4135, 4138,$		$1364, 1365, 1367, 1368, 1373, 1374, 1376, 1377,$
16	$\{83, 84, 164, 165\}$		$5350, 5353, 5593, 5596, 6079, 6082, 6322, 6325\}$		$1418, 1419, 1421, 1422, 1427, 1428, 1430, 1431,$
17	$\{85, 88, 166, 169\}$	161	$\{3169, 3178, 3412, 3421, 3898, 3907, 4141, 4150,$		$1445, 1446, 1448, 1449, 1454, 1455, 1457, 1458,$
18	$\{91, 100, 172, 181\}$		$5356, 5365, 5599, 5608, 6085, 6094, 6328, 6337\}$		$1823, 1824, 1826, 1827, 1832, 1833, 1835, 1836,$
19	$\{109, 136, 190, 217\}$	162	$\{3187, 3214, 3430, 3457, 3916, 3943, 4159, 4186,$		$1850, 1851, 1853, 1854, 1859, 1860, 1862, 1863,$
20	$\{245, 246, 488, 489\}$		$5374, 5401, 5617, 5644, 6103, 6130, 6346, 6373\}$		$1904, 1905, 1907, 1908, 1913, 1914, 1916, 1917,$
21	$\{247, 250, 490, 493\}$	163	$\{3241, 3322, 3484, 3565, 3970, 4051, 4213, 4294,$		$1931, 1932, 1934, 1935, 1940, 1941, 1943, 1944,$
22	$\{253, 262, 496, 505\}$		$5428, 5509, 5671, 5752, 6157, 6238, 6400, 6481\}$		$2066, 2067, 2069, 2070, 2075, 2076, 2078, 2079,$
23	$\{271, 298, 514, 541\}$	$\vdots$	$\vdots$		$2093, 2094, 2096, 2097, 2102, 2103, 2105, 2106,$
24	$\{325, 406, 568, 649\}$	219	$\{3268, 3295, 3349, 3376, 3511, 3538, 3592, 3619,$		$2147, 2148, 2150, 2151, 2156, 2157, 2159, 2160,$
25	$\{731, 732, 1460, 1461\}$		$3997, 4024, 4078, 4105, 4240, 4267, 4321, 4348,$		$2174, 2175, 2177, 2178, 2183, 2184, 2186, 2187\}$
26	$\{733, 736, 1462, 1465\}$		$5455, 5482, 5536, 5563, 5698, 5725, 5779, 5806,$	256	$\emptyset$
27	$\{739, 748, 1468, 1477\}$		$6184, 6211, 6265, 6292, 6427, 6454, 6508, 6535\}$		
28	$\{757, 784, 1486, 1513\}$	$\vdots$	$\vdots$		

Table C.8 Module/item mapping L^{A}_{cw}

core $c=1$, $c=2$, and $c=3$															
w	L^{A}_{cw}	w	L^{A}_{cw}	w	L^{A}_{cw}	w	L^{A}_{cw}	w	L^{A}_{cw}	w	L^{A}_{cw}	w	L^{A}_{cw}	w	L^{A}_{cw}
1	1	33	.	65	.	97	33	129	.	161	.	193	.	225	.
2	2	34	.	66	.	98	35	130	.	162	.	194	.	226	.
3	3	35	.	67	.	99	.	131	34	163	.	195	.	227	48
4	4	36	.	68	.	100	.	132	36	164	40	196	.	228	.
5	5	37	.	69	.	101	.	133	.	165	.	197	.	229	.
6	6	38	17	70	.	102	.	134	.	166	42	198	.	230	.
7	.	39	18	71	.	103	32	135	.	167	43	199	.	231	.
8	.	40	20	72	.	104	.	136	.	168	.	200	.	232	50
9	.	41	23	73	.	105	37	137	.	169	45	201	.	233	49
10	7	42	19	74	.	106	.	138	39	170	.	202	.	234	.
11	8	43	21	75	.	107	38	139	.	171	.	203	.	235	.
12	11	44	24	76	.	108	.	140	.	172	.	204	.	236	.
13	9	45	22	77	.	109	.	141	.	173	.	205	.	237	.
14	12	46	25	78	28	110	.	142	.	174	.	206	.	238	.
15	14	47	27	79	.	111	.	143	.	175	.	207	.	239	.
16	10	48	.	80	.	112	.	144	.	176	.	208	.	240	.
17	13	49	.	81	.	113	.	145	.	177	.	209	.	241	.
18	15	50	.	82	.	114	.	146	.	178	.	210	.	242	.
19	16	51	.	83	.	115	.	147	.	179	.	211	.	243	.
20	.	52	.	84	.	116	.	148	.	180	.	212	.	244	.
21	.	53	.	85	.	117	.	149	.	181	.	213	.	245	.
22	.	54	.	86	.	118	.	150	.	182	.	214	.	246	.
23	.	55	26	87	.	119	.	151	.	183	.	215	.	247	.
24	.	56	.	88	.	120	.	152	.	184	.	216	.	248	1
25	.	57	.	89	.	121	.	153	.	185	41	217	.	249	2
26	.	58	.	90	.	122	.	154	.	186	.	218	.	250	3
27	.	59	.	91	.	123	.	155	.	187	.	219	.	251	4
28	.	60	.	92	.	124	.	156	.	188	44	220	47	252	5
29	.	61	.	93	.	125	.	157	.	189	46	221	.	253	6
30	.	62	.	94	29	126	.	158	.	190	.	222	.	254	7
31	.	63	.	95	30	127	.	159	.	191	.	223	.	255	8
32	.	64	.	96	31	128	.	160	.	192	.	224	.	256	.

Dots denote zero values. The values of L^{A}_{cw} with $1 \le w \le 247$ are module indices and with $248 \le w \le 255$ are item indices.

C.5 Optimal solutions of state limited flexible planning

Table C.9 Optimal solution with four states

variables representing the interfaces									
Q_1^{C}	30	Q_1^{I}	246	Q_1^{M}	30	Q_1^{R}	36,213	Q_4^{R}	0
Q_2^{C}	216	Q_2^{I}	152	Q_2^{M}	90	Q_2^{R}	70,679	Q_1^{D}	32
Q_3^{C}	31	Q_3^{I}	213			Q_3^{R}	0	Q_2^{D}	6,000

integer variables

	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
				$r=1$			$r=2$			$d=1$			$d=2$		
	c			c			c			c			c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13	97	13	17	119	18	.	.	.	.	.	.	.	.	.
B	13	97	13	17	119	18	.	.	.	.	.	.	.	.	.
C	.	.	.	29	213	30	.	.	.	1	.	1	.	.	.
D	.	.	.	29	122	30	.	91	.	1	.	1	.	.	.
E	29	123	.	1	.	31	.	.	.	.	.	.	.	.	.
F	.	.	.	.	123	31	.	.	.	.	.	.	.	.	.
G	.	213	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	213	.	.	.	.	30	.	.

	Y_{cm}^{M}			Y_{cmr}^{R}					
				$r=1$			$r=2$		
	c			c			c		
m	1	2	3	1	2	3	1	2	3
32 (CDGH)	.	.	.	.	.	.	.	3	.
47 (GH)	.	.	30	.	.	.	.	.	1
48 (FG)	.	.	.	.	.	.	30	.	.
49 (EF)	.	90	.	.	3	.	.	.	.

Dots denote zero values.

Table C.10 Optimal solution with five and six states

Five states

variables representing the interfaces with **five states**

Q_1^C	30	Q_1^I	244	Q_1^M	30	Q_1^R	35,772	Q_4^R	0
Q_2^C	217	Q_2^I	153	Q_2^M	90	Q_2^R	71,508	Q_1^D	32
Q_3^C	31	Q_3^I	214			Q_3^R	0	Q_2^D	6,000

integer variables

	X_{ci}^I			X_{cir}^R						X_{cid}^D					
	c			$r=1$, c			$r=2$, c			$d=1$, c			$d=2$, c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13	96	13	.	118	18	17	.	.	.	.	.	.	.	.
B	13	96	13	.	118	13	17	.	5	.	.	.	.	.	.
C	.	.	.	29	207	30	.	7	.	1	.	1	.	.	.
D	.	.	.	.	214	30	29	.	.	1	.	1	.	.	.
E	29	124	.	1	1	31	.	.	.	.	.	.	.	.	.
F	.	.	.	.	122	31	.	.	.	.	.	.	.	.	.
G	.	214	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	214	.	.	.	.	30	.	.

		Y_{cm}^M			Y_{cmr}^R					
		c			$r=1$, c			$r=2$, c		
m		1	2	3	1	2	3	1	2	3
6	(ABCDFGH)	.	.	.	.	.	.	.	3	.
47	(GH)	.	.	30	.	.	.	.	.	1
48	(FG)	.	.	.	.	.	.	30	.	.
49	(EF)	.	90	.	.	2	.	.	.	.

Six states

variables representing the interfaces with **six states**

Q_1^C	30	Q_1^I	246	Q_1^M	30	Q_1^R	35,993	Q_4^R	0
Q_2^C	217	Q_2^I	153	Q_2^M	90	Q_2^R	71,265	Q_1^D	32
Q_3^C	31	Q_3^I	214			Q_3^R	0	Q_2^D	6,000

integer variables

	X_{ci}^I			X_{cir}^R						X_{cid}^D					
	c			$r=1$, c			$r=2$, c			$d=1$, c			$d=2$, c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	13	97	13	5	118	18	2	.	.	.	.	.	.	.	.
B	13	97	13	.	118	18	7	.	.	.	.	.	.	.	.
C	.	.	.	29	164	30	.	50	.	1	.	1	.	.	.
D	.	.	.	29	214	30	.	.	.	1	.	1	.	.	.
E	29	124	.	1	.	31	.	.	.	.	.	.	.	.	.
F	.	.	.	.	122	31	.	.	.	.	.	.	.	.	.
G	.	214	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	214	.	.	.	.	30	.	.

		Y_{cm}^M			Y_{cmr}^R					
		c			$r=1$, c			$r=2$, c		
m		1	2	3	1	2	3	1	2	3
6	(ABCDFGH)	.	.	.	.	.	.	.	2	.
32	(CDGH)	.	.	.	.	.	.	.	1	.
39	(ABFG)	.	.	.	.	.	.	10	.	.
47	(GH)	.	.	30	.	.	.	.	.	1
48	(FG)	.	.	.	.	.	.	20	.	.
49	(EF)	.	90	.	.	3	.	.	.	.

Dots denote zero values.

Table C.11 Optimal solution with seven and eight states

Variables representing the interfaces with seven states

Q_1^{C}	30	Q_1^{I}	233	Q_1^{M}	30	Q_1^{R}	35,727	Q_4^{R}	0
Q_2^{C}	218	Q_2^{I}	154	Q_2^{M}	90	Q_2^{R}	72,040	Q_1^{D}	32
Q_3^{C}	31	Q_3^{I}	215			Q_3^{R}	0	Q_2^{D}	6,000

Integer variables (seven states)

	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
	c			$r=1$, c			$r=2$, c			$d=1$, c			$d=2$, c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	.	97	13	.	105	.	.	13	17	.	.	.	.	.	.
B	13	97	13	.	118	17	.	.	.	.	.	.	.	.	.
C	.	.	.	29	215	30	.	.	.	1	.	1	.	.	.
D	.	.	.	29	214	30	.	1	.	1	.	1	.	.	.
E	29	125	.	1	1	30	.	.	.	.	.	.	.	.	.
F	.	.	.	.	123	30	.	.	.	.	.	.	.	.	.
G	.	215	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	215	.	.	.	.	30	.	.

		Y_{cm}^{M}			Y_{cmr}^{R}					
		c			$r=1$, c			$r=2$, c		
m		1	2	3	1	2	3	1	2	3
6	(ABCDFGH)	.	.	.	.	.	.	.	3	.
14	(ABEFGH)	.	.	.	.	.	.	.	.	1
39	(ABFG)	.	.	.	.	.	.	17	.	.
46	(AFG)	.	.	.	.	.	.	13	.	.
47	(GH)	.	.	30	.	.	.	.	.	.
49	(EF)	.	90	.	.	2	.	.	.	.

Variables representing the interfaces with eight states

Q_1^{C}	30	Q_1^{I}	235	Q_1^{M}	30	Q_1^{R}	35,677	Q_4^{R}	0
Q_2^{C}	218	Q_2^{I}	154	Q_2^{M}	90	Q_2^{R}	72,068	Q_1^{D}	32
Q_3^{C}	31	Q_3^{I}	215			Q_3^{R}	0	Q_2^{D}	6,000

Integer variables (eight states)

	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
	c			$r=1$, c			$r=2$, c			$d=1$, c			$d=2$, c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	.	98	13	.	119	2	.	.	15	.	.	.	.	.	.
B	13	98	13	.	119	1	.	.	16	.	.	.	.	.	.
C	.	.	.	29	212	30	.	3	.	1	.	1	.	.	.
D	.	.	.	29	214	30	.	1	.	1	.	1	.	.	.
E	29	125	.	1	.	30	.	.	.	.	.	.	.	.	.
F	.	.	.	.	123	30	.	.	.	.	.	.	.	.	.
G	.	215	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	215	.	.	.	.	30	.	.

		Y_{cm}^{M}			Y_{cmr}^{R}					
		c			$r=1$, c			$r=2$, c		
m		1	2	3	1	2	3	1	2	3
1	(ABCDEFGH)	.	.	.	.	.	.	.	1	.
14	(ABEFGH)	.	.	.	.	.	.	.	.	1
19	(CDFGH)	.	.	.	.	.	.	.	2	.
39	(ABFG)	.	.	.	.	.	.	17	.	.
46	(AFG)	.	.	.	.	.	.	13	.	.
47	(GH)	.	.	30	.	.	.	.	.	.
49	(EF)	.	90	.	.	2	.	.	.	.

Dots denote zero values.

Table C.12 Optimal solution with nine and ten states

variables representing the interfaces with **seven states**

Q_1^{C}	30	Q_1^{I}	238	Q_1^{M}	30	Q_1^{R}	35,677	Q_4^{R}	0
Q_2^{C}	218	Q_2^{I}	154	Q_2^{M}	90	Q_2^{R}	72,035	Q_1^{D}	32
Q_3^{C}	31	Q_3^{I}	215			Q_3^{R}	0	Q_2^{D}	6,000

integer variables

	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
	c			$r=1$ c			$r=2$ c			$d=1$ c			$d=2$ c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	5	97	13	.	118	17	.	.	.	.	.	.	.	.	.
B	13	97	13	.	118	16	.	.	1	.	.	.	.	.	.
C	.	.	.	29	167	30	.	48	.	1	.	1	.	.	.
D	.	.	.	29	215	30	.	.	.	1	.	1	.	.	.
E	29	125	.	.	1	30	.	.	.	.	.	.	.	.	.
F	.	.	.	.	123	30	.	.	.	.	.	.	.	.	.
G	.	215	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	215	.	.	.	.	30	.	.

		Y_{cm}^{M}			Y_{cmr}^{R}					
		c			$r=1$ c			$r=2$ c		
m		1	2	3	1	2	3	1	2	3
6	(ABCDFGH)	.	.	.	.	.	.	.	3	.
14	(ABEFGH)	.	.	.	.	.	.	.	.	1
28	(ABEFG)	.	.	.	.	.	.	1	.	.
39	(ABFG)	.	.	.	.	.	.	16	.	.
46	(AFG)	.	.	.	.	.	.	8	.	.
47	(GH)	.	.	30	.	.	.	.	.	.
48	(FG)	.	.	.	.	.	.	5	.	.
49	(EF)	.	90	.	.	2	.	.	.	.

variables representing the interfaces with **seven states**

Q_1^{C}	30	Q_1^{I}	241	Q_1^{M}	30	Q_1^{R}	35,627	Q_4^{R}	0
Q_2^{C}	218	Q_2^{I}	154	Q_2^{M}	90	Q_2^{R}	72,052	Q_1^{D}	32
Q_3^{C}	31	Q_3^{I}	215			Q_3^{R}	0	Q_2^{D}	6,000

integer variables

	X_{ci}^{I}			X_{cir}^{R}						X_{cid}^{D}					
	c			$r=1$ c			$r=2$ c			$d=1$ c			$d=2$ c		
i	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
A	7	97	13	.	115	.	.	3	17	.	.	.	.	.	.
B	13	98	13	.	108	16	.	11	1	.	.	.	.	.	.
C	.	.	.	29	215	30	.	.	.	1	.	1	.	.	.
D	.	.	.	28	215	29	1	.	1	1	.	1	.	.	.
E	29	125	.	.	.	30	.	.	.	.	.	.	.	.	.
F	.	.	.	.	123	30	.	.	.	.	.	.	.	.	.
G	.	215	.	.	.	.	.	.	.	.	.	.	.	.	.
H	.	.	.	.	.	.	.	215	.	.	.	.	30	.	.

		Y_{cm}^{M}			Y_{cmr}^{R}					
		c			$r=1$ c			$r=2$ c		
m		1	2	3	1	2	3	1	2	3
1	(ABCDEFGH)	.	.	.	.	.	.	.	1	.
13	(ACDFGH)	.	.	.	.	.	.	.	2	.
14	(ABEFGH)	.	.	.	.	.	.	.	.	1
28	(ABEFG)	.	.	.	.	.	.	1	.	.
39	(ABFG)	.	.	.	.	.	.	9	.	.
44	(BFG)	.	.	.	.	.	.	7	.	.
46	(AFG)	.	.	.	.	.	.	13	.	.
47	(GH)	.	.	30	.	.	.	.	.	.
49	(EF)	.	90	.	.	2	.	.	.	.

Dots denote zero values.

C.6 Module and state definition for the two examples

For a better understanding the disassembly state graphs of the two small examples (Fig. 4.13 and 4.14) are repeated here. Based on these graphs the module definition matrices δ_{cmi} for each core as well as the state definition matrices $\gamma_{cms}^{\mathrm{M}}$ and $\gamma_{cis}^{\mathrm{I}}$ are developed. We start with the module definition for both examples (cores) in Table C.13. Only the relevant indices m and i are listed. The state definitions are displayed in two separate Tables C.14

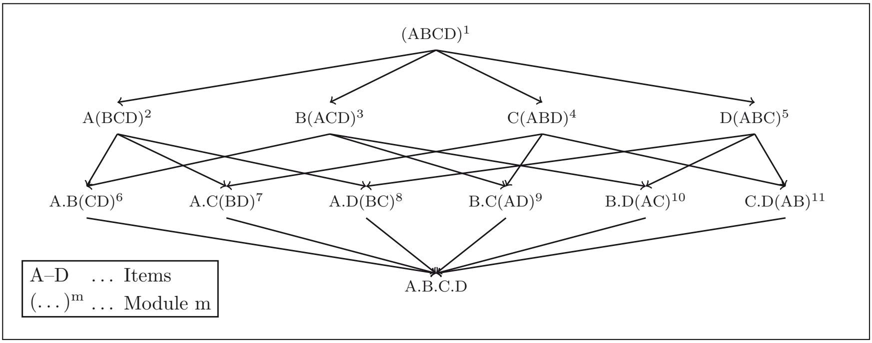


Fig. C.5 Disassembly state graph of example one

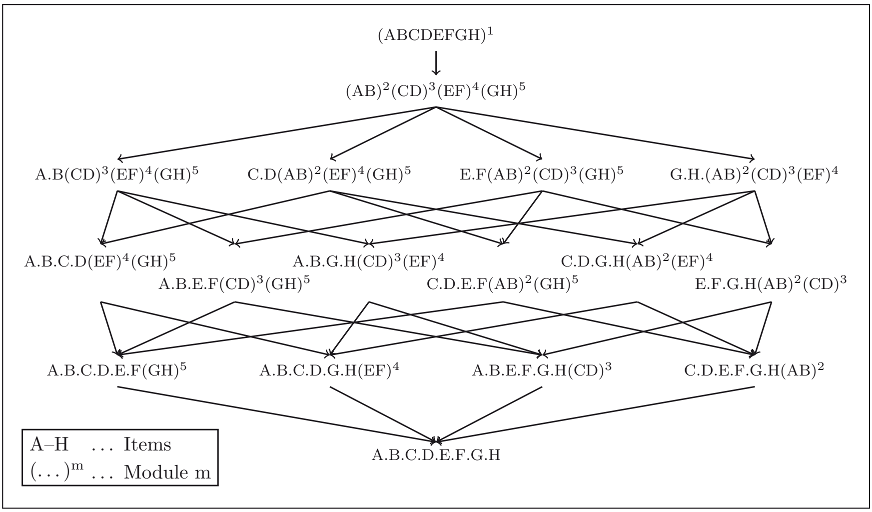


Fig. C.6 Disassembly state graph of example two

Table C.13 Module definition

	δ_{cmi}											
	$c = 1$				$c = 2$							
	item i				item i							
m	A	B	C	D	A	B	C	D	E	F	G	H
1	1	1	1	1	1	1	1	1	1	1	1	1
2	.	1	1	1	1	1	.	.	.	.	.	.
3	1	.	1	1	.	.	1	1	.	.	.	.
4	1	1	.	1	.	.	.	.	1	1	.	.
5	1	1	1	.	.	.	.	.	.	.	1	1
6	.	.	1	1								
7	.	1	.	1								
8	.	1	1	.								
9	1	.	.	1								
10	1	.	1	.								
11	1	1	.	.								

Dots denote zero values.

Table C.14 State definition of example one

			state s											
			1	2	3	4	5	6	7	8	9	10	11	12
$\gamma^{\mathrm{M}}_{1,m,s}$	m	1	1	.	.	.	.	.	.	.	.	.	.	.
		2	.	1	.	.	.	.	.	.	.	.	.	.
		3	.	.	1	.	.	.	.	.	.	.	.	.
		4	.	.	.	1	.	.	.	.	.	.	.	.
		5	.	.	.	.	1	.	.	.	.	.	.	.
		6	.	.	.	.	.	1	.	.	.	.	.	.
		7	.	.	.	.	.	.	1	.	.	.	.	.
		8	.	.	.	.	.	.	.	1	.	.	.	.
		9	.	.	.	.	.	.	.	.	1	.	.	.
		10	.	.	.	.	.	.	.	.	.	1	.	.
		11	.	.	.	.	.	.	.	.	.	.	1	.
$\gamma^{\mathrm{I}}_{1,i,s}$	i	A	.	1	.	.	.	1	1	1	.	.	.	1
		B	.	.	1	.	.	1	.	.	1	1	.	1
		C	.	.	.	1	.	.	1	.	1	.	1	1
		D	.	.	.	.	1	.	.	1	.	1	1	1

Dots denote zero values.

and C.15 for example one and two, respectively. In Table C.14 the allocation of one module per state becomes obvious. State $s = 1$ represents no disassembly operation and states $s = 12$ and $s = 17$ represent the complete disassembly for example one and two, respectively. Both tables contain the

Table C.15 State definition of example two

			state s																
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$\gamma^{\mathrm{M}}_{2,m,s}$	m	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
		2	.	1	.	1	1	1	.	.	.	1	1	1	.	.	.	1	.
		3	.	1	1	.	1	1	.	1	1	.	.	1	.	.	1	.	.
		4	.	1	1	1	.	1	1	.	1	.	1	.	.	1	.	.	.
		5	.	1	1	1	1	.	1	1	.	1	.	.	1	.	.	.	.
$\gamma^{\mathrm{I}}_{2,i,s}$	i	A	.	.	1	.	.	.	1	1	1	.	.	.	1	1	1	.	1
		B	.	.	1	.	.	.	1	1	1	.	.	.	1	1	1	.	1
		C	.	.	.	1	.	.	1	.	.	1	1	.	1	1	.	1	1
		D	.	.	.	1	.	.	1	.	.	1	1	.	1	1	.	1	1
		E	.	.	.	.	1	.	.	1	.	1	.	1	1	.	1	1	1
		F	.	.	.	.	1	.	.	1	.	1	.	1	1	.	1	1	1
		G	.	.	.	.	.	1	.	.	1	.	1	1	.	1	1	1	1
		H	.	.	.	.	.	1	.	.	1	.	1	1	.	1	1	1	1

Dots denote zero values.

binary matrices, which is a different form of presenting basically the same information as in Table 4.19 on page 227. Thus, both types are included in this work.

C.7 State quantity determination for example one

The example can be found in Sect. 4.4 and more specific in Eq. (4.148) on page 242. For a better understanding the system of equations is repeated here with the assumed solution vector $(X\ Y)^{\mathrm{T}}$.

$$\begin{pmatrix} 0&1&0&0&0&1&1&1&0&0&0&1\\ 0&0&1&0&0&1&0&0&1&1&0&1\\ 0&0&0&1&0&0&1&0&1&0&1&1\\ 0&0&0&0&1&0&0&1&0&1&1&1\\ 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0 \end{pmatrix} \begin{pmatrix} Q_1^{\mathrm{S}}\\ Q_2^{\mathrm{S}}\\ Q_3^{\mathrm{S}}\\ Q_4^{\mathrm{S}}\\ Q_5^{\mathrm{S}}\\ Q_6^{\mathrm{S}}\\ Q_7^{\mathrm{S}}\\ Q_8^{\mathrm{S}}\\ Q_9^{\mathrm{S}}\\ Q_{10}^{\mathrm{S}}\\ Q_{11}^{\mathrm{S}}\\ Q_{12}^{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} X_{\mathrm{A}}\\ X_{\mathrm{B}}\\ X_{\mathrm{C}}\\ X_{\mathrm{D}}\\ Y_1\\ Y_2\\ Y_3\\ Y_4\\ Y_5\\ Y_6\\ Y_7\\ Y_8\\ Y_9\\ Y_{10}\\ Y_{11} \end{pmatrix} = \begin{pmatrix} 2\\0\\3\\6\\1\\2\\0\\3\\6\\0\\0\\0\\0\\0\\0 \end{pmatrix} \tag{C.50}$$

The set of states is $S = \{1, \ldots, 12\}$ and the priorities are 1, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, and 4 for the states 1 through 12, respectively. Following the algorithm in Fig. 4.15 on page 246 we start with the elimination of the rows with a zero value on the right hand side. These rows are $\tilde{J} = \{2, 7, 10, 11, 12, 13, 14, 15\}$. The corresponding selected states are $\tilde{S} = \{3, 6, 7, 8, 9, 10, 11, 12\}$. After the elimination the system is reduced to four states and seven rows.

$$\begin{pmatrix} 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1\\ 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{pmatrix} \begin{pmatrix} Q_1^{\mathrm{S}}\\ Q_2^{\mathrm{S}}\\ Q_4^{\mathrm{S}}\\ Q_5^{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} X_{\mathrm{A}}\\ X_{\mathrm{C}}\\ X_{\mathrm{D}}\\ Y_1\\ Y_2\\ Y_4\\ Y_5 \end{pmatrix} = \begin{pmatrix} 2\\3\\6\\1\\2\\3\\6 \end{pmatrix} \tag{C.51}$$

The set S is not empty and $\mathbf{r}$ has no elements with a value of zero. But, rows with a row sum of one exist. Accordingly, the values of the quantity variables Q_1^{S}, Q_2^{S}, Q_4^{S}, and Q_5^{S} are set to the values one, two, three, and six. All selected states are removed from S and the set S is empty. Hence, the algorithm ends. The same result as with the closed form expression is found.

C.8 Incoming units and their assignment for cores 2 and 3

Table C.16 Listing of incoming units of core 2

	item									item									item							
unit	A	B	C	D	E	F	G	H	unit	A	B	C	D	E	F	G	H	unit	A	B	C	D	E	F	G	H
1	○	●	●	●	●	●	●	●	38	○	●	●	●	●	●	●	●	75	○	●	●	○	●	●	●	●
2	●	●	○	○	●	●	●	●	39	●	○	●	●	●	○	●	●	76	●	○	○	●	●	●	●	●
3	○	●	●	●	●	●	●	●	40	○	○	○	●	●	○	●	●	77	○	●	●	○	●	●	●	●
4	○	●	●	○	●	●	●	●	41	○	●	○	○	●	●	●	●	78	○	●	○	○	●	●	●	●
5	○	○	○	●	●	●	●	●	42	●	○	●	○	●	●	●	●	79	○	●	○	○	●	●	●	●
6	○	●	○	○	●	●	●	●	43	○	●	○	○	●	●	●	●	80	○	○	○	○	●	●	●	●
7	●	●	○	●	●	●	●	●	44	●	●	○	●	●	●	●	●	81	●	○	●	○	●	●	●	●
8	○	○	●	○	●	●	●	●	45	○	○	●	○	●	●	●	●	82	○	○	●	●	●	●	●	●
9	○	●	●	○	●	●	●	●	46	●	●	●	○	●	●	●	●	83	○	○	○	○	●	●	●	●
10	○	○	●	●	●	●	●	●	47	○	●	○	●	●	●	●	●	84	●	○	○	○	●	●	●	●
11	○	○	●	○	●	●	●	●	48	○	○	●	○	●	●	●	●	85	○	●	○	●	●	●	●	●
12	○	○	●	●	●	●	●	●	49	●	○	●	○	●	●	●	●	86	○	●	○	●	●	●	●	●
13	○	●	●	○	●	●	●	●	50	●	○	○	●	●	●	●	●	87	●	○	●	○	●	●	●	○
14	●	○	○	●	●	●	●	●	51	○	○	●	●	●	●	●	●	88	○	○	○	○	●	●	●	○
15	●	●	○	○	●	●	●	●	52	○	○	○	○	●	●	●	●	89	○	●	●	●	●	○	●	●
16	○	○	○	●	●	●	●	●	53	○	○	○	●	●	●	●	●	90	○	●	●	●	●	●	●	●
17	●	●	○	●	●	○	●	●	54	●	○	○	●	●	●	●	●	91	●	●	●	○	●	●	●	●
18	●	○	○	○	●	●	●	●	55	○	○	○	○	●	●	●	●	92	○	●	●	○	●	●	●	●
19	●	○	●	○	●	●	●	●	56	○	○	●	●	●	●	●	●	93	○	○	○	●	●	●	●	●
20	●	●	○	○	●	●	●	●	57	○	●	○	●	●	●	●	●	94	●	○	○	○	●	●	●	●
21	●	○	●	○	●	●	●	●	58	●	●	○	●	●	●	●	●	95	○	●	○	●	●	●	●	●
22	●	○	●	○	●	●	●	●	59	○	●	○	○	●	●	●	●	96	●	●	○	○	●	●	●	●
23	○	○	●	○	●	●	●	●	60	●	○	○	○	●	●	●	●	97	●	●	○	○	●	●	●	●
24	●	●	○	●	●	●	●	●	61	●	●	○	●	●	●	●	●	98	○	○	●	○	●	●	●	●
25	○	○	○	●	●	●	●	●	62	○	○	○	○	●	●	●	●	99	○	●	●	●	●	●	●	●
26	●	●	○	●	●	●	●	●	63	●	●	●	○	●	●	●	●	100	○	○	○	○	●	●	●	●
27	○	○	○	○	●	○	●	○	64	○	○	●	○	●	●	●	●	101	●	○	●	●	●	●	●	●
28	●	●	○	○	●	●	●	●	65	●	●	●	●	●	●	●	●	102	●	●	●	○	●	●	●	●
29	○	○	●	●	●	●	●	●	66	○	●	●	●	●	●	●	●	103	●	●	●	●	●	●	●	●
30	●	○	○	●	●	●	●	●	67	○	●	○	○	●	●	●	●	104	○	●	○	○	●	●	●	○
31	○	○	●	●	●	●	●	●	68	○	○	●	○	●	●	●	○	105	○	○	○	○	●	●	●	●
32	○	○	○	○	●	●	●	●	69	○	●	○	○	●	●	●	●	106	●	●	●	○	●	●	●	●
33	●	○	●	●	●	●	●	●	70	●	●	○	○	●	●	●	●	107	●	●	●	○	●	●	●	●
34	●	●	○	○	●	●	●	●	71	○	○	●	●	●	●	●	●	108	○	●	●	●	●	●	●	●
35	○	○	●	○	●	●	●	●	72	○	○	○	○	●	●	●	●	109	○	○	○	●	●	●	●	●
36	●	●	○	●	●	●	●	●	73	●	●	○	●	●	●	●	●	110	○	●	○	●	●	●	●	●
37	●	○	●	●	●	●	●	●	74	●	●	●	●	●	●	●	●	111	●	○	○	●	●	●	●	●

The symbols ●, ○, and × denote the condition of an item that allows distribution, recycling, and disposal, recycling and disposal, as well as disposal only, respectively.

Table C.17 Listing of incoming units of core 2 (cont.)

	item									item									item							
unit	A	B	C	D	E	F	G	H	unit	A	B	C	D	E	F	G	H	unit	A	B	C	D	E	F	G	H
112	●	●	●	●	●	●	●	●	148	●	○	○	○	●	●	●	●	184	●	○	○	○	●	●	●	●
113	●	●	●	●	●	●	●	●	149	●	○	●	●	●	●	●	○	185	○	○	●	○	●	●	●	●
114	●	○	●	○	●	●	●	●	150	○	●	●	●	●	●	●	●	186	●	○	○	○	●	●	●	●
115	○	○	●	●	●	●	●	●	151	○	●	○	○	●	●	●	●	187	○	○	○	○	●	●	○	●
116	○	●	○	○	●	●	●	●	152	●	○	○	○	●	●	●	●	188	●	○	●	○	●	●	●	●
117	○	●	○	●	●	○	●	●	153	○	○	○	●	●	●	●	○	189	●	○	●	●	●	●	●	●
118	○	○	○	○	●	●	●	●	154	●	●	●	○	●	●	●	●	190	●	○	○	○	●	●	○	●
119	○	●	●	○	●	●	●	●	155	○	○	○	●	●	●	●	●	191	○	●	●	○	●	●	●	●
120	○	○	○	○	●	●	●	●	156	○	○	○	●	●	●	●	●	192	●	○	○	●	●	○	●	●
121	●	○	●	○	●	●	●	●	157	○	○	○	○	●	●	●	●	193	○	●	●	○	●	●	●	○
122	●	○	○	○	●	●	●	●	158	○	○	○	●	●	●	●	●	194	○	○	○	●	●	●	●	●
123	●	○	○	○	●	○	●	●	159	●	●	●	○	●	●	●	●	195	○	○	○	○	○	●	●	●
124	●	●	○	○	●	●	●	●	160	●	●	○	●	●	●	●	●	196	●	○	○	●	●	●	●	●
125	●	●	○	●	●	●	●	○	161	●	○	○	●	●	●	●	●	197	○	○	○	○	●	●	●	●
126	●	●	○	○	●	●	●	●	162	●	○	○	○	●	●	●	●	198	○	○	●	○	●	●	●	●
127	○	●	○	○	●	●	●	●	163	●	○	●	○	●	●	●	●	199	●	●	●	●	●	●	●	●
128	○	○	●	○	●	●	●	●	164	○	●	●	○	●	●	●	●	200	○	●	●	○	●	●	●	●
129	●	○	○	○	●	●	●	●	165	○	●	●	●	●	●	●	○	201	○	●	○	●	●	●	●	●
130	●	○	○	○	●	●	●	●	166	○	○	●	○	●	●	●	●	202	●	●	●	●	●	●	●	●
131	○	●	●	○	●	●	●	●	167	●	●	●	○	●	●	●	●	203	○	●	●	○	●	●	●	○
132	○	○	●	●	●	○	●	●	168	○	○	●	●	●	●	●	●	204	●	●	○	○	●	●	●	●
133	●	●	○	○	●	●	●	○	169	○	●	○	●	●	●	●	●	205	○	●	○	○	●	○	●	●
134	●	○	●	●	●	●	●	●	170	●	●	○	●	●	●	●	●	206	○	●	○	○	●	●	●	●
135	●	●	●	○	●	○	●	●	171	○	●	○	●	●	●	●	●	207	●	○	●	●	●	●	●	●
136	●	○	○	●	●	●	●	●	172	○	○	○	●	●	●	●	●	208	○	○	○	●	●	●	●	●
137	○	○	○	●	●	●	●	●	173	○	●	○	○	●	●	●	●	209	○	●	●	●	●	●	●	●
138	○	●	●	●	●	●	●	●	174	○	●	○	●	●	●	●	●	210	●	○	●	●	●	●	●	●
139	●	○	○	●	●	●	●	●	175	●	○	○	○	○	●	●	●	211	○	○	●	●	●	●	●	●
140	●	○	○	●	●	●	●	●	176	○	●	○	○	●	●	●	●	212	●	○	●	●	●	●	●	●
141	●	●	●	○	●	●	●	●	177	○	○	○	○	●	●	●	●	213	○	○	○	○	●	●	●	●
142	○	○	○	○	●	●	●	●	178	○	●	○	●	●	●	●	●	214	●	○	○	○	●	●	●	●
143	○	○	○	○	●	●	●	●	179	●	○	○	●	●	●	●	●	215	●	○	●	●	●	●	●	●
144	○	○	●	○	●	●	●	●	180	○	○	●	○	●	●	●	●	216	●	○	●	●	●	●	●	●
145	○	●	○	●	●	●	●	●	181	●	○	●	○	●	●	●	●	217	○	○	●	●	●	●	●	●
146	●	○	○	○	●	●	●	○	182	○	○	○	●	●	●	●	●	218	●	○	○	○	●	●	●	●
147	●	●	●	●	●	●	●	●	183	○	○	○	○	●	●	●	●									

The symbols $\bullet$, $\circ$, and $\times$ denote the condition of an item that allows distribution, recycling, and disposal, recycling and disposal, as well as disposal only, respectively.

Table C.18 State assignment for core 2

	state priority $\pi^{\mathrm{S}}(s)$					selected unit	item usage u								module usage u			
	s						i								m			
unit	1	6	10	58	60	s	A	B	C	D	E	F	G	H	1	6	10	49
1	4.74	9.71	9.71	23.77	19.04	58	r	i	r	r			i	r				i
2	4.75	9.71	14.16	28.17	23.49	58	i	i	r	r			i	r				i
3	4.75	9.72	9.72	23.69	19.05	58	r	i	r	r			i	r				i
4	4.76	9.72	9.72	23.65	19.05	58	r	i	r	r			i	r				i
5	4.76	9.73	9.72	19.23	14.67	58	r	r	r	r			i	r				i
6	4.76	9.73	9.72	23.56	19.05	58	r	i	r	r			i	r				i
7	4.76	9.73	14.18	27.97	23.50	58	i	i	r	r			i	r				i
8	4.77	9.74	9.73	19.11	14.68	58	r	r	r	r			i	r				i
9	4.77	9.74	9.73	23.43	19.05	58	r	i	r	r			i	r				i
10	4.77	9.74	9.74	19.02	14.68	58	r	r	r	r			i	r				i
11	4.77	9.74	9.74	18.97	14.68	58	r	r	r	r			i	r				i
12	4.77	9.74	9.73	18.92	14.68	58	r	r	r	r			i	r				i
13	4.77	9.74	9.73	23.25	19.05	58	r	i	r	r			i	r				i
14	4.77	9.74	14.20	23.30	19.14	58	i	r	r	r			i	r				i
15	4.78	9.74	14.20	27.63	23.52	58	i	i	r	r			i	r				i
16	4.79	9.75	9.75	18.75	14.69	58	r	r	r	r			i	r				i
17	4.79	9.75	14.21	18.57	23.53	60	i	i	r	r	i	r	i	r				
18	4.79	9.75	14.21	23.16	19.15	58	i	r	r	r			i	r				i
19	4.79	9.76	14.21	23.12	19.15	58	i	r	r	r			i	r				i
20	4.80	9.76	14.21	27.44	23.53	58	i	i	r	r			i	r				i
21	4.81	9.77	14.22	23.03	19.16	58	i	r	r	r			i	r				i
22	4.81	9.77	14.22	22.98	19.16	58	i	r	r	r			i	r				i
23	4.82	9.78	9.78	18.49	14.72	58	r	r	r	r			i	r				i
24	4.81	9.78	14.22	27.26	23.54	58	i	i	r	r			i	r				i
25	4.82	9.79	9.78	18.39	14.73	58	r	r	r	r			i	r				i
26	4.82	9.79	14.23	27.16	23.55	58	i	i	r	r			i	r				i
27	4.83	9.80	9.79	9.78	14.73	60	r	r	r	r	i	r	i	r				
28	4.83	9.79	14.23	27.11	23.55	58	i	i	r	r			i	r				i
29	4.84	9.80	9.80	18.25	14.74	58	r	r	r	r			i	r				i
30	4.83	9.80	14.24	22.64	19.18	58	i	r	r	r			i	r				i
31	4.84	9.80	9.80	18.15	14.74	58	r	r	r	r			i	r				i
32	4.84	9.80	9.80	18.09	14.74	58	r	r	r	r			i	r				i
33	4.84	9.80	14.24	22.48	19.18	58	i	r	r	r			i	r				i
34	4.84	9.80	14.24	26.81	23.56	58	i	i	r	r			i	r				i
35	4.85	9.81	9.81	17.94	14.75	58	r	r	r	r			i	r				i
36	4.85	9.81	14.25	26.71	23.57	58	i	i	r	r			i	r				i
37	4.86	9.82	14.26	22.28	19.20	58	i	r	r	r			i	r				i
38	4.87	9.83	9.82	22.17	19.14	58	r	i	r	r			i	r				i
39	4.87	9.83	14.27	14.25	19.20	60	i	r	r	r	i	r	i	r				
40	4.87	9.83	9.83	9.81	14.76	60	r	r	r	r	i	r	i	r				

The values i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Table C.19 State assignment for core 2 (cont.)

	state priority $\pi^{\mathrm{S}}(s)$					selected unit	item usage u								module usage u			
	s						i								m			
unit	1	6	10	58	60	s	A	B	C	D	E	F	G	H	1	6	10	49
41	4.86	9.82	9.82	22.11	19.14	58	r	i	r	r			i	r				i
42	4.87	9.83	14.27	22.12	19.20	58	i	r	r	r			i	r				i
43	4.87	9.83	9.83	22.01	19.15	58	r	i	r	r			i	r				i
44	4.88	9.84	14.27	26.39	23.59	58	i	i	r	r			i	r				i
45	4.89	9.85	9.85	17.52	14.78	58	r	r	r	r			i	r				i
46	4.89	9.85	14.28	26.28	23.59	58	i	i	r	r			i	r				i
47	4.90	9.86	9.86	21.79	19.16	58	r	i	r	r			i	r				i
48	4.91	9.86	9.86	17.36	14.79	58	r	r	r	r			i	r				i
49	4.91	9.86	14.30	21.74	19.23	58	i	r	r	r			i	r				i
50	4.91	9.87	14.30	21.68	19.23	58	i	r	r	r			i	r				i
51	4.92	9.87	9.87	17.18	14.80	58	r	r	r	r			i	r				i
52	4.92	9.87	9.87	17.12	14.80	58	r	r	r	r			i	r				i
53	4.91	9.87	9.87	17.06	14.80	58	r	r	r	r			i	r				i
54	4.91	9.87	14.31	21.43	19.24	58	i	r	r	r			i	r				i
55	4.92	9.88	9.87	16.93	14.80	58	r	r	r	r			i	r				i
56	4.92	9.88	9.87	16.87	14.80	58	r	r	r	r			i	r				i
57	4.92	9.87	9.87	21.19	19.19	58	r	i	r	r			i	r				i
58	4.92	9.88	14.32	25.57	23.64	58	i	i	r	r			i	r				i
59	4.94	9.89	9.89	21.07	19.20	58	r	i	r	r			i	r				i
60	4.94	9.90	14.34	21.07	19.27	58	i	r	r	r			i	r				i
61	4.95	9.90	14.34	25.38	23.65	58	i	i	r	r			i	r				i
62	4.96	9.92	9.91	16.50	14.84	58	r	r	r	r			i	r				i
63	4.96	9.92	14.35	25.25	23.66	58	i	i	r	r			i	r				i
64	4.97	9.93	9.92	16.37	14.85	58	r	r	r	r			i	r				i
65	4.97	9.93	14.36	25.12	23.67	58	i	i	r	r			i	r				i
66	4.99	9.94	9.94	20.62	19.23	58	r	i	r	r			i	r				i
67	4.99	9.95	9.94	20.55	19.23	58	r	i	r	r			i	r				i
68	5.00	9.95	9.95	16.11	14.87	58	r	r	r	r			i	r				i
69	5.00	9.95	9.95	20.40	19.24	58	r	i	r	r			i	r				i
70	5.01	9.96	14.40	24.78	23.69	58	i	i	r	r			i	r				i
71	5.02	9.97	9.97	15.91	14.89	58	r	r	r	r			i	r				i
72	5.02	9.97	9.97	15.83	14.89	58	r	r	r	r			i	r				i
73	5.02	9.97	14.42	24.57	23.70	58	i	i	r	r			i	r				i
74	5.03	9.98	14.43	24.50	23.71	58	i	i	r	r			i	r				i
75	5.05	10.00	10.00	19.98	19.27	58	r	i	r	r			i	r				i
76	5.06	10.01	14.45	20.00	19.37	58	i	r	r	r			i	r				i
77	5.06	10.01	10.01	19.83	19.28	58	r	i	r	r			i	r				i
78	5.07	10.02	10.02	19.75	19.28	58	r	i	r	r			i	r				i
79	5.08	10.03	10.02	19.67	19.28	58	r	i	r	r			i	r				i
80	5.09	10.03	10.03	15.26	14.95	58	r	r	r	r			i	r				i

The values i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Table C.20 State assignment for core 2 (cont.)

	state priority $\pi^{S}(s)$					selected unit	item usage u								module usage u			
	s						i								m			
unit	1	6	10	58	60	s	A	B	C	D	E	F	G	H	1	6	10	49
81	5.09	10.04	14.49	19.64	19.40	58	i	r	r	r			i	r				i
82	5.10	10.04	10.04	15.10	14.95	58	r	r	r	r			i	r				i
83	5.10	10.04	10.04	15.02	14.95	58	r	r	r	r			i	r				i
84	5.10	10.04	14.50	19.39	19.41	60	i	r	r	r	i	r	i	r				
85	5.10	10.04	10.04	19.29	19.30	60	r	i	r	r	i	r	i	r				
86	5.10	10.04	10.04	19.29	19.30	60	r	i	r	r	i	r	i	r				
87	5.10	10.04	14.50	19.42	19.41	58	i	r	r	r			i	r				i
88	5.11	10.05	10.05	14.88	14.96	60	r	r	r	r	i	r	i	r				
89	5.10	10.05	10.04	14.36	19.30	60	r	i	r	r	i	r	i	r				
90	5.10	10.05	10.04	19.22	19.29	60	r	i	r	r	i	r	i	r				
91	5.10	10.05	14.51	23.69	23.75	60	i	i	r	r	i	r	i	r				
92	5.11	10.05	10.05	19.23	19.28	60	r	i	r	r	i	r	i	r				
93	5.11	10.05	10.05	14.91	14.96	60	r	r	r	r	i	r	i	r				
94	5.10	10.05	14.51	19.38	19.42	60	i	r	r	r	i	r	i	r				
95	5.11	10.05	10.04	19.24	19.27	60	r	i	r	r	i	r	i	r				
96	5.11	10.05	14.51	23.72	23.74	60	i	i	r	r	i	r	i	r				
97	5.12	10.06	14.52	23.72	23.74	60	i	i	r	r	i	r	i	r				
98	5.12	10.07	10.06	14.96	14.96	60	r	r	r	r	i	r	i	r				
99	5.12	10.06	10.05	19.27	19.26	58	r	i	r	r			i	r				i
100	5.13	10.07	10.06	14.87	14.96	60	r	r	r	r	i	r	i	r				
101	5.12	10.06	14.53	19.35	19.43	60	i	r	r	r	i	r	i	r				
102	5.12	10.06	14.52	23.66	23.73	60	i	i	r	r	i	r	i	r				
103	5.13	10.07	14.53	23.66	23.73	60	i	i	r	r	i	r	i	r				
104	5.14	10.08	10.07	19.21	19.27	60	r	i	r	r	i	r	i	r				
105	5.14	10.08	10.07	14.92	14.97	60	r	r	r	r	i	r	i	r				
106	5.13	10.07	14.53	23.68	23.72	60	i	i	r	r	i	r	i	r				
107	5.14	10.08	14.54	23.69	23.72	60	i	i	r	r	i	r	i	r				
108	5.15	10.09	10.08	19.23	19.25	60	r	i	r	r	i	r	i	r				
109	5.16	10.09	10.09	14.97	14.98	60	r	r	r	r	i	r	i	r				
110	5.15	10.08	10.08	19.24	19.24	60	r	i	r	r	i	r	i	r				
111	5.15	10.08	14.55	19.45	19.44	58	i	r	r	r			i	r				i
112	5.16	10.09	14.55	23.61	23.71	60	i	i	r	r	i	r	i	r				
113	5.17	10.10	14.56	23.62	23.70	60	i	i	r	r	i	r	i	r				
114	5.18	10.11	14.56	19.37	19.45	60	i	r	r	r	i	r	i	r				
115	5.18	10.11	10.11	14.93	15.00	60	r	r	r	r	i	r	i	r				
116	5.18	10.11	10.10	19.18	19.24	60	r	i	r	r	i	r	i	r				
117	5.18	10.11	10.10	14.32	19.23	60	r	i	r	r	i	r	i	r				
118	5.18	10.11	10.10	14.94	14.99	60	r	r	r	r	i	r	i	r				
119	5.17	10.10	10.09	19.18	19.22	60	r	i	r	r	i	r	i	r				
120	5.17	10.10	10.09	14.95	14.98	60	r	r	r	r	i	r	i	r				

The values i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Table C.21 State assignment for core 2 (cont.)

	state priority $\pi^{S}(s)$					selected unit	item usage u								module usage u			
	s						i								m			
unit	1	6	10	58	60	s	A	B	C	D	E	F	G	H	1	6	10	49
121	5.16	10.09	14.56	19.43	19.44	60	i	r	r	r	i	r	i	r				
122	5.17	10.09	14.56	19.43	19.44	60	i	r	r	r	i	r	i	r				
123	5.17	10.09	14.56	14.53	19.43	60	i	r	r	r	i	r	i	r				
124	5.17	10.09	14.55	23.68	23.67	58	i	i	r	r			i	r				i
125	5.19	10.12	14.57	23.56	23.68	60	i	i	r	r	i	r	i	r				
126	5.21	10.13	14.57	23.57	23.67	60	i	i	r	r	i	r	i	r				
127	5.22	10.14	10.13	19.13	19.22	60	r	i	r	r	i	r	i	r				
128	5.22	10.14	10.14	14.93	15.01	60	r	r	r	r	i	r	i	r				
129	5.21	10.13	14.58	19.38	19.45	60	i	r	r	r	i	r	i	r				
130	5.22	10.13	14.58	19.39	19.44	60	i	r	r	r	i	r	i	r				
131	5.22	10.14	10.13	19.16	19.21	60	r	i	r	r	i	r	i	r				
132	5.22	10.14	10.13	10.10	15.00	60	r	r	r	r	i	r	i	r				
133	5.21	10.13	14.57	23.61	23.64	60	i	i	r	r	i	r	i	r				
134	5.23	10.14	14.58	19.42	19.44	60	i	r	r	r	i	r	i	r				
135	5.23	10.14	14.57	18.74	23.63	60	i	i	r	r	i	r	i	r				
136	5.24	10.16	14.58	19.43	19.44	60	i	r	r	r	i	r	i	r				
137	5.25	10.16	10.15	15.01	15.01	58	r	r	r	r			i	r				i
138	5.25	10.16	10.15	19.07	19.20	60	r	i	r	r	i	r	i	r				
139	5.25	10.16	14.59	19.32	19.44	60	i	r	r	r	i	r	i	r				
140	5.26	10.16	14.59	19.33	19.44	60	i	r	r	r	i	r	i	r				
141	5.26	10.17	14.58	23.52	23.62	60	i	i	r	r	i	r	i	r				
142	5.28	10.18	10.17	14.94	15.02	60	r	r	r	r	i	r	i	r				
143	5.27	10.17	10.16	14.94	15.01	60	r	r	r	r	i	r	i	r				
144	5.26	10.16	10.15	14.94	14.99	60	r	r	r	r	i	r	i	r				
145	5.25	10.15	10.14	19.13	19.17	60	r	i	r	r	i	r	i	r				
146	5.25	10.15	14.58	19.39	19.42	60	i	r	r	r	i	r	i	r				
147	5.25	10.15	14.57	23.57	23.59	60	i	i	r	r	i	r	i	r				
148	5.27	10.17	14.58	19.41	19.41	60	i	r	r	r	i	r	i	r				
149	5.27	10.17	14.57	19.42	19.41	58	i	r	r	r			i	r				i
150	5.29	10.19	10.18	19.03	19.18	60	r	i	r	r	i	r	i	r				
151	5.30	10.19	10.18	19.03	19.16	60	r	i	r	r	i	r	i	r				
152	5.30	10.19	14.60	19.30	19.42	60	i	r	r	r	i	r	i	r				
153	5.31	10.20	10.19	14.90	15.01	60	r	r	r	r	i	r	i	r				
154	5.30	10.18	14.59	23.47	23.56	60	i	i	r	r	i	r	i	r				
155	5.32	10.20	10.19	14.93	15.01	60	r	r	r	r	i	r	i	r				
156	5.31	10.19	10.18	14.93	14.99	60	r	r	r	r	i	r	i	r				
157	5.29	10.18	10.17	14.93	14.98	60	r	r	r	r	i	r	i	r				
158	5.28	10.16	10.15	14.93	14.96	60	r	r	r	r	i	r	i	r				
159	5.27	10.15	14.57	23.52	23.53	60	i	i	r	r	i	r	i	r				
160	5.29	10.17	14.58	23.52	23.51	58	i	i	r	r			i	r				i

The values i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Table C.22 State assignment for core 2 (cont.)

	state priority $\pi^{\mathrm{S}}(s)$					selected unit	item usage u								module usage u			
	s						i								m			
unit	1	6	10	58	60	s	A	B	C	D	E	F	G	H	1	6	10	49
161	5.33	10.21	14.61	19.22	19.40	60	i	r	r	r	i	r	i	r				
162	5.34	10.21	14.60	19.22	19.39	60	i	r	r	r	i	r	i	r				
163	5.34	10.21	14.59	19.23	19.38	60	i	r	r	r	i	r	i	r				
164	5.35	10.22	10.21	18.99	19.12	60	r	i	r	r	i	r	i	r				
165	5.36	10.22	10.21	18.98	19.10	60	r	i	r	r	i	r	i	r				
166	5.36	10.23	10.21	14.89	14.99	60	r	r	r	r	i	r	i	r				
167	5.35	10.21	14.60	23.38	23.46	60	i	i	r	r	i	r	i	r				
168	5.38	10.23	10.22	14.93	14.99	60	r	r	r	r	i	r	i	r				
169	5.37	10.22	10.21	19.00	19.04	60	r	i	r	r	i	r	i	r				
170	5.37	10.22	14.62	23.40	23.42	60	i	i	r	r	i	r	i	r				
171	5.40	10.25	10.24	19.00	19.00	58	r	i	r	r			i	r				i
172	5.43	10.28	10.26	14.77	15.01	60	r	r	r	r	i	r	i	r				
173	5.42	10.26	10.25	18.75	18.97	60	r	i	r	r	i	r	i	r				
174	5.43	10.27	10.25	18.73	18.93	60	r	i	r	r	i	r	i	r				
175	5.44	$-\infty$	$-\infty$	14.63	$-\infty$	58	i	r	r	r			i	r				r
176	5.43	10.28	10.27	18.73	18.90	60	r	i	r	r	i	r	i	r				
177	5.44	10.29	10.27	14.84	14.99	60	r	r	r	r	i	r	i	r				
178	5.42	10.27	10.26	18.71	18.83	60	r	i	r	r	i	r	i	r				
179	5.43	10.28	14.71	19.31	19.41	60	i	r	r	r	i	r	i	r				
180	5.44	10.28	10.27	14.88	14.96	60	r	r	r	r	i	r	i	r				
181	5.43	10.26	14.70	19.33	19.38	60	i	r	r	r	i	r	i	r				
182	5.44	10.27	10.26	14.91	14.93	60	r	r	r	r	i	r	i	r				
183	5.42	10.25	10.23	14.91	14.90	58	r	r	r	r			i	r				i
184	5.44	10.26	14.70	19.04	19.35	60	i	r	r	r	i	r	i	r				
185	5.45	10.26	10.25	14.60	14.89	60	r	r	r	r	i	r	i	r				
186	5.43	10.24	14.68	19.04	19.31	60	i	r	r	r	i	r	i	r				
187	5.45	10.25	10.23	$-\infty$	$-\infty$	6					i					r		
188	5.39	$-\infty$	14.62	19.05	19.27	60	i	r	r	r	i	r	i	r				
189	5.41	$-\infty$	14.61	19.05	19.25	60	i	r	r	r	i	r	i	r				
190	5.42	$-\infty$	14.59	$-\infty$	$-\infty$	10	i				i						r	
191	5.40	$-\infty$	$-\infty$	18.62	18.75	60	r	i	r	r	i	r	i	r				
192	5.42	$-\infty$	$-\infty$	14.47	19.20	60	i	r	r	r	i	r	i	r				
193	5.43	$-\infty$	$-\infty$	18.59	18.68	60	r	i	r	r	i	r	i	r				
194	5.45	$-\infty$	$-\infty$	14.73	14.78	60	r	r	r	r	i	r	i	r				
195	5.43	$-\infty$	$-\infty$	10.04	$-\infty$	58	r	r	r	r			i	r				r
196	5.35	$-\infty$	$-\infty$	19.14	19.12	58	i	r	r	r			i	r				i
197	5.42	$-\infty$	$-\infty$	14.24	14.72	60	r	r	r	r	i	r	i	r				
198	5.39	$-\infty$	$-\infty$	14.20	14.65	60	r	r	r	r	i	r	i	r				
199	5.35	$-\infty$	$-\infty$	22.52	22.94	60	i	i	r	r	i	r	i	r				
200	5.43	$-\infty$	$-\infty$	18.04	18.43	60	r	i	r	r	i	r	i	r				

The values i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Table C.23 State assignment for core 2 (cont.)

	state priority $\pi^{\mathrm{S}}(s)$					selected unit	item usage u								module usage u			
	s						i								m			
unit	1	6	10	58	60	s	A	B	C	D	E	F	G	H	1	6	10	49
201	5.45	$-\infty$	$-\infty$	17.95	18.31	60	r	i	r	r	i	r	i	r				
202	5.48	$-\infty$	$-\infty$	22.31	22.62	60	i	i	r	r	i	r	i	r				
203	5.58	$-\infty$	$-\infty$	17.75	18.02	60	r	i	r	r	i	r	i	r				
204	5.62	$-\infty$	$-\infty$	22.02	22.24	60	i	i	r	r	i	r	i	r				
205	5.74	$-\infty$	$-\infty$	$-\infty$	17.49	60	r	i	r	r	i	r	i	r				
206	5.80	$-\infty$	$-\infty$	16.82	16.97	60	r	i	r	r	i	r	i	r				
207	5.87	$-\infty$	$-\infty$	18.99	19.07	60	i	r	r	r	i	r	i	r				
208	5.95	$-\infty$	$-\infty$	14.52	14.51	58	r	r	r	r			i	r				i
209	6.05	$-\infty$	$-\infty$	$-\infty$	15.95	60	r	i	r	r	i	r	i	r				
210	6.17	$-\infty$	$-\infty$	$-\infty$	18.99	60	i	r	r	r	i	r	i	r				
211	6.33	$-\infty$	$-\infty$	$-\infty$	14.33	60	r	r	r	r	i	r	i	r				
212	6.37	$-\infty$	$-\infty$	$-\infty$	18.70	60	i	r	r	r	i	r	i	r				
213	6.61	$-\infty$	$-\infty$	$-\infty$	13.92	60	r	r	r	r	i	r	i	r				
214	6.72	$-\infty$	$-\infty$	$-\infty$	18.18	60	i	r	r	r	i	r	i	r				
215	7.19	$-\infty$	$-\infty$	$-\infty$	17.69	60	i	r	r	r	i	r	i	r				
216	8.00	$-\infty$	$-\infty$	$-\infty$	16.85	60	i	r	r	r	i	r	i	r				
217	9.75	$-\infty$	$-\infty$	$-\infty$	$-\infty$	1									r			
218	$-\infty$	$-\infty$	$-\infty$	$-\infty$	21.74	60	i	r	r	r	i	r	i	r				

The values i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

Table C.24 Listing of incoming units of core 3

	item									item									item							
unit	A	B	C	D	E	F	G	H	unit	A	B	C	D	E	F	G	H	unit	A	B	C	D	E	F	G	H
1	●	●	●	●	●	●	●	●	12	○	○	○	○	●	●	●	●	23	●	●	○	●	●	●	●	●
2	○	○	○	○	●	●	●	●	13	○	○	○	○	●	●	●	●	24	○	○	×	×	○	○	●	○
3	●	●	●	○	●	●	●	●	14	○	○	●	○	●	●	●	●	25	●	○	○	○	●	●	●	●
4	●	●	○	○	●	●	●	●	15	○	●	○	●	●	●	●	●	26	●	○	●	○	●	●	●	●
5	○	●	●	○	●	●	●	●	16	○	○	○	●	●	●	●	●	27	●	●	○	○	●	●	●	●
6	●	○	●	○	●	●	●	●	17	○	○	●	●	●	●	●	●	28	○	●	○	●	●	●	●	●
7	○	○	○	●	●	●	●	●	18	○	○	●	●	●	●	●	●	29	○	●	●	○	●	●	●	●
8	○	●	○	○	●	●	●	●	19	○	●	○	○	●	●	●	●	30	○	○	●	○	●	●	●	●
9	○	●	●	●	●	●	●	●	20	●	○	○	○	●	●	●	●	31	○	○	○	○	●	○	●	●
10	●	●	○	○	●	●	●	●	21	●	○	○	●	●	●	●	●									
11	●	○	●	●	●	●	●	●	22	●	○	○	●	●	●	●	●									

The symbols •, ○, and × denote the condition of an item that allows distribution, recycling, and disposal, recycling and disposal, as well as disposal only, respectively.

Table C.25 State assignment for core 3

	state priority $\pi^{\mathrm{S}}(s)$		selected unit	item usage u						module usage u	
	s			i						m	
unit	15	56	s	A	B	C	D	E	F	14	47
1	5.27	32.68	56	i	i	r	r	r	r		i
2	5.31	24.54	56	r	r	r	r	r	r		i
3	5.29	32.59	56	i	i	r	r	r	r		i
4	5.33	32.50	56	i	i	r	r	r	r		i
5	5.39	28.47	56	r	i	r	r	r	r		i
6	5.40	28.46	56	i	r	r	r	r	r		i
7	5.42	24.49	56	r	r	r	r	r	r		i
8	5.39	28.33	56	r	i	r	r	r	r		i
9	5.41	28.22	56	r	i	r	r	r	r		i
10	5.43	32.03	56	i	i	r	r	r	r		i
11	5.50	28.24	56	i	r	r	r	r	r		i
12	5.53	24.36	56	r	r	r	r	r	r		i
13	5.50	24.28	56	r	r	r	r	r	r		i
14	5.47	24.18	56	r	r	r	r	r	r		i
15	5.44	27.74	56	r	i	r	r	r	r		i
16	5.47	24.01	56	r	r	r	r	r	r		i
17	5.43	23.88	56	r	r	r	r	r	r		i
18	5.38	23.73	56	r	r	r	r	r	r		i
19	5.33	27.14	56	r	i	r	r	r	r		i
20	5.36	27.41	56	i	r	r	r	r	r		i
21	5.40	27.14	56	i	r	r	r	r	r		i
22	5.44	26.81	56	i	r	r	r	r	r		i
23	5.49	29.86	56	i	i	r	r	r	r		i
24	3.94	$-\infty$	15			d	d			r	
25	$-\infty$	27.19	56	i	r	r	r	r	r		i
26	$-\infty$	26.54	56	i	r	r	r	r	r		i
27	$-\infty$	28.80	56	i	i	r	r	r	r		i
28	$-\infty$	26.53	56	r	i	r	r	r	r		i
29	$-\infty$	25.27	56	r	i	r	r	r	r		i
30	$-\infty$	23.07	56	r	r	r	r	r	r		i
31	$-\infty$	22.61	56	r	r	r	r	r	r		i

The values i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively. Values are rounded to two digits. Values with less than two post decimal digits indicate an exact value without rounding being necessary.

C.9 Incoming units and their assignment with LP for cores 2 and 3

The state and usage assignment for the cores 2 and 3 are listed in the sequel. Thereby, the order of incoming units is identical to the ones in Tables C.16 and C.24.

In total, one feasible state assignment exists for core 3. For core 2 approximately $6.26 \cdot 10^{59}$ feasible state assignments exist. The estimated number of assignments for core 2 is based on the assumption, that state 58 and 60 can more or less be substituted, because items E and F are very often in a condition for distribution (205 units of the required 215 units). 90 times the module $m = 49$ is required for distribution so that 115 times state 60 can be selected of the 205 times, where item E and F are in a distributable condition. With these 90 (state 58) and 115 (state 60) times $\frac{(90+115)!}{90! \cdot 115!}$ permutations are possible. Note that all other items, modules, and states are neglected for this estimation.

Table C.26 State assignment for core 2 using LP

	sel. state	item usage u								mod. usage u					sel. state	item usage u								mod. usage u			
		i								m						i								m			
unit	s	A	B	C	D	E	F	G	H	1	6	10	49	unit	s	A	B	C	D	E	F	G	H	1	6	10	49
1	58	r	i	r	r			i	r				i	21	58	i	r	r	r			i	r				i
2	60	i	i	r	r	i	r	i	r					22	60	i	r	r	r	i	r	i	r				
3	58	r	i	r	r			i	r				i	23	60	r	r	r	r	i	r	i	r				
4	60	r	i	r	r	i	r	i	r					24	58	i	i	r	r			i	r				i
5	58	r	r	r	r			i	r				i	25	58	r	r	r	r			i	r				i
6	60	r	i	r	r	i	r	i	r					26	60	i	i	r	r	i	r	i	r				
7	60	i	i	r	r	i	r	i	r					27	60	r	r	r	r	i	r	i	r				
8	58	r	r	r	r			i	r				i	28	58	i	i	r	r			i	r				i
9	58	r	i	r	r			i	r				i	29	60	r	r	r	r	i	r	i	r				
10	60	r	r	r	r	i	r	i	r					30	58	i	r	r	r			i	r				i
11	58	r	r	r	r			i	r				i	31	58	r	r	r	r			i	r				i
12	60	r	r	r	r	i	r	i	r					32	60	r	r	r	r	i	r	i	r				
13	60	r	i	r	r	i	r	i	r					33	58	i	r	r	r			i	r				i
14	58	i	r	r	r			i	r				i	34	60	i	i	r	r	i	r	i	r				
15	58	i	i	r	r			i	r				i	35	60	r	r	r	r	i	r	i	r				
16	60	r	r	r	r	i	r	i	r					36	60	i	i	r	r	i	r	i	r				
17	60	i	i	r	r	i	r	i	r					37	58	i	r	r	r			i	r				i
18	60	i	r	r	r	i	r	i	r					38	60	r	i	r	r	i	r	i	r				
19	58	i	r	r	r			i	r				i	39	60	i	r	r	r	i	r	i	r				
20	60	i	i	r	r	i	r	i	r					40	60	r	r	r	r	i	r	i	r				

The i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively.

Table C.27 State assignment for core 2 using LP (cont.)

	sel. state	item usage u								mod. usage u			
		i								m			
unit	s	A	B	C	D	E	F	G	H	1	6	10	49
41	60	r	i	r	r	i	r	i	r				
42	60	i	r	r	r	i	r	i	r				
43	58	r	i	r	r			i	r				i
44	60	i	i	r	r	i	r	i	r				
45	60	r	r	r	r	i	r	i	r				
46	60	i	i	r	r	i	r	i	r				
47	60	r	i	r	r	i	r	i	r				
48	60	r	r	r	r	i	r	i	r				
49	58	i	r	r	r			i	r				i
50	58	i	r	r	r			i	r				i
51	60	r	r	r	r	i	r	i	r				
52	58	r	r	r	r			i	r				i
53	60	r	r	r	r	i	r	i	r				
54	60	i	r	r	r	i	r	i	r				
55	60	r	r	r	r	i	r	i	r				
56	60	r	r	r	r	i	r	i	r				
57	58	r	i	r	r			i	r				i
58	60	i	i	r	r	i	r	i	r				
59	60	r	i	r	r	i	r	i	r				
60	58	i	r	r	r			i	r				i
61	60	i	i	r	r	i	r	i	r				
62	58	r	r	r	r			i	r				i
63	60	i	i	r	r	i	r	i	r				
64	58	r	r	r	r			i	r				i
65	60	i	i	r	r	i	r	i	r				
66	58	r	i	r	r			i	r				i
67	60	r	i	r	r	i	r	i	r				
68	58	r	r	r	r			i	r				i
69	60	r	i	r	r	i	r	i	r				
70	60	i	i	r	r	i	r	i	r				
71	60	r	r	r	r	i	r	i	r				
72	60	r	r	r	r	i	r	i	r				
73	58	i	i	r	r			i	r				i
74	60	i	i	r	r	i	r	i	r				
75	58	r	i	r	r			i	r				i
76	60	i	r	r	r	i	r	i	r				
77	60	r	i	r	r	i	r	i	r				
78	58	r	i	r	r			i	r				i
79	60	r	i	r	r	i	r	i	r				
80	58	r	r	r	r			i	r				i
81	60	i	r	r	r	i	r	i	r				
82	58	r	r	r	r			i	r				i
83	58	r	r	r	r			i	r				i
84	60	i	r	r	r	i	r	i	r				
85	60	r	i	r	r	i	r	i	r				
86	58	r	i	r	r			i	r				i
87	60	i	r	r	r	i	r	i	r				
88	60	r	r	r	r	i	r	i	r				
89	60	r	i	r	r	i	r	i	r				
90	60	r	i	r	r	i	r	i	r				
91	60	i	i	r	r	i	r	i	r				
92	58	r	i	r	r			i	r				i
93	58	r	r	r	r			i	r				i
94	60	i	r	r	r	i	r	i	r				
95	60	r	i	r	r	i	r	i	r				
96	60	i	i	r	r	i	r	i	r				
97	60	i	i	r	r	i	r	i	r				
98	60	r	r	r	r	i	r	i	r				
99	58	r	i	r	r			i	r				i
100	58	r	r	r	r			i	r				i
101	58	i	r	r	r			i	r				i
102	58	i	i	r	r			i	r				i
103	58	i	i	r	r			i	r				i
104	60	r	i	r	r	i	r	i	r				
105	60	r	r	r	r	i	r	i	r				
106	60	i	i	r	r	i	r	i	r				
107	58	i	i	r	r			i	r				i
108	58	r	i	r	r			i	r				i
109	58	r	r	r	r			i	r				i
110	58	r	i	r	r			i	r				i
111	60	i	r	r	r	i	r	i	r				
112	58	i	i	r	r			i	r				i
113	58	i	i	r	r			i	r				i
114	58	i	r	r	r			i	r				i
115	60	r	r	r	r	i	r	i	r				
116	58	r	i	r	r			i	r				i
117	60	r	i	r	r	i	r	i	r				
118	60	r	r	r	r	i	r	i	r				
119	60	r	i	r	r	i	r	i	r				
120	58	r	r	r	r			i	r				i
121	58	i	r	r	r			i	r				i
122	60	i	r	r	r	i	r	i	r				
123	60	i	r	r	r	i	r	i	r				
124	60	i	i	r	r	i	r	i	r				
125	60	i	i	r	r	i	r	i	r				
126	60	i	i	r	r	i	r	i	r				
127	58	r	i	r	r			i	r				i
128	60	r	r	r	r	i	r	i	r				
129	60	i	r	r	r	i	r	i	r				
130	60	i	r	r	r	i	r	i	r				

The i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively.

Table C.28 State assignment for core 2 using LP (cont.)

	sel. state	item usage u								mod. usage u					sel. state	item usage u								mod. usage u			
		i								m						i								m			
unit	s	A	B	C	D	E	F	G	H	1	6	10	49	unit	s	A	B	C	D	E	F	G	H	1	6	10	49
131	60	r	i	r	r	i	r	i	r					175	58	i	r	r	r			i	r				r
132	58	r	r	r	r			i	r				r	176	58	r	i	r	r			i	r				i
133	58	i	i	r	r			i	r				i	177	60	r	r	r	r	i	r	i	r				
134	60	i	r	r	r	i	r	i	r					178	60	r	i	r	r	i	r	i	r				
135	60	i	i	r	r	i	r	i	r					179	60	i	r	r	r	i	r	i	r				
136	58	i	r	r	r			i	r				i	180	60	r	r	r	r	i	r	i	r				
137	58	r	r	r	r			i	r				i	181	58	i	r	r	r			i	r				i
138	58	r	i	r	r			i	r				i	182	58	r	r	r	r			i	r				i
139	58	i	r	r	r			i	r				i	183	60	r	r	r	r	i	r	i	r				
140	60	i	r	r	r	i	r	i	r					184	60	i	r	r	r	i	r	i	r				
141	58	i	i	r	r			i	r				i	185	60	r	r	r	r	i	r	i	r				
142	58	r	r	r	r			i	r				i	186	60	i	r	r	r	i	r	i	r				
143	58	r	r	r	r			i	r				i	187	6					i					r		
144	60	r	r	r	r	i	r	i	r					188	60	i	r	r	r	i	r	i	r				
145	60	r	i	r	r	i	r	i	r					189	58	i	r	r	r			i	r				i
146	58	i	r	r	r			i	r				i	190	10	i				i						r	
147	58	i	i	r	r			i	r				i	191	60	r	i	r	r	i	r	i	r				
148	58	i	r	r	r			i	r				i	192	60	i	r	r	r	i	r	i	r				
149	58	i	r	r	r			i	r				i	193	58	r	i	r	r			i	r				i
150	58	r	i	r	r			i	r				i	194	58	r	r	r	r			i	r				i
151	60	r	i	r	r	i	r	i	r					195	1									r			
152	60	i	r	r	r	i	r	i	r					196	60	i	r	r	r	i	r	i	r				
153	58	r	r	r	r			i	r				i	197	58	r	r	r	r			i	r				i
154	60	i	i	r	r	i	r	i	r					198	60	r	r	r	r	i	r	i	r				
155	58	r	r	r	r			i	r				i	199	60	i	i	r	r	i	r	i	r				
156	60	r	r	r	r	i	r	i	r					200	58	r	i	r	r			i	r				i
157	60	r	r	r	r	i	r	i	r					201	60	r	i	r	r	i	r	i	r				
158	60	r	r	r	r	i	r	i	r					202	58	i	i	r	r			i	r				i
159	58	i	i	r	r			i	r				i	203	60	r	i	r	r	i	r	i	r				
160	60	i	i	r	r	i	r	i	r					204	58	i	i	r	r			i	r				i
161	60	i	r	r	r	i	r	i	r					205	60	r	i	r	r	i	r	i	r				
162	58	i	r	r	r			i	r				i	206	58	r	i	r	r			i	r				i
163	58	i	r	r	r			i	r				i	207	58	i	r	r	r			i	r				i
164	58	r	i	r	r			i	r				i	208	58	r	r	r	r			i	r				i
165	60	r	i	r	r	i	r	i	r					209	60	r	i	r	r	i	r	i	r				
166	60	r	r	r	r	i	r	i	r					210	60	i	r	r	r	i	r	i	r				
167	58	i	i	r	r			i	r				i	211	60	r	r	r	r	i	r	i	r				
168	60	r	r	r	r	i	r	i	r					212	58	i	r	r	r			i	r				i
169	58	r	i	r	r			i	r				i	213	60	r	r	r	r	i	r	i	r				
170	60	i	i	r	r	i	r	i	r					214	58	i	r	r	r			i	r				i
171	60	r	i	r	r	i	r	i	r					215	60	i	r	r	r	i	r	i	r				
172	58	r	r	r	r			i	r				i	216	60	i	r	r	r	i	r	i	r				
173	60	r	i	r	r	i	r	i	r					217	60	r	r	r	r	i	r	i	r				
174	58	r	i	r	r			i	r				i	218	60	i	r	r	r	i	r	i	r				

The i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively.

Table C.29 State assignment for core 3

unit	sel. state s	item usage u						mod. usage u	
		i						m	
		A	B	C	D	E	F	14	47
1	56	i	i	r	r	r	r		i
2	56	r	r	r	r	r	r		i
3	56	i	i	r	r	r	r		i
4	56	i	i	r	r	r	r		i
5	56	r	i	r	r	r	r		i
6	56	i	r	r	r	r	r		i
7	56	r	r	r	r	r	r		i
8	56	r	i	r	r	r	r		i
9	56	r	i	r	r	r	r		i
10	56	i	i	r	r	r	r		i
11	56	i	r	r	r	r	r		i
12	56	r	r	r	r	r	r		i
13	56	r	r	r	r	r	r		i
14	56	r	r	r	r	r	r		i
15	56	r	i	r	r	r	r		i
16	56	r	r	r	r	r	r		i
17	56	r	r	r	r	r	r		i
18	56	r	r	r	r	r	r		i
19	56	r	i	r	r	r	r		i
20	56	i	r	r	r	r	r		i
21	56	i	r	r	r	r	r		i
22	56	i	r	r	r	r	r		i
23	56	i	i	r	r	r	r		i
24	15			d	d			r	
25	56	i	r	r	r	r	r		i
26	56	i	r	r	r	r	r		i
27	56	i	i	r	r	r	r		i
28	56	r	i	r	r	r	r		i
29	56	r	i	r	r	r	r		i
30	56	r	r	r	r	r	r		i
31	56	r	r	r	r	r	r		i

The i, r, and d denote the usage of an item/module for distribution, recycling, and disposal, respectively.

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